03-03-26 MTH261

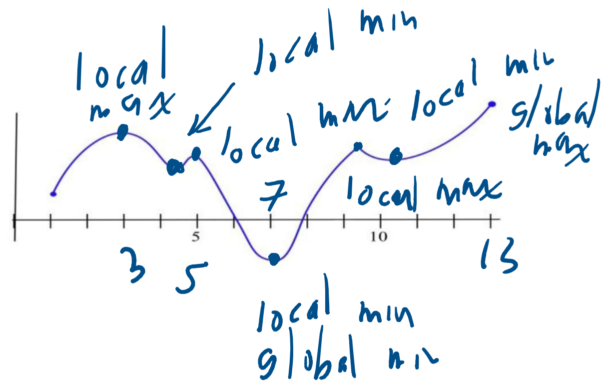
Section 2.7: Optimization
page 139: 1, 3, 7, 9, 11, 12, 13, 15, 2

Section 2.8: Curve Sketching
page 146: 1, 11, 13

2.7: 1

2.7 Exercises

1. Find all of the critical points of the function shown and identify them as local max, local min, or neither. Find the global max and min on the interval.



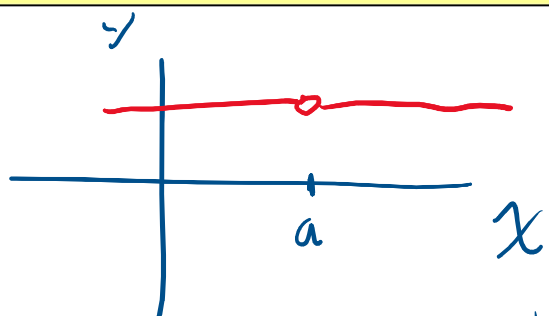
2.8
Memorize

Definitions: The function f is **increasing on (a,b)** if $a < x_1 < x_2 < b$ implies $f(x_1) < f(x_2)$.
The function f is **decreasing on (a,b)** if $a < x_1 < x_2 < b$ implies $f(x_1) > f(x_2)$.

Memorize

First Derivative Information about Shape

For a function f which is differentiable on an interval (a,b);
(i) if f is increasing on (a,b), then $f'(x) \geq 0$ for all x in (a,b)
(ii) if f is decreasing on (a,b), then $f'(x) \leq 0$ for all x in (a,b)
(iii) if f is constant on (a,b), then $f'(x) = 0$ for all x in (a,b).



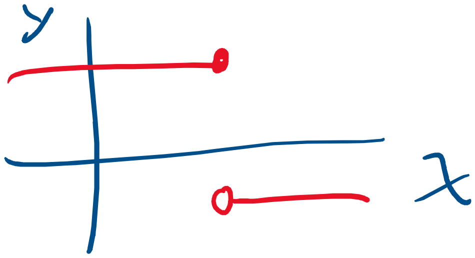
$f(x) = \begin{cases} b & x \neq a \\ \text{not defined} & \text{if } x = a \end{cases}$

$f'(x)$ dne at $x = a$

so $f'(x) = 0$ all x is not true

... constant on its domain

so $f'(x)=0$ all x is not true
but $f(x)$ is constant on its domain



$f'(x)=0$ on its domain
but $f(x) \neq$ constant

If $f'(x) \geq 0$ on (a, b)
does that imply that $f(x)$ is increasing on (a, b) ?

If f'(x)=0 then also $f'(x) \geq 0$, but now f(x) is constant, not increasing.

Memorize

First Derivative Information about Shape (part 2)

For a function f which is differentiable on an interval I;

(i) if $f'(x) > 0$ for all x in the interval I, then f is increasing on I,
(ii) if $f'(x) < 0$ for all x in the interval I, then f is decreasing on I,
(iii) if $f'(x) = 0$ for all x in the interval I, then f is constant on I.

Memorize

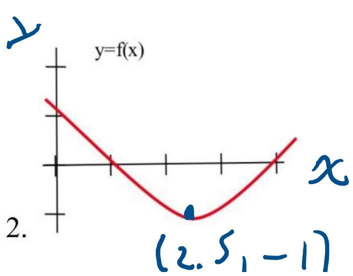
Second Derivative Information about Shape

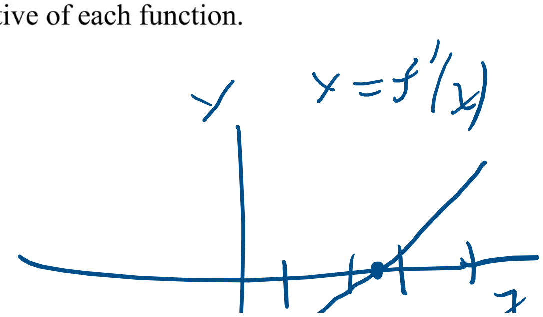
(i) if f is concave up on (a,b), then $f''(x) \geq 0$ for all x in (a,b)
(ii) if f is concave down on (a,b), then $f''(x) \leq 0$ for all x in (a,b)

The converse of both of these are also true:
(i) if $f''(x) \geq 0$ for all x in (a,b), then f is concave up on (a,b)
(ii) if $f''(x) \leq 0$ for all x in (a,b), then f is concave down on (a,b)

2.8

In problems 2–4, sketch the graph of the derivative of each function.

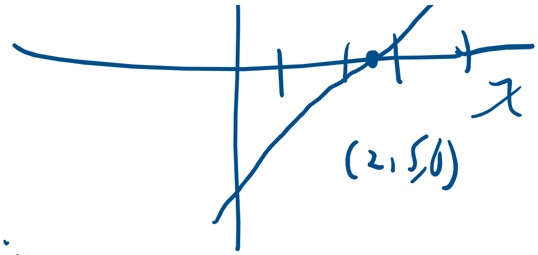
2.




2. 

(2.5, −1)
local
and global min

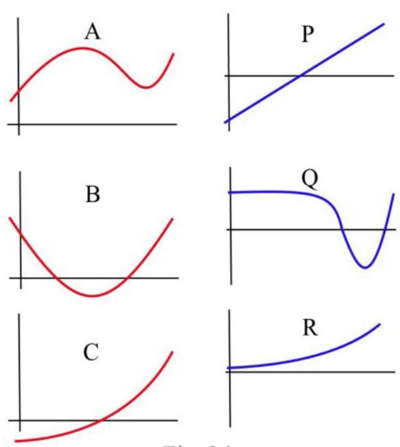
concave up on domain
decreasing on (−.1, 2.5)
increasing on (2.5, 4.2)



$f''(x) > 0$

8. In the graphs to the right, match the graphs of the functions with those of their derivatives

Functions f Derivatives f'



A B C
P R Q
Q R P
Q P R
Q P R

S?

In problems 10 – 14 , use information from the derivatives of each function to help you graph the function. Find all local maximums and minimums of each function.

12. $h(x) = x^4 - 8x^2 + 3$

Give intervals where h(x) is increasing and decreasing.
Give intervals where h(x) is concave up and concave down.
Find any inflection point(s) or say they don't exist.

$$h'(x) = 4x^3 - 16x$$
$$h''(x) = 12x^2 - 16$$

$h'(x)$ is defined for all x
set $h'(x) = 0$ and solve for x

$$4x^3 - 16x = 0$$

$$x^3 - 4x = 0$$

$4x^3 - 16x = 0$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$\boxed{x = 0}$ $\quad x^2 - 4 = 0$

critical points

$x^2 = 4$

$\boxed{x = \pm 2}$

$h''(0) = 12(0^2) - 16 = -16 < 0$

$\boxed{\begin{array}{l} \text{local max at } x = 0 \\ \qquad h(0) = 3 \end{array}}$

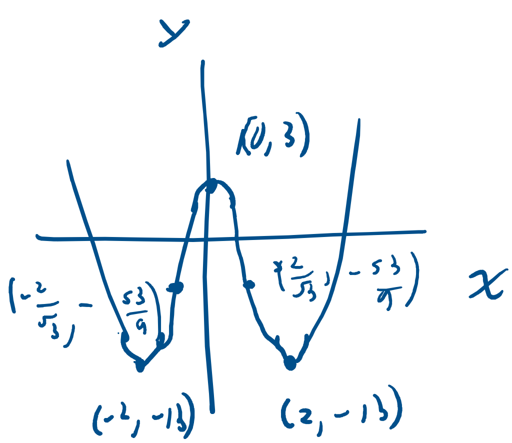
$h''(\pm 2) = 12(\pm 2)^2 - 16$

$= 48 - 16 = 32 > 0$

local min at $x = \pm 2$

$h(\pm 2) = (\pm 2)^4 - 8(\pm 2)^2 + 3$

$= 16 - 32 + 3 = -13$



$h''(x) = 0$

$12x^2 - 16 = 0$

$12x^2 = 16$

$x^2 = \frac{16}{12} = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$h\left(\pm \frac{2}{\sqrt{3}}\right) = \left(\pm \frac{2}{\sqrt{3}}\right)^4 - 8\left(\pm \frac{2}{\sqrt{3}}\right)^2 + 3$

$= \frac{16}{9} - 8\left(\frac{4}{3}\right) + 3$

$= \frac{16}{9} - \frac{96}{9} + \frac{27}{9} = \frac{43 - 96}{9}$

$= \boxed{-\frac{53}{9}}$

$= \boxed{-\frac{53}{9}}$

2/sqrt(3)=1.154700538379252
-53/9=-5.8889

concave up $f''(x) > 0$

$12x^2 - 16 > 0$

$12x^2 > 16$

$x^2 > \frac{4}{3}$

$|x| > \frac{2}{\sqrt{3}}$

$(-\infty, -\frac{2}{\sqrt{3}}), (\frac{2}{\sqrt{3}}, \infty)$

concave down

$|x| < \frac{2}{\sqrt{3}}$

$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

$h(x)$ increasing

$h'(x) > 0$

$4x^3 - 16x > 0$

$x^3 - 4x > 0$

$x(x^2 - 4) > 0$

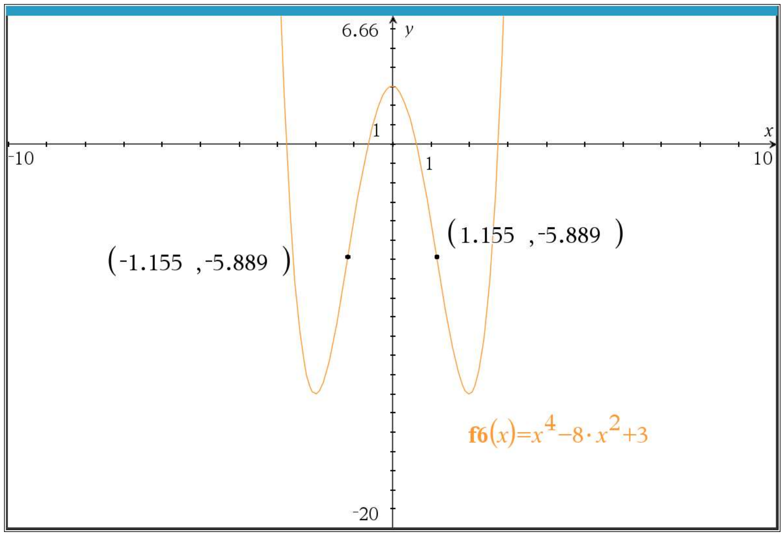
$x(x+2)(x-2) > 0$

solve this inequality

$h(x)$ decreasing

$x(x+2)(x-2) < 0$

solve this inequality



This graph agrees with our calculations and sketch.