02-26-26 MTH 261

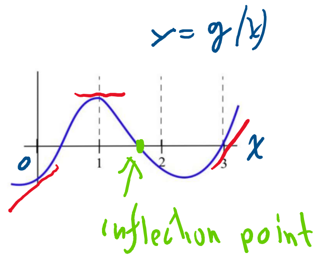
Chapter 2
Section 6: Second Derivative and Concavity
page 126: 1, 5, 15 17, 21

Section 7: Optimization
page 139: 1, 3, 7, 9, 11, 12, 13, 15, 23

2.6:15

15. Fill in the table with "+", "–", or "0" for the function shown.

x	g(x)	g '(x)	g "(x)
0	−	+	+
1	+	0	−
2	−	−	?
3	0	+	+



2.6: 21

In problems 16 – 22 , find the derivative and second derivative of each function.

21. $f(x) = \sqrt{x^2 + 6x - 1}$

$$f(x) = (x^2 + 6x - 1)^{\frac{1}{2}}$$

$$f'(x) = \left(\frac{1}{2}\right)(x^2 + 6x - 1)^{-\frac{1}{2}}(2x + 6)$$

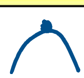
$$f'(x) = \frac{x+3}{\sqrt{x^2+6x-1}} = (x+3)(x^2+6x-1)^{-\frac{1}{2}}$$

2.7
Memorize

Definitions: f has a **local maximum** at a if $f(a) \geq f(x)$ for all x near a
f has a **local minimum** at a if $f(a) \leq f(x)$ for all x near a
f has a **local extreme** at a if f(a) is a **local maximum or minimum.**
The plurals of these are maxima and minima. We often simply say "max" or "min;" it saves a lot of syllables.
Some books say "relative" instead of "local."
The process of finding maxima or minima is called **optimization.**

A point is a local max (or min) if it is higher (lower) than all the **nearby points**. These points come from the shape of the graph.

Local max = top of small hill



Local min = bottom of small valley

Memorize

Definitions: f has a **global maximum** at a if $f(a) \geq f(x)$ for all x in the domain of f.
f has a **global minimum** at a if $f(a) \leq f(x)$ for all x in the domain of f.
f has a **global extreme** at a if f(a) is a **global maximum or minimum.**
Some books say "absolute" instead of "global"

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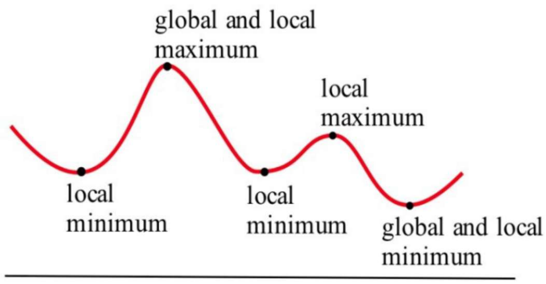
A point is a global max (or min) if it is higher (lower) than every point on the graph. These points come from the shape of the graph **and** the window through which we view the graph.



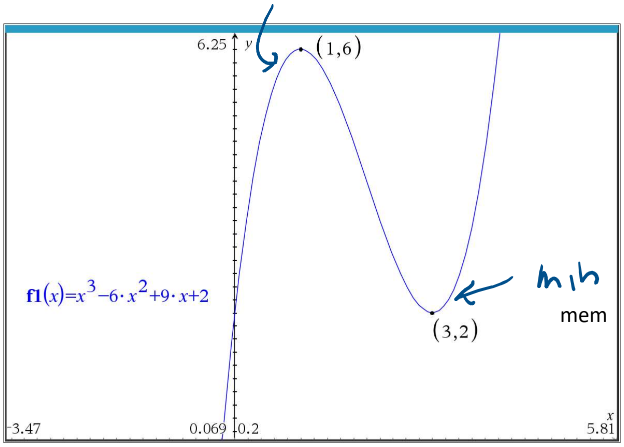
Memorize

Definition: A **critical number** for a function f is a value x = a in the domain of f where either **$f'(a) = 0$** or **$f'(a)$ is undefined.**

Definition: A **critical point** for a function f is a point (a, f(a)) where a is a critical number of f.

Useful Fact: A local max or min of f can only occur at a critical point.

2nd calc - max - left - right - guess

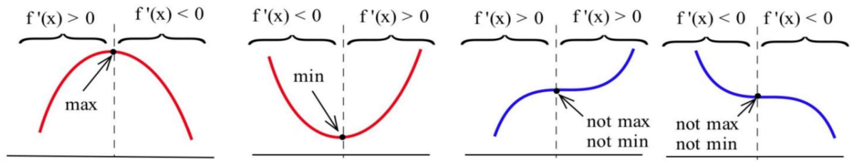


Memorize

The First Derivative Test for Extremes:
Find the critical points of f.

For each critical number c, examine the sign of f' to the left and to the right of c. What happens to the sign as you move from left to right?

If f '(x) changes from positive to negative at x = c, then f has a local **maximum** at (c, f(c)).
If f '(x) changes from negative to positive at x = c, then f has a local **minimum** at (c, f(c)).
If f '(x) does not change sign at x = c, then (c, f(c)) is **neither** a local max nor a local min.

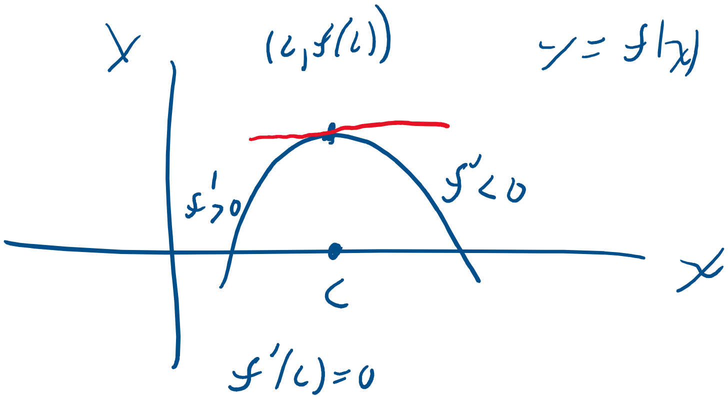


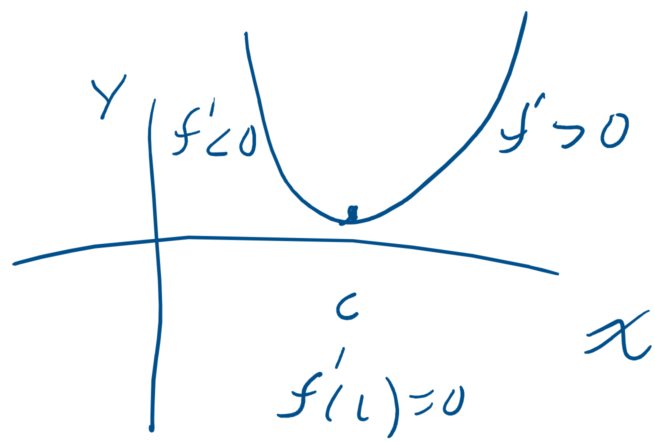
Memorize

The Second Derivative Test for Extremes:

Find all critical points of f. For those critical points where f'(c) = 0, find f''(c).

(a) If f''(c) < 0 then f is concave down and has a local maximum at x = c.
(b) If f''(c) > 0 then f is concave up and has a local minimum at x = c.
(c) If f''(c) = 0 then f may have a local maximum, a minimum or neither at x = c.





f' goes from + to 0 to −

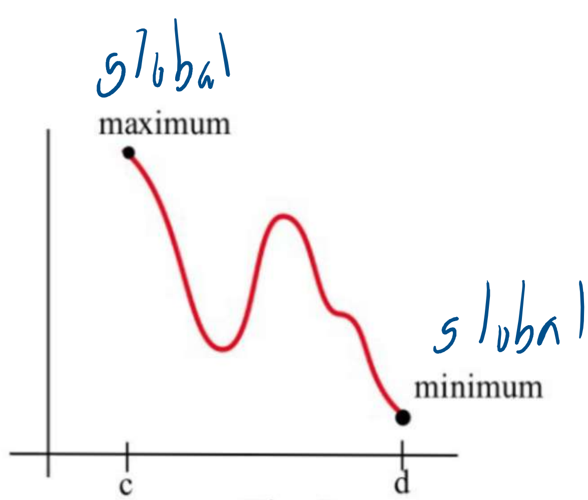
f' decreasing

$\Rightarrow f'' < 0$

f' goes from − to 0 to +

f' increasing

$f'' > 0$



Memorize the procedure

To find Global Extremes:
The only places where a function can have a global extreme are critical points or endpoints.

(a) If the function has only one critical point, and it's a local extreme, then it is also the global extreme.
(b) If there are endpoints, find the global extremes by comparing y-values at all the critical points and at the endpoints.
(c) When in doubt, graph the function to be sure.

In problems 9 – 16, find all critical points and global extremes of each function on the given intervals.

14. $f(x) = 2 - x^3$ on $[-2, 1]$.

Also find any local max or min

$f'(x) = -3x^2$ defined on $[-2, 1]$

set $f'(x) = 0$, solve for x

$-3x^2 = 0$

$x^2 = 0$

$\boxed{x = 0}$ critical point

$f''(x) = -6x$

$f''(0) = 0$ 2nd deriv test inconclusive

$f'(x) = -3x^2$

$x < 0 \Rightarrow f'(x) < 0$

$x > 0 \Rightarrow f'(x) < 0$

$x = 0$ is neither local max or local min

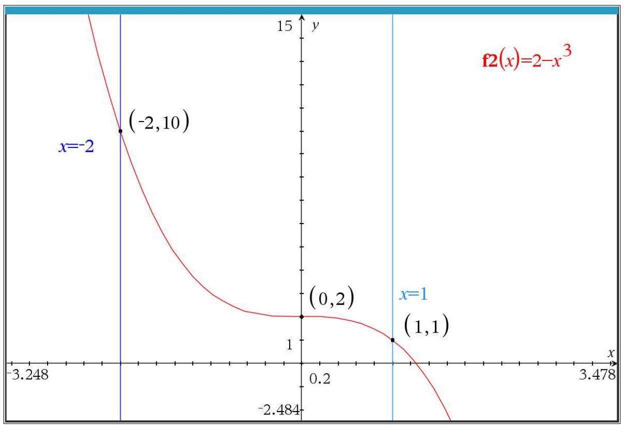
$f(-2) = 2 - (-2)^3 = 10$ global max

$f(1) = 2 - (1^3) = 1$ global min

$f(0) = 2 - 0^3 = 2$

$f(0) = 2 - 0^3 = 2$

f has global max $= 10$ at $x = -2$
f has global min $= 1$ at $x = 1$



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1. Evaluate $\lim_{x \to 4} (5x - 6)$

$= 5(4) - 6 = 20 - 6 = \boxed{14}$

2. Let $f(x) = 3$
Find $f'(x)$ by taking the limit of the difference quotient.

$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x}$

$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$

$= \frac{3 - 3}{h} = \frac{0}{h} = 0$

$\lim_{h \to 0} (0) = 0$

$$\lim_{h\to 0}(0) = 0$$

$$\boxed{\therefore\ f'(x) = 0}$$