02-24-26 MTH 261

Chapter 2
Section 6: Second Derivative and Concavity
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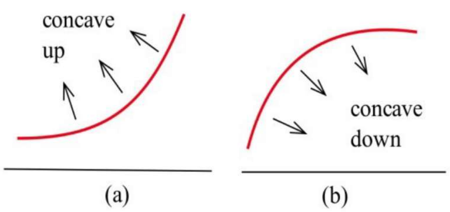
Exam 1		stem & leaf		
52.46667	mean			A- 1
20.6012	st.dev	9	2	B - 0
56	median	8		C - 0
19	min	7		D - 6
92	max	6	035688	F - 8
15	count	5	66	
		4	445	
		3		
		2	01	
		1	9	

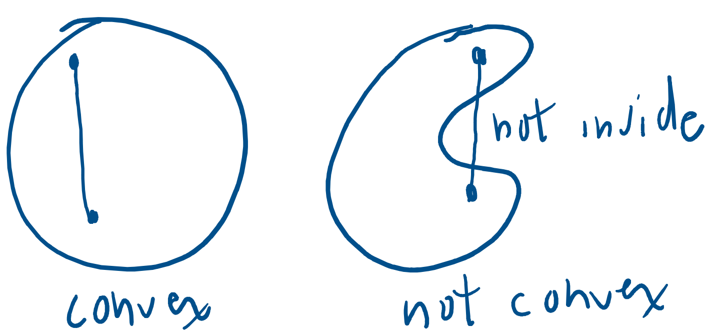
2.6
Memorize



holds water

spill water

Supplied
Definition a figure is convex if the line segment joining any two points in the figure lies entirely in the figure.



Memorize

Second Derivative Let $y = f(x)$

The **second derivative of f** is the derivative of $y' = f'(x)$.

Using prime notation, this is $f''(x)$ or y''. You can read this aloud as “y double prime.”

Using Leibniz notation, the second derivative is written $\dfrac{d^2y}{dx^2}$ or $\dfrac{d^2f}{dx^2}$. This is read aloud as “the second derivative of f.

If $f''(x)$ is positive on an interval, the graph of $y = f(x)$ is **concave up** on that interval**.**
We can say that *f* is increasing (or decreasing) **at an increasing rate**.
If $f''(x)$ is negative on an interval, the graph of $y = f(x)$ is **concave down** on that interval**.**
We can say that *f* is increasing (or decreasing) **at a decreasing rate.**

Memorize

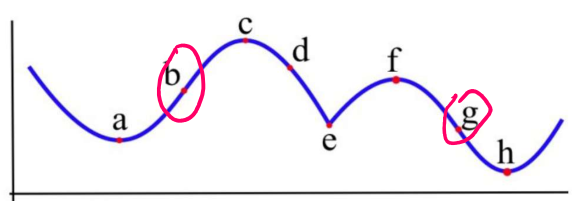
Let f(t) = position of point at time t
Then $f'(t)$ = velocity of the point at time t
And $f''(t)$ = acceleration of the point at time t

Memorize

Definition: An **inflection point** is a point on the graph of a function where the concavity of the function changes, from concave up to down or from concave down to up.

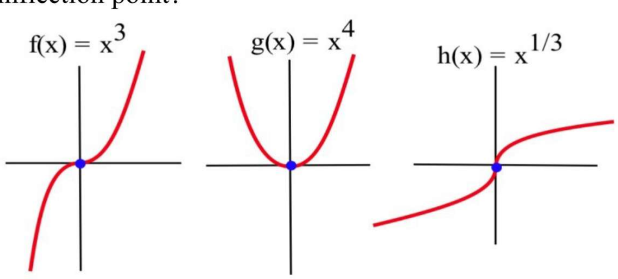
Example 3

Which of the labeled points in the graph below are inflection points?



Example 4

Let $f(x) = x^3$, $g(x) = x^4$ and $h(x) = x^{1/3}$. For which of these functions is the point (0,0) an inflection point?



Graphically, it is clear that the concavity of $f(x) = x^3$ and $h(x) = x^{1/3}$ changes at (0,0), so (0,0) is an inflection point for f and h. The function $g(x) = x^4$ is concave up everywhere so (0,0) is not an inflection point of g.

$f'(x) = 3x^2$ | $g'(x) = 4x^3$ | $h'(x) = \left(\frac{1}{3}\right)x^{-2/3}$

$f''(x) = 6x$ | $g''(x) = 12x^2$ | $h''(x) = \left(-\frac{2}{9}\right)x^{-5/3}$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f''(0) = 6(0) = 0$

inflection point

$g'(x) = 4x^3$

$g''(x) = 12x^2$

$g''(0) = 12(0^2) = 0$

no inflection point

$h'(x) = \left(\frac{1}{3}\right)x^{-2/3}$

$h''(x) = \left(-\frac{2}{9}\right)x^{-\frac{5}{3}}$

$h''(0) =$ not defined

inflection point

If $f''(x) = 0$ or is not defined this gives us a candidate for an inflection point

2.6: 2

In problems 1 and 2, each quotation is a statement about a quantity of something changing over time. Let f(t) represent the quantity at time t. For each quotation, tell what f represents and whether the first and second derivatives of f are positive or negative.

2. (a) "The child's temperature is still rising, but slower than it was a few hours ago."
   (b) "The number of whales is decreasing, but at a slower rate than last year."
   (c) "The number of people with the flu is rising and at a faster rate than last month."

(2 a) Let $f(t)$ = child's temperature at time t (hours)

$f'(t) > 0$

$f''(t) < 0$

concave down

(2 b) $f(t)$ = number of whales at time t (years)

$f'(t) < 0$

$f''(t) < 0$

$f''(t) < 0$