02-17-26 MTH 261

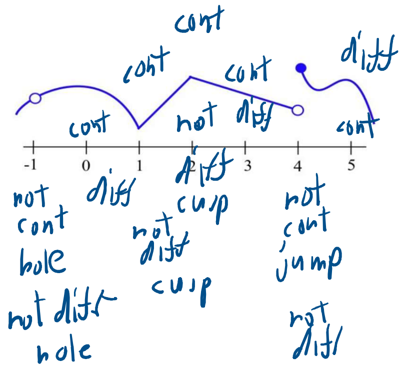
Section 5: Chain Rule
page 121: 1, 3, 7, 9, 13

Exam 1, Thursday, 02/19/26, 1.1 - 1.8, 2.1 -2.5,
focus on chapter 2

2.5: 1

2.5 Exercises

1. The graph of $y = f(x)$ is shown.
   (a) At which integers is f continuous?
   (b) At which integers is f differentiable?



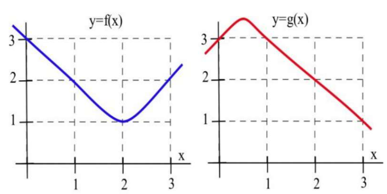
memorize

Definition: f(x) is continuous at x = c if $\lim_{x \to c} f(x) = f(c)$

2.5:5

5. Use the graphs to estimate the values of g(x), **g '(x)**, (f∘g)(x), **f '(** g(x) **)**, and **(f∘g) '(** x **)** at x = 1.

6. Use the graphs to estimate the values of g(x), **g '(x)**, (f∘g)(x), **f '(** g(x) **)**, and **(f∘g) '(** x **)** for x = 2.



$g(1) = 3$

$g'(1) = -1$

$(f \circ g)(1) = f(g(1)) = f(3) = \boxed{2}$

$f'(g(1)) = f'(3) = \boxed{1}$

2.5: 9

In problems 7 – 12 , find the derivative of each function.

9. $f(x) = x \cdot (3x + 7)^5$

$x * (3x+7)^5$
$= 243x^6 + 2835x^5 + 13\,230x^4 + 30\,870x^3 + 36\,015x^2 + 16\,807x$
$\frac{d}{dx}(243x^6 + 2835x^5 + 13\,230x^4 + 30\,870x^3 + 36\,015x^2 + 16\,807x) =$
$(3x+7)^4(18x+7) = 1458x^5 + 14\,175x^4 + 52\,920x^3 + 92\,610x^2 + 72\,030x + 16\,807$

$\frac{df}{dx} = x \frac{d}{dx}(3x+7)^5 + (3x+7)^5 \frac{dx}{dx}$

$$\frac{df}{dx} = x\frac{d}{dx}(3x+7)^5 + (3x+7)^5\frac{dx}{dx}$$

$$= x(5)(3x+7)^4(3) + (3x+7)^5(1)$$

$$= 15x(3x+7)^4 + (3x+7)^5$$

$$= 15x\underline{(3x+7)}^4 + (3x+7)\underline{(3x+7)}^4$$

$$= (3x+7)^4(15x+3x+7)$$

$$= \boxed{(3x+7)^4(18x+7)}$$

$$\boxed{\begin{gathered}ab+ac\\ a(b+c)\end{gathered}}$$

2.3

2. Find (a) $D(x^{12})$ (b) $\frac{d}{dx}(\sqrt[7]{x})$ (c) $D(\frac{1}{x^3})$ (d) $\frac{d\,x^e}{dx}$

(a) $D(x^{12}) = 12x^{12-1} = 12x^{11}$

(b) $\frac{d}{dx}(\sqrt[7]{x}) = \frac{d}{dx}(x^{\frac{1}{7}}) = (\frac{1}{7})x^{\frac{1}{7}-1}$

$$= (\tfrac{1}{7})x^{\frac{1}{7}-\frac{7}{7}} = \boxed{(\tfrac{1}{7})x^{-\frac{6}{7}}} = \frac{1}{7\sqrt[]{x^{6/7}}}$$

$$\frac{d}{dx}(x^e) = \boxed{ex^{e-1}} \qquad \frac{d}{dx}(e^x) = \boxed{e^x}$$

2.4:4

Absolutely! No problem—let's say you have a function like f of x equals the sine of two x squared plus three x. The chain rule is going to come into play when you're differentiating something like that. Let me know how that sounds and we can dig into the details whenever you're ready!

$$f(x) = \sin(2x^2+3x)$$

$$f'(x) = [\cos(2x^2+3x)][4x+3]$$

$$f'(x) = [\cos(2x^2+3x)][4x+3]$$
$$= \boxed{(4x+3)\cos(2x^2+3x)}$$

$$f(x) = \frac{e^{2x}}{x^2}$$

$$f'(x) = \frac{x^2 \frac{d}{dx}(e^{2x}) - e^{2x}\frac{d}{dx}(x^2)}{x^4}$$

$$= \frac{x^2(2)e^{2x} - e^{2x}(2x)}{x^4}$$

$$= \frac{2xe^{2x}(x-1)}{x^4}$$

$$= \boxed{\frac{2e^{2x}(x-1)}{x^3}}$$

$$\frac{d}{dx}e^{u(x)}$$

$$= e^{u(x)} \cdot u'(x)$$

Let $y = e^u$

$u = 2x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= e^u \frac{d}{dx}(2x)$$
$$= e^{2x}(2)$$
$$= 2e^{2x}$$

---

after class notes

$$(e^x)' = \frac{d}{dx}(e^x) = e^x$$

---

chain rule

Chain rule

Let $y = f(u)$

$u = u(x)$

$$\frac{dy}{dx} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$$

$$\frac{dy}{dx} = f'(u) \cdot u'(x)$$

$y = (3x^2 - 5)^4$

Find $\frac{dy}{dx}$

write $y = f \circ g$

Let $g(x) = 3x^2 - 5$

Let $f(g) = g^4$

$$\frac{dy}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$= 4g^3 \cdot (6x)$$

$$= 4(3x^2 - 5)^3 (6x)$$

$$= 24x(3x^2 - 5)^3$$

$$\frac{dy}{dx} = \frac{d}{dx}(y)$$

$f(x) = \sqrt{x^2 + 6x - 1}$

Find $f'(x)$

$$f(x) = (x^2 + 6x - 1)^{\frac{1}{2}}$$

$$f'(x) = \left(\frac{1}{2}\right)(x^2 + 6x - 1)^{-\frac{1}{2}} \frac{d}{dx}(x^2 + 6x - 1)$$

$$= \left(\frac{1}{2}\right)(x^2 + 6x - 1)^{-\frac{1}{2}}(2x + 6)$$

$= \left(\frac{1}{2}\right)\left(x^2+6x-1\right)^{-\frac{1}{2}}\left(2x+6\right)$

$= \left(\frac{1}{2}\right)(2x+6)\left(x^2+6x-1\right)^{-\frac{1}{2}}$

$$\boxed{\begin{aligned} f'(x) &= (x+3)\left(x^2+6x-1\right)^{-\frac{1}{2}} \\ f'(x) &= \frac{x+3}{\sqrt{x^2+6x-1}} \end{aligned}}$$

2.4

11. Find (a) $\frac{d}{dt}\left(te^t\right)$, (b) $d\left(e^x\right)^5$

(a) $t\frac{d}{dt}\left(e^t\right) + e^t\frac{d}{dt}(t)$

$= te^t + e^t(1)$

$= \boxed{e^t(t+1)}$

$\frac{d}{dx}\left(k \cdot f(x)\right)$ $\quad k = \text{constant}$

$= k\, f'(x) + (f(x))\frac{dk}{dx}$

$= kf'(x) + (f(x))(0)$

$= kf'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{3}\right) = \frac{1}{3}\left(f'(x)\right)$

constant multiple rule

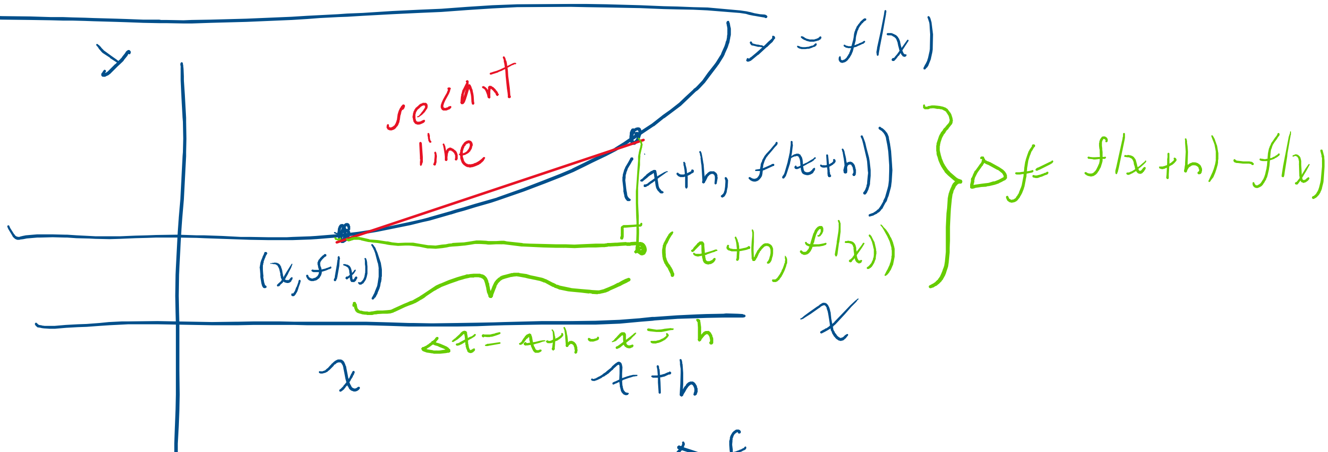
$= 3f'(x) - (f(x))\frac{d}{dx}3$

$$= \frac{3f'(x) - (f(x))\frac{d}{dx}3}{3^2}$$

quotient rule

$$= \frac{3f'(x) - (f(x))(0)}{3^2}$$

$$= \frac{3f'(x) - 0}{3^2} = \frac{3f'(x)}{3^2} = \left(\frac{1}{3}\right) f'(x)$$



$$\text{slope of secant line} = \frac{\Delta f}{\Delta x}$$

$$\lim_{h \to 0} \frac{\Delta f}{\Delta x} = f'(x) = \text{slope of tangent line}$$