02-12-26 MTH 261

Chapter 2

Section 3: Power and Sum Rules for Derivatives
page 105: 1, 3, 7, 9, 11, 12

Section 4: Product and Quotient Rules
page 112: 1, 4, 5, 7, 9, 10

Section 5: Chain Rule
page 121: 1, 3, 7, 9, 13

Exam 1, Thursday, 02/19/26, 1.1 - 1.8, 2.1 -2.5, focus on chapter 2

2.4:4

4. Calculate $\frac{d}{dx}((x-5)(3x+7))$ by (a) using the product rule and (b) expanding the product and then differentiating. Verify that both methods give the same result.

(a)

$$\frac{d}{dx}((x-5)(3x+7)) = (x-5)\frac{d}{dx}(3x+7) + (3x+7)\frac{d}{dx}(x-5)$$

$$= (x-5)(3) + (3x+7)(1)$$

$$= 3x - 15 + 3x + 7$$

$$= \boxed{6x-8}$$

(b)

$$(x-5)(3x+7)$$

$$= 3x^2 + 7x - 15x - 35$$

$$= \boxed{3x^2 - 8x - 35}$$

$$= \boxed{3x^2 - 8x - 35}$$

$$\frac{d}{dx}(3x^2 - 8x - 35)$$

$$= 6x - 8 - 0$$

$$= \boxed{6x - 8}$$

2.4: 10b

(b) $\frac{d}{dx}(e^x)^3$.

$$= 3(e^x)^2 \frac{d}{dx}(e^x)$$

$$= 3(e^x)^2(e^x)$$

$$= \boxed{3e^{3x}}$$

TI-nspire check

$\frac{d}{dx}((e^x)^3)$	$3 \cdot e^{3 \cdot x}$

2.5

Memorize

Derivative Rules: Chain Rule

In what follows, *f* and *g* are differentiable functions with $y = f(u)$ and $u = g(x)$

(h) **Chain Rule (Leibniz notation):**

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Notice that the du's seem to cancel. This is one advantage of the Leibniz notation; it can remind you of how the chain rule chains together.

(h) **Chain Rule (using prime notation):**

$$f'(x) = f'(u) \cdot g'(x) = f'(g(x)) \cdot g'(x)$$

(h) **Chain Rule (in words):**

The derivative of a composition is the derivative of the outside, with the inside staying the same, TIMES the derivative of what's inside.

Example 5

The table gives values for f , f ' , g and g ' at a number of points. Use these values to determine (f∘g)(x) and **(** f∘g **) '**(x) at $x = -1$ and 0.

$(f \circ g)'(x)$

x	f(x)	g(x)	f'(x)	g'(x)	(f∘g)(x)	(g∘f)(x)
-1	2	3	1	0		
0	-1	1	3	2		
1	1	0	-1	3		
2	3	-1	0	1		
3	0	2	2	-1		

Your Name MTH 261 quiz 3 write each problem.
Calculator OK

1. Let $f(x) = x^2 + 4$. Find $\frac{df}{dx}$ by using the limit of the difference quotient.

$$\frac{df}{dx} = \lim_{h \to 0} \frac{\Delta f}{\Delta x}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 + 4] - [x^2 + 4]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4} - \cancel{x^2} - \cancel{4}}{h} = \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2x + h} \quad f'(x) = \lim_{h \to 0} (2x + h) = 2x + 0 = \boxed{2x}$$

Of course, we can see this result easily by using the sum rule, the power rule, and the fact that the derivative of a constant is zero.

2. Let $g(x) = 9x^4 - 8x^2 + 7$. Find $g'(x)$ using rules of differentiation. Show all intermediate steps.

$$g'(x) = \frac{d}{dx}(9x^4) - \frac{d}{dx}(8x^2) + \frac{d}{dx}(7)$$
$$= 9\frac{d}{dx}(x^4) - 8\frac{d}{dx}(x^2) + 0$$
$$= 9(4x^3) - (8)(2x)$$
$$= \boxed{36x^3 - 16x}$$

3. Let $h(x) = (2x+4)^{20}(x^2-1)$. Find $h'(x)$.

$$h'(x) = (2x+4)^{20}(2x) + (x^2-1)20(2x+4)^{19}(2)$$
$$= \boxed{2x(2x+4)^{20} + 40(x^2-1)(2x+4)^{19}}$$
$$= (2x+4)^{19}\left(2x(2x+4) + 40(x^2-1)\right)$$
$$= (2x+4)^{19}(4x^2 + 8x + 40x^2 - 40)$$
$$= \boxed{(2x+4)^{19}(44x^2 + 8x - 40)}$$

chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

TI-Nspire check

$\frac{d}{dx}\left((2\cdot x+4)^{20}\cdot(x^2-1)\right)$	$2097152\cdot(x+2)^{19}\cdot(11\cdot x^2+2\cdot x-10)$
$2097152\cdot(x+2)^{19}\cdot(11\cdot x^2+2\cdot x-10)-(2\cdot x+4)^{19}\cdot(44\cdot x^2+8\cdot x-40)$	0

4. Let $k(x) = \frac{x^3}{x}$.

4a) Find $k'(x)$ with the quotient rule.

$$k'(x) = \frac{x(3x^2) - x^3(1)}{x^2} = \frac{3x^3 - x^3}{x^2} = \frac{2x^3}{x^2}$$

$$\boxed{k'(x) = 2x}$$

4b) Find $k'(x)$ without the quotient rule.

$$k(x) = x^2$$

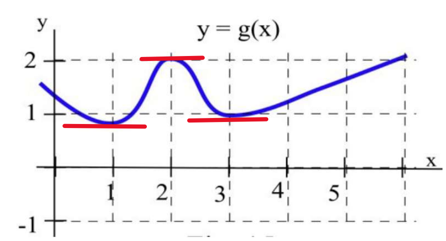
$$\boxed{k'(x) = 2x}$$

After class question

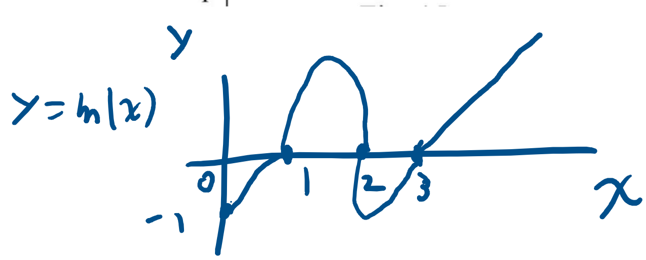
13. (a) At what values of x does the graph of g have a horizontal tangent line?

(b) At what value(s) of x is the value of g the largest? smallest?

(c) Sketch the graph of m(x) = the slope of the line tangent to the graph of g at the point (x,y).



Note that where the Tangent line is horizontal, m(x) = 0



Note that where the
Tangent line is horizontal,
m(x) = 0