02-10-26 MTH 261

Section 3: Power and Sum Rules for Derivatives
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Section 4: Product and Quotient Rules
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2.3 Exercises

1. Fill in the values in the table for $\frac{d}{dx}(3f(x))$, $\frac{d}{dx}(2f(x)+g(x))$, and $\frac{d}{dx}(3g(x)-f(x))$.

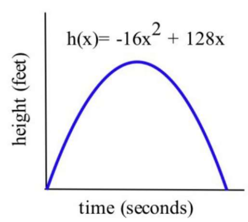
x	f(x)	f '(x)	g(x)	g '(x)	$\frac{d}{dx}(3f(x))$	$\frac{d}{dx}(2f(x)+g(x))$	$\frac{d}{dx}(3g(x)-f(x))$
0	3	–2	–4	3	$(3)(-2)=-6$		
1	2	–1	1	0	$(3)(-1)=-3$		
2	4	2	3	1	$(3)(2)=6$		

$$\frac{d}{dx}(3f(x)) = 3\frac{d}{dx}f(x) = 3f'(x)$$

2.3: 12

12. An arrow shot straight up from ground level with an initial velocity of 128 feet per second will be at height $h(x) = -16x^2 + 128x$ feet at x seconds.

(a) Determine the velocity of the arrow when $x = 0$, 1 and 2 seconds.
(b) What is the velocity of the arrow, $v(x)$, at any time x?
(c) At what time x will the velocity of the arrow be 0?
(d) What is the greatest height the arrow reaches?
(e) How long will the arrow be aloft?
(f) Use the answer for the velocity in part (b) to determine the acceleration, $a(x) = v'(x)$, at any time x.



(a) $h'(x) = -32x + 128 = v(x)$

$v(0) = h'(0) = -32(0) + 128 = \boxed{128 \frac{ft}{sec}}$

$v(1) = h'(1) = -32(1) + 128 = \boxed{96 \frac{ft}{sec}}$

$v(2) = h'(2) = -32(2) + 128 = \boxed{64 \frac{ft}{sec}}$

(b) $v(x) = h'(x) = -32x + 128$

(b) $v(x) = h'(x) = -32x + 128$

(c) Solve $v(x) = 0$ for x

$$-32x + 128 = 0$$

$$32x = 128$$

$$x = \frac{128}{32} = 4$$

The velocity is zero ft/sec, 4 seconds after the arrow is released.

(d)

What is the maximum value of h(x)?

$$h(4) = -16(4^2) + 128(4) = 256$$

-16*4^2+128*4=256

The maximum height of the arrow is 256 feet.

(e) when is $h(x) = 0$?

$$h(x) = -16x^2 + 128x = 0$$

$$\Rightarrow \quad x^2 - 8x = 0$$

$$\Rightarrow \quad x(x-8) = 0$$

$x = 0$ or $x - 8 = 0$

launch $\qquad x = 8$

hits ground

The arrow was aloft for 8 seconds.

(f) $a(x) = v'(x) = \frac{d}{dx}(-32x + 128)$

$$= -32 \frac{ft}{sec^2}$$

note $a(x) = h''(x)$

note $a(x) = h''(x)$

2.4

Memorize

Derivative Rules: Product and Quotient Rules

In what follows, f and g are differentiable functions of x.

(a) **Product Rule:**

$$\frac{d}{dx}(fg) = f'g + fg'$$

The derivative of the first factor times the second left alone, plus the first left alone times the derivative of the second.

The product rule can extend to a product of several functions; the pattern continues – take the derivative of each factor in turn, multiplied by all the other factors left alone, and add them up.

(b) **Quotient Rule:**

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

The numerator of the result resembles the product rule, but there is a minus instead of a plus; the minus sign goes with the g'. The denominator is simply the square of the original denominator – no derivatives there.

Let $f(x) = x$
$g(x) = x$

$$\frac{d}{dx}\big((f\cdot g)(x)\big) = \frac{d}{dx}\big(f(x)\cdot g(x)\big)$$

$$= \frac{d}{dx}(x\cdot x) = \frac{d}{dx}(x^2) = \boxed{2x}$$

$$\frac{d}{dx}\big((fg)(x)\big) = \frac{d}{dx}(f\cdot g)$$

$$= fg' + gf'$$

$$= (x)(1) + (x)(1)$$

$$= x + x$$

$$= \boxed{2x}$$

$f(x) = x$
$g(x) = x$
$f'(x) = 1$
$g'(x) = 1$

Let $f(x) = x$, $g(x) = x$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(\frac{x}{x}\right) = \frac{d}{dx}(1) = \boxed{0}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx}\left(\frac{\ }{g}\right) = \frac{\ \ }{g^2}$$

$$= \frac{x(1) - x(1)}{x^2}$$

$$= \frac{x - x}{x^2} = \frac{0}{x^2} = \boxed{0}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(f \cdot \frac{1}{g}\right)$$

$$f \frac{d}{dx}\left(\frac{1}{g}\right) + \frac{1}{g}\frac{df}{dx}$$

$$f \frac{d}{dx}\left(g^{-1}\right) + \frac{1}{g}\frac{df}{dx}$$

$$f(-1)(g^{-2})g' + \frac{1}{g}f'$$ [chain rule to be learned later]

$$-\frac{fg'}{g^2} + \frac{f'}{g}$$

$$-\frac{fg'}{g^2} + \frac{f'g}{g^2}$$

$$\frac{f'g - fg'}{g^2}$$

Here, we derived the quotient rule from the product rule!

2.4

6. If the quotient of f and g is a constant ($\frac{f(x)}{g(x)} = k$ for all x), then how are $g \cdot f'$ and $f \cdot g'$ related?

(b) Quotient Rule:

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}(k) = 0$$

Differentiate $\frac{f}{g}$ in 2 ways

$$\frac{f'g - fg'}{g^2} = 0$$

$$f'g - fg' = 0$$

$$\boxed{f'g = fg'}$$

Find $\frac{d}{dx}\left[(3x-5)(2x+4)\right]$

$$\frac{d}{dx}(fg) = fg' + f'g$$

$$= \frac{d}{dx}\left(6x^2 + 2x - 20\right)$$

$$= 12x + 2$$