02-03-26 MTH 261

Chapter 2: The Derivative
Precalculus Idea: Slope and Rate of Change
Section 1: Limits and Continuity
page 78: 1, 5, 7, 9

Section 2: The Derivative
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Your Name MTH 261 quiz 1
Open homework notebook; closed everything else.
No calculator

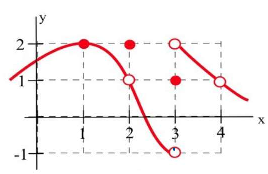
2.1 : 1

2.1 Exercises

1. Use the graph to determine the following limits.

(a) $\lim_{x\to 1} f(x) = 2$ (b) $\lim_{x\to 2} f(x) = 1$

(c) $\lim_{x\to 3} f(x)$ dne (d) $\lim_{x\to 4} f(x) = 1$



$\lim_{x\to 3^-} f(x) = -1$

$\lim_{x\to 3^+} f(x) = 2$

$-1 \neq 2$

$\therefore \lim_{x\to 3} f(x)$ dne

2.1: 5a

5. Evaluate (a) $\lim_{x\to 1} \dfrac{x^2+3x+3}{x-2}$

$= \dfrac{1^2 + 3(1) + 3}{1-2}$

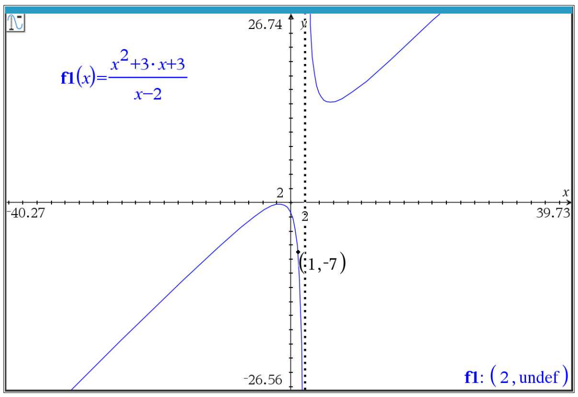
$= \dfrac{1+3+3}{-1}$

$= \dfrac{7}{-1} = -7$

-1

$= \frac{7}{-1} = -7$

(b) $\lim_{x \to 2} \frac{x^2 + 3x + 3}{x - 2} = \frac{4+6+3}{0}$ not defined



2.2
Memorize

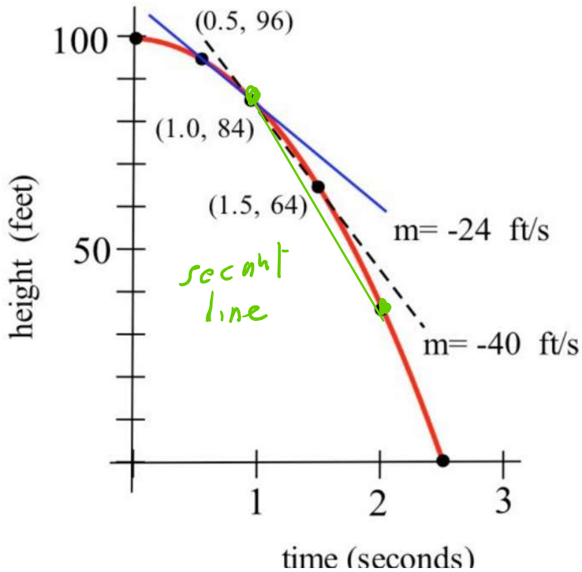
$$\textbf{Average velocity} = \frac{\textbf{distance fallen}}{\textbf{total time}} = \frac{\Delta\ \textbf{position}}{\Delta\ \textbf{time}}$$

Δ = change

Memorize

Average velocity $= \frac{\Delta \text{ position}}{\Delta \text{ time}}$ **= slope of the secant line through 2 points.**

Instantaneous velocity = slope of the line tangent to the graph.



Memorize

The Derivative:

The **derivative** of a function f at a point (x, f(x)) is the instantaneous rate of change.
The **derivative** is the slope of the tangent line to the graph of f at the point (x, f(x)).
The **derivative** is the slope of the curve f(x) at the point (x, f(x)).
A function is called **differentiable** at (x, f(x)) if its derivative exists at (x, f(x)).

Notation for the Derivative:

The **derivative of y = f(x) with respect to x** is written as
$f\,'(x)$ (read aloud as "f prime of x"), or y' ("y prime")
or $\frac{dy}{dx}$ (read aloud as "dee why dee ex"), or $\frac{df}{dx}$
The notation that resembles a fraction is called **Leibniz notation**. It displays not only the name of the function (f or y), but also the name of the variable (in this case, x). It looks like a fraction because the derivative is a slope. In fact, this is simply $\frac{\Delta y}{\Delta x}$ written in Roman letters instead of Greek letters.

Verb forms:

We **find the derivative** of a function, or **take the derivative** of a function, or **differentiate** a function.
We use an adaptation of the $\frac{dy}{dx}$ notation to mean "find the derivative of f(x):"

$$\frac{d}{dx}(f(x)) = \frac{df}{dx}$$

Formal Algebraic Definition:

$$f\,'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Practical Definition:

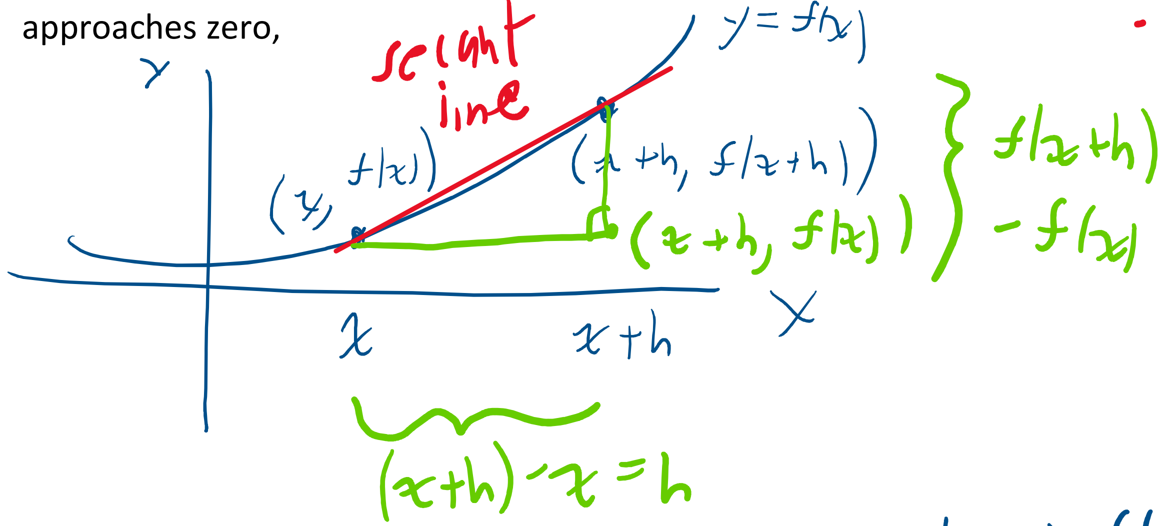
The derivative can be approximated by looking at an average rate of change, or the slope of a secant line, over a very tiny interval. The tinier the interval, the closer this is to the true instantaneous rate of change, slope of the tangent line, or slope of the curve.

Looking Ahead:

We will have methods for computing exact values of derivatives from formulas soon. If the function is given to you as a table or graph, you will still need to approximate this way.

The difference quotient is the slope of a secant line joining two points on a curve.

The first derivative is the slope of the tangent line,
or the limit of the difference quotient as the denominator
approaches zero,



$$\text{slope of secant line} = \frac{\Delta f}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$

Let $f(x) = 3x + 4$.
Find $\frac{df}{dx}$ when $x = 2$

$$\frac{df}{dx} = \lim_{h\to 0} \frac{\Delta f}{\Delta x} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[3(x+h)+4] - [3x+4]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x} + 3h \cancel{+4} - \cancel{3x} \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 3 = \boxed{3}$$

$$\boxed{f'(x) = 3}$$

$$\boxed{f'(2) = 3}$$

$$\boxed{f'(2) = 3}$$

Alternative presentation

$f(x) = 3x + 4$

Find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{[3(x+h)+4] - [3x+4]}{h}$$

$$= \frac{3x + 3h + 4 - 3x - 4}{h}$$

$$= \frac{3h}{h}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 3}$$ simplified difference quotient

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x} = \lim_{h \to 0} (3) = 3$$

$$\boxed{\therefore f'(x) = 3}$$

$f(x) = x^2$

Find $f'(x)$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$\frac{\quad}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2x + h}$$

$$f'(x) = \lim_{h \to 0} (2x+h) = \lim_{h \to 0} (2x) + \lim_{h \to 0} (h)$$

$$= 2x + 0$$

$$= \boxed{2x = f'(x)}$$

$$(x+h)^{100} = h^{100} + 100h^{99}x + 4950h^{98}x^2 + 161\,700h^{97}x^3 + 3921\,225h^{96}x^4 + 75\,287\,520h^{95}x^5 + 1192\,052\,400h^{94}x^6 + 16\,007\,560\,800h^{93}x^7 + 186\,087\,894\,300h^{92}x^8 + 1902\,231\,808\,400h^{91}x^9 + 17\,310\,309\,456\,440h^{90}x^{10} + 141\,629\,804\,643\,600h^{89}x^{11} + 1050\,421\,051\,106\,700h^{88}x^{12} + 7110\,542\,499\,799\,200h^{87}x^{13} + 44\,186\,942\,677\,323\,600h^{86}x^{14} + 253\,338\,471\,349\,988\,640h^{85}x^{15} + 1345\,860\,629\,046\,814\,650h^{84}x^{16} + 6650\,134\,872\,937\,201\,800h^{83}x^{17} + 30\,664\,510\,802\,988\,208\,300h^{82}x^{18} + 132\,341\,572\,939\,212\,267\,400h^{81}x^{19} + 535\,983\,370\,403\,809\,682\,970h^{80}x^{20} + 2041\,841\,411\,062\,132\,125\,600h^{79}x^{21} + 7332\,066\,885\,177\,656\,269\,200h^{78}x^{22} + 24\,865\,270\,306\,254\,660\,391\,200h^{77}x^{23} + 79\,776\,075\,565\,900\,368\,755\,100h^{76}x^{24} + 242\,519\,269\,720\,337\,121\,015\,504h^{75}x^{25} + 699\,574\,816\,500\,972\,464\,467\,800h^{74}x^{26} + 1917\,353\,200\,780\,443\,050\,763\,600h^{73}x^{27} + 4998\,813\,702\,034\,726\,525\,205\,100h^{72}x^{28} + 12\,410\,847\,811\,948\,286\,545\,336\,800h^{71}x^{29} + 29\,372\,339\,821\,610\,944\,823\,963\,760h^{70}x^{30} + 66\,324\,638\,306\,863\,423\,796\,047\,200h^{69}x^{31} + 143\,012\,501\,349\,174\,257\,560\,226\,775h^{68}x^{32} + 294\,692\,427\,022\,540\,894\,366\,527\,900h^{67}x^{33} + 580\,717\,429\,720\,889\,409\,486\,981\,450h^{66}x^{34} + 1095\,067\,153\,187\,962\,886\,461\,165\,020h^{65}x^{35} + 1977\,204\,582\,144\,932\,989\,443\,770\,175h^{64}x^{36} + 3420\,029\,547\,493\,938\,143\,902\,737\,600h^{63}x^{37} + 5670\,048\,986\,634\,686\,922\,786\,117\,600h^{62}x^{38} + 9013\,924\,030\,034\,630\,492\,634\,340\,800h^{61}x^{39} + 13\,746\,234\,145\,802\,811\,501\,267\,369\,720h^{60}x^{40} + 20\,116\,440\,213\,369\,968\,050\,635\,175\,200h^{59}x^{41} + 28\,258\,808\,871\,162\,574\,166\,368\,460\,400h^{58}x^{42} + 38\,116\,532\,895\,986\,727\,945\,334\,202\,400h^{57}x^{43} + 49\,378\,235\,797\,073\,715\,747\,364\,762\,200h^{56}x^{44} + 61\,448\,471\,214\,136\,179\,596\,720\,592\,960h^{55}x^{45} + 73\,470\,998\,190\,814\,997\,343\,905\,056\,800h^{54}x^{46} + 84\,413\,487\,283\,064\,039\,501\,507\,937\,600h^{53}x^{47} + 93\,206\,558\,875\,049\,876\,949\,581\,681\,100h^{52}x^{48} + 98\,913\,082\,887\,808\,032\,681\,188\,722\,800h^{51}x^{49} + 100\,891\,344\,545\,564\,193\,334\,812\,497\,256h^{50}x^{50} + 98\,913\,082\,887\,808\,032\,681\,188\,722\,800h^{49}x^{51} + 93\,206\,558\,875\,049\,876\,949\,581\,681\,100h^{48}x^{52} + 84\,413\,487\,283\,064\,039\,501\,507\,937\,600h^{47}x^{53} + 73\,470\,998\,190\,814\,997\,343\,905\,056\,800h^{46}x^{54} + 61\,448\,471\,214\,136\,179\,596\,720\,592\,960h^{45}x^{55} + 49\,378\,235\,797\,073\,715\,747\,364\,762\,200h^{44}x^{56} + 38\,116\,532\,895\,986\,727\,945\,334\,202\,400h^{43}x^{57} + 28\,258\,808\,871\,162\,574\,166\,368\,460\,400h^{42}x^{58} + 20\,116\,440\,213\,369\,968\,050\,635\,175\,200h^{41}x^{59} + 13\,746\,234\,145\,802\,811\,501\,267\,369\,720h^{40}x^{60} + 9013\,924\,030\,034\,630\,492\,634\,340\,800h^{39}x^{61} + 5670\,048\,986\,634\,686\,922\,786\,117\,600h^{38}x^{62} + 3420\,029\,547\,493\,938\,143\,902\,737\,600h^{37}x^{63} + 1977\,204\,582\,144\,932\,989\,443\,770\,175h^{36}x^{64} + 1095\,067\,153\,187\,962\,886\,461\,165\,020h^{35}x^{65} + 580\,717\,429\,720\,889\,409\,486\,981\,450h^{34}x^{66} + 294\,692\,427\,022\,540\,894\,366\,527\,900h^{33}x^{67} + 143\,012\,501\,349\,174\,257\,560\,226\,775h^{32}x^{68} + 66\,324\,638\,306\,863\,423\,796\,047\,200h^{31}x^{69} + 29\,372\,339\,821\,610\,944\,823\,963\,760h^{30}x^{70} + 12\,410\,847\,811\,948\,286\,545\,336\,800h^{29}x^{71} + 4998\,813\,702\,034\,726\,525\,205\,100h^{28}x^{72} + 1917\,353\,200\,780\,443\,050\,763\,600h^{27}x^{73} + 699\,574\,816\,500\,972\,464\,467\,800h^{26}x^{74} + 242\,519\,269\,720\,337\,121\,015\,504h^{25}x^{75} + 79\,776\,075\,565\,900\,368\,755\,100h^{24}x^{76} + 24\,865\,270\,306\,254\,660\,391\,200h^{23}x^{77} + 7332\,066\,885\,177\,656\,269\,200h^{22}x^{78} + 2041\,841\,411\,062\,132\,125\,600h^{21}x^{79} + 535\,983\,370\,403\,809\,682\,970h^{20}x^{80} + 132\,341\,572\,939\,212\,267\,400h^{19}x^{81} + 30\,664\,510\,802\,988\,208\,300h^{18}x^{82} + 6650\,134\,872\,937\,201\,800h^{17}x^{83} + 1345\,860\,629\,046\,814\,650h^{16}x^{84} + 253\,338\,471\,349\,988\,640h^{15}x^{85} + 44\,186\,942\,677\,323\,600h^{14}x^{86} + 7110\,542\,499\,799\,200h^{13}x^{87} + 1050\,421\,051\,106\,700h^{12}x^{88} + 141\,629\,804\,643\,600h^{11}x^{89} + 17\,310\,309\,456\,440h^{10}x^{90} + 1902\,231\,808\,400h^9x^{91} + 186\,087\,894\,300h^8x^{92} + 16\,007\,560\,800h^7x^{93} + 1192\,052\,400h^6x^{94} + 75\,287\,520h^5x^{95} + 3921\,225h^4x^{96} + 161\,700h^3x^{97} + 4950h^2x^{98} + 100hx^{99} + x^{100}$$

$(x+h)^{100} = x^{100} + 100\,h\,x^{99}$ + terms with h raised to powers ≥ 2

terms with $h, h^2, h^3, \dots$

$$\frac{(x+h)^{100} - x^{100}}{h} = \frac{\cancel{x^{100}} + 100\cancel{h}x^{99} + \text{terms (with } h^n,\ n \geq 2) - \cancel{x^{100}}}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{(x+h)^{100} - x^{100}}{h} = 100x^{99} + 0$$

$$= 100x^{99}$$