01-29-26 MTH 261

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1.8

Rewrite each equation in exponential form

2. $\log(r) = s$

$10^s = r$

$a^b =$ a^b

Rewrite each equation in logarithmic form.

8. $e^y = x$

$\ln(x) = y$

$\ln(x) = \log_e(x)$

In TI-Nspire, I entered log(10). It was automatically changed to the following.

$\log_{10}(100)$ 2

Scientific Notebook interprets log as ln.

$\log(100) = \ln 100 \approx 4.6052$

Here, I had to enter the base 10 explicitly to get the common log.

$\log_{10}(100) = 2$

Solve each equation for the variable.

9. $5^x = 14$

Exact answer and decimal rounded to the nearest hundredth.

$\log_5(5^x) = \log_5(14)$

$\boxed{x = \log_5(14)} \approx 1.64$

$\left(\log_5(14)\right)$ ▸Decimal	1.63974

$\ln(5^x) = \ln(14)$

$x \ln(5) = \ln(14)$

$\boxed{x = \frac{\ln(14)}{\ln(5)}}$

$\frac{\ln(14)}{\ln(5)}$ ▸Decimal	1.63974
$\frac{\log_{10}(14)}{\log_{10}(5)}$ ▸Decimal	1.63974

Find the domain of each function.

33. $f(x) = \log(2x+5)$

$2x + 5 > 0$

$2x > -5$

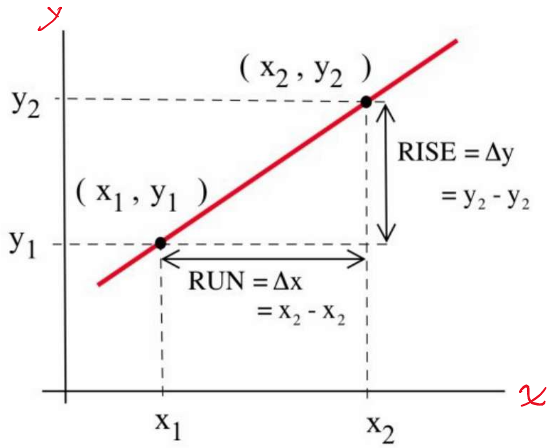
$\boxed{x > -\frac{5}{2}}$

domain $\left(-\frac{5}{2}, \infty\right)$

$= \{x \mid x > -\frac{5}{2}\}$

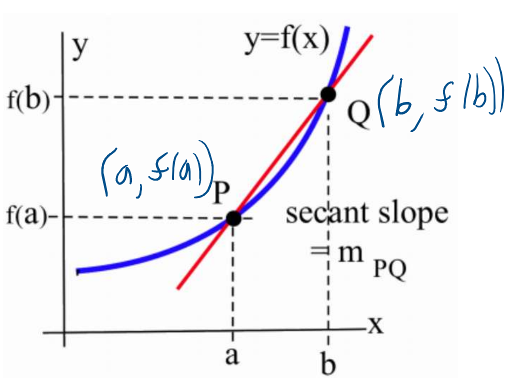
Chapter 2: The Derivative

Precalculus Idea: Slope and Rate of Change



Memorize

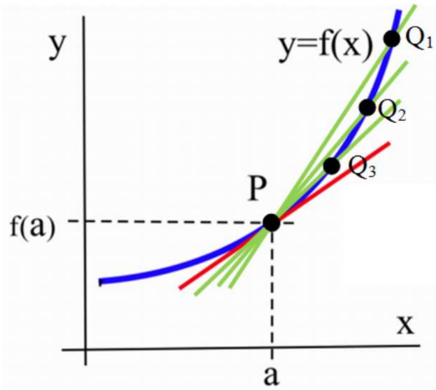
Definition: A **secant line** is a line between two points on a curve, as in the figure to the right.



Δ = change

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Can't-quite-do-it-yet Definition: A **tangent line** is a line at one point on a curve …. that does its best to be the curve at that point?



2.1

Section 1: Limits and Continuity

Memorize

> We say **"the limit of f(x), as x approaches c, is L "** and we write
> $\lim_{x \to c} f(x)$ **= L.** (The symbol " $\to$ " means "approaches" or "gets very close to.")

For MTH 261, this is enough. However, for more precise and advanced calculus, there is a formal definition of limit, which can be used to prove theorems of calculus.

The difficulty with our intuitive definition of limit, is that "approaches" or "gets close to" are not precise enough for rigorous mathematical proofs.

Example 1

Use the graph of y = f(x) in Fig. 2 to determine the following limits:

(a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 2} f(x)$

(c) $\lim_{x \to 3} f(x)$ (d) $\lim_{x \to 4} f(x)$

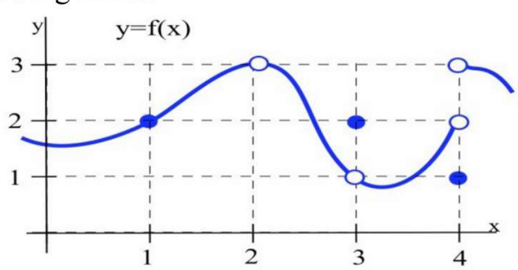


Fig. 2

(a) $\lim_{x \to 1} f(x) = 2$.

When x is very close to 1, the values of f(x) are very close to y = 2. In this example, it happens that f(1) = 2, but that is <u>irrelevant</u> for the limit. The only thing that matters is what happens for x close to 1 but $x \neq 1$.

(b) f(2) is undefined, but we only care about the behavior of f(x) for x <u>close to</u> 2 and <u>not equal to</u> 2. When x is close to 2, the values of f(x) are close to 3. If we restrict x close enough to 2, the values of y will be as close to 3 as we want, so $\lim_{x \to 2} f(x) = 3$.

(c) When x is close to 3 (or as x approaches the value 3), the values of f(x) are close to 1 (or approach the value 1), so $\lim_{x \to 3} f(x) = 1$. For this limit it is completely irrelevant that f(3) = 2, We only care about what happens to f(x) for x close to and not equal to 3.

(d) This one is harder and we need to be careful. When x is close to 4 and slightly **less than** 4 (x is just to the left of 4 on the x–axis), then the values of f(x) are close to 2. But if x is close to 4 and slightly **larger than** 4 then the values of f(x) are close to 3. If we only know that x is very close to 4, then we cannot say whether y = f(x) will be close to 2 or close to 3 — it depends on whether x is on the right or the left side of 4. In this situation, the f(x) values are not close to a <u>single</u> number so we say $\lim_{x \to 4} f(x)$ **does not exist**. It is irrelevant that f(4) = 1. The limit, as x approaches 4, would still be undefined if f(4) was 3 or 2 or anything else.

$\lim_{x \to 4} f(x)$ dne (does not exist)

$\lim_{x \to 4^-} f(x) = 2$

approach 4 from left

$\lim_{x \to 4^+} f(x) = 3$

from right

$2 \neq 3$

$\therefore$ $\lim_{x \to 4} f(x)$ dne

Therefore

memorize

Definition of Left and Right Limits:

The **left limit** as x approaches c of f(x) is L if the values of f(x) get as close to L as we want when x is very close to and **left of c**, x < **c**: $\lim_{x \to c^-} f(x) = L$.

The **right limit**, written with $\mathbf{x \to c^+}$, requires that x lie to the **right of c**, x > **c**: $\lim_{x \to c^+} f(x) = L$

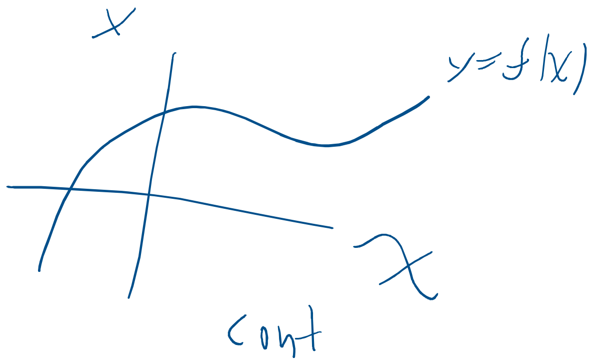
Memorize

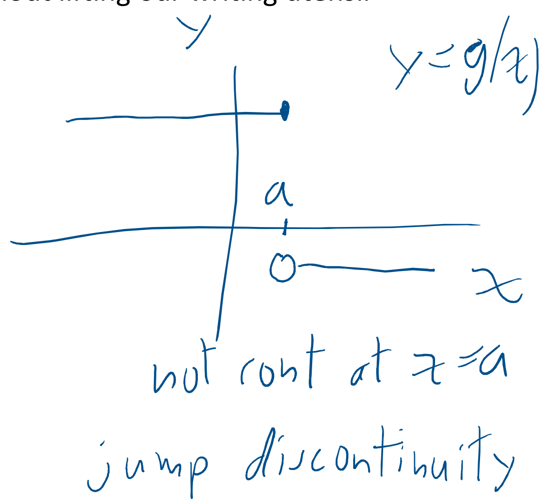
Definition of Continuity at a Point

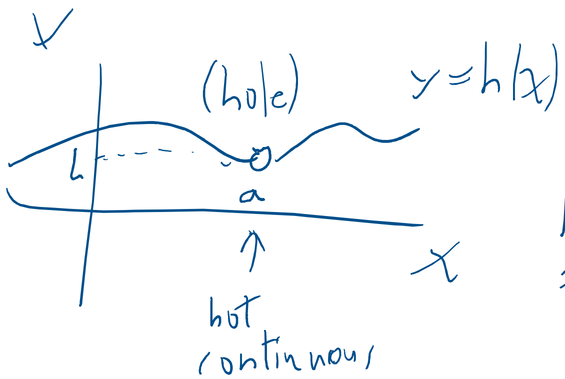
A function **f is continuous at x = a** if and only if $\lim_{x \to a} \mathbf{f(x)} = \mathbf{f(a)}$.

If a function is continuous at every point on some interval, we say it is continuous.

Informal definition: we can draw the graph without lifting our writing utensil







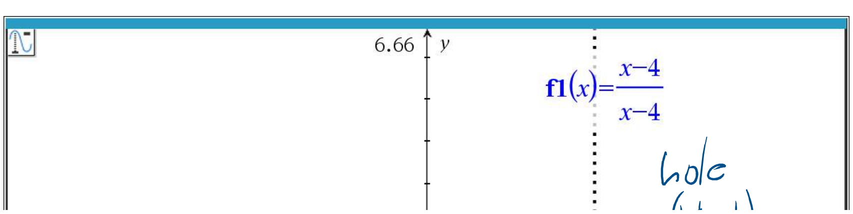
$$\lim_{x \to a} h(x) = L$$

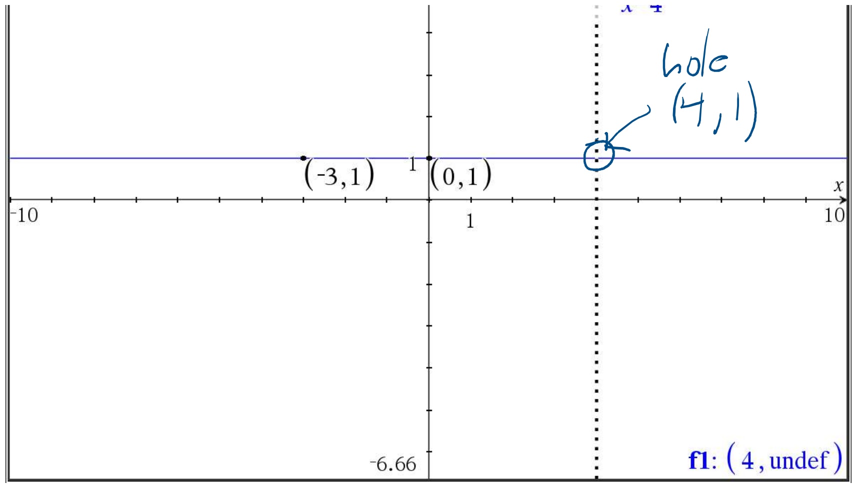
$$\lim_{x \to 3} (7x-1) = 7(3) - 1 = 21 - 1 = \boxed{20}$$

$$\lim_{x \to 4} \frac{x-4}{x-4}$$

$$\frac{x-4}{x-4} = 1 \text{ for } x \neq 4$$

$$\frac{4-4}{4-4} = \frac{0}{0}$$ not defined





$$\lim_{x \to 4} f(x) = 1$$

$f(4)$ not defined

$\therefore f(x)$ is not continuous at $x = 4$