12-04-25 MTH 167-004N

8 Systems of Equations and Matrices
8.1 Systems of Linear Equations: Gaussian Elimination
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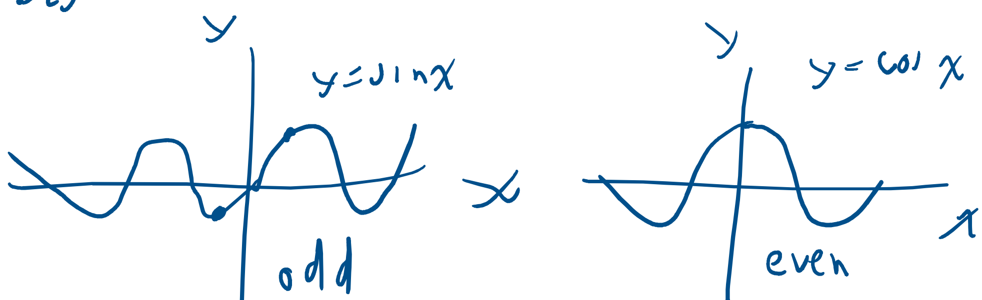
10.4.1 Exercises

In Exercises 1 - 6, use the Even / Odd Identities to verify the identity. Assume all quantities are defined.

1. $\sin(3\pi - 2\theta) = -\sin(2\theta - 3\pi)$

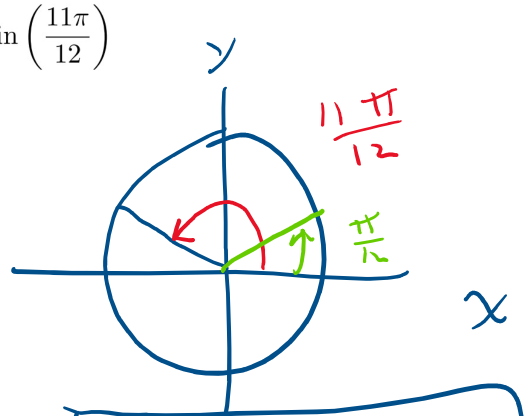
Def $f(x)$ is even if $f(-x) = f(x)$
Def $f(x)$ is odd if $f(-x) = -f(x)$



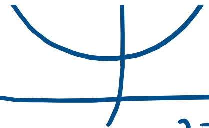
$\sin(3\pi - 2\theta)$
$= -\sin(-3\pi + 2\theta)$ [because sin is odd]
$= -\sin(2\theta - 3\pi)$ ✓

In Exercises 7 - 21, use the Sum and Difference Identities to find the exact value. You may have need of the Quotient, Reciprocal or Even / Odd Identities as well.

14. $\sin\left(\frac{11\pi}{12}\right)$



$\frac{11}{12} = \frac{10}{12} + \frac{1}{12} = \frac{5}{6} + \frac{1}{12}$
$= \frac{9}{12} + \frac{2}{12} = \frac{3}{4} + \frac{1}{6}$
$= \frac{8}{12} + \frac{3}{12} = \frac{2}{3} + \frac{1}{4}$
$= \frac{7}{12} + \frac{4}{12} = \frac{7}{12} + \frac{1}{3}$



$$\boxed{\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}}$$

$$= \frac{7}{12} + \frac{4}{12} = \frac{7}{12} + \frac{1}{3}$$

$$= \frac{6}{12} + \frac{5}{12} = \frac{1}{2} + \frac{5}{12}$$

$$\sin\left(\frac{\pi}{12}\right) = ?$$

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$

$$= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{-2}{2\sqrt{2}} =$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\begin{array}{r} 60 \\ -45 \\ \hline 15 \end{array}$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

Step 5: Simplify

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

✅ Final Answer:

$$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

We can evaluate $\sin\left(\frac{\pi}{12}\right)$ exactly using angle subtraction:

Step 1: Recognize the angle

$$\frac{\pi}{12} = 15^\circ$$

Step 2: Use the identity

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ)$$

Step 3: Apply the sine difference formula

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

So:

$$\sin(45^\circ - 30^\circ) = \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$$

Step 4: Substitute known values

- $\sin(45^\circ) = \frac{\sqrt{2}}{2}$
- $\cos(30^\circ) = \frac{\sqrt{3}}{2}$
- $\cos(45^\circ) = \frac{\sqrt{2}}{2}$
- $\sin(30^\circ) = \frac{1}{2}$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

Step 5: Simplify

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

✅ Final Answer:

$$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

10.4

In Exercises 26 - 38, verify the identity.

33. $\dfrac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \dfrac{1 + \cot(\alpha)\tan(\beta)}{1 - \cot(\alpha)\tan(\beta)}$

$$\left(\frac{1 + \cot(\alpha)\tan(\beta)}{1 - \cot(\alpha)\tan(\beta)}\right)\left(\frac{1 + \cot(\alpha)\tan(\beta)}{1 + \cot(\alpha)\tan(\beta)}\right)$$

$$\frac{(1 + \cot(\alpha)\tan(\beta))^2}{1 - (\cot(\alpha)\tan(\beta))^2}$$

$$\frac{1 + 2\cot(\alpha)\tan(\beta) + \cot^2(\alpha)\tan^2(\beta)}{1 - (\cot(\alpha)\tan(\beta))^2}$$

dead end?

33. $\dfrac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \dfrac{1 + \cot(\alpha)\tan(\beta)}{1 - \cot(\alpha)\tan(\beta)}$

33. $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{1+\cot(\alpha)\tan(\beta)}{1-\cot(\alpha)\tan(\beta)}$

$$\frac{1 + \frac{\cos(\alpha)}{\sin(\alpha)}\frac{\sin(\beta)}{\cos(\beta)}}{1 - \frac{\cos(\alpha)}{\sin(\alpha)}\frac{\sin(\beta)}{\cos(\beta)}}$$

$$\frac{\frac{\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)}{\cancel{\sin(\alpha)\cos(\beta)}}}{\frac{\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)}{\cancel{\sin(\alpha)\cos(\beta)}}}$$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$$

4.2: 14

In Exercises 1 - 16, use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

14. $f(x) = \dfrac{-x^3+4x}{x^2-9}$

$f(x)$ is not defined if $x^2 - 9 = 0$

$x^2 = 9$

$\boxed{x = \pm 3}$

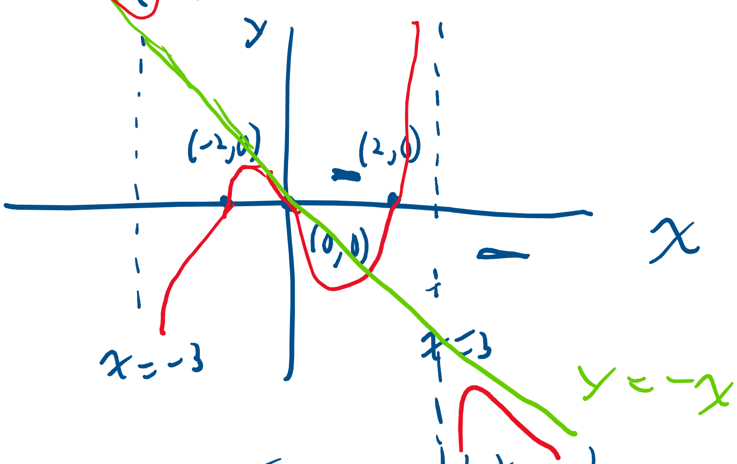
$$f(x) = \frac{-x(x^2-4)}{(x+3)(x-3)}$$

$$\boxed{f(x) = \frac{-x(x+2)(x-2)}{(x+3)(x-3)}}$$

$f(\pm 3)$ not defined

$f(\pm 3)$ not defined

$f(0) = f(\pm 2) = 0 \quad \nexists$ vert. asymp



slant asymptote because

deg of num = 3

deg of denom = 2

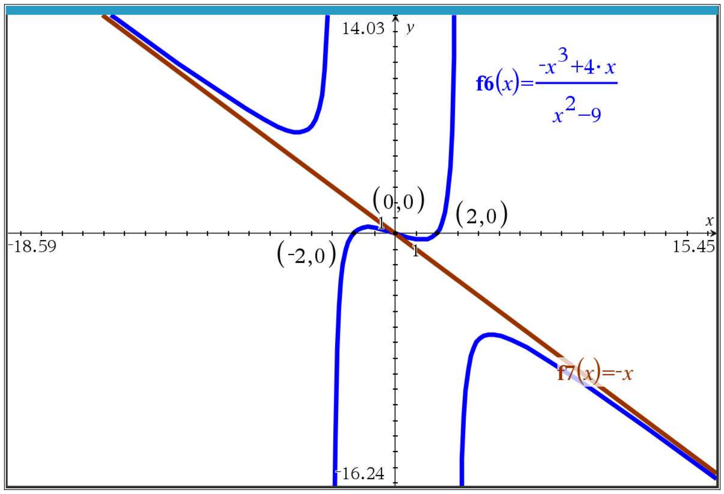
$3 - 2 = 1$

$$\begin{array}{r|l} & -x - \frac{5x}{x^2-9} \\ x^2-9 & -x^3 + 0x^2 + 4x + 0 \\ & \underline{-x^3 + 0x^2 + 9x} \\ & \qquad\qquad -5x \end{array}$$

$\boxed{y = -x}$ slant asymptote

$$f(1) = \frac{-(1^3) + 4(1)}{1^2 - 9} = \frac{3}{-8} < 0$$

$$f(4) = \frac{-4^3 + 4^h}{16 - 9} < 0$$


14.03
y
$f6(x)=\frac{-x^3+4\cdot x}{x^2-9}$
(0,0)
(2,0)
x
-18.59
(-2,0)
1
1
15.45
$f7(x)=-x$
-16.24