11.3 The Law of Cosines
11.3.1 Exercises
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11.4 Polar Coordinates
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8 Systems of Equations and Matrices
8.1 Systems of Linear Equations: Gaussian Elimination
8.1.1 Exercises
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8.2 Systems of Linear Equations: Augmented Matrices
8.2.1 Exercises
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Omit vectors

8.1

$$2x - 5y = 1$$
$$x + 5y = 2$$

Use any convenient combination of substitution and elimination.

Add the equations to eliminate y

$$3x = 3$$
$$\boxed{x = 1}$$

Substitute x into either equation to solve for y

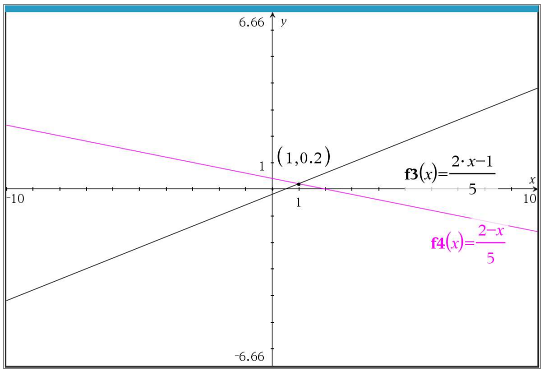
$$1 + 5y = 2$$
$$5y = 1$$
$$\boxed{y = \frac{1}{5}}$$

check $2(1) - 5\left(\frac{1}{5}\right) \stackrel{?}{=} 1$

$$2 - 1 \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

$$1 + 5\left(\frac{1}{5}\right) \stackrel{?}{=} 2$$
$$1 + 1 \stackrel{?}{=} 2$$
$$2 = 2 \checkmark$$

$$2x - 5y = 1 \Rightarrow y = \frac{2x - 1}{5}$$
$$x + 5y = 2 \Rightarrow y = \frac{2 - x}{5}$$



Memorize

Definition 8.1. A **linear equation in two variables** is an equation of the form $a_1x + a_2y = c$ where a_1, a_2 and c are real numbers and at least one of a_1 and a_2 is nonzero.

Memorize

Definition 8.2. A **linear equation in n variables**, $x_1, x_2, \ldots, x_n$, is an equation of the form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = c$ where $a_1, a_2, \ldots a_n$ and c are real numbers and at least one of a_1, $a_2, \ldots, a_n$ is nonzero.

Memorize

Theorem 8.1. Given a system of equations, the following moves will result in an equivalent system of equations.

- Interchange the position of any two equations.
- Replace an equation with a nonzero multiple of itself.[a]
- Replace an equation with itself plus a nonzero multiple of another equation.

[a] That is, an equation which results from multiplying both sides of the equation by the same nonzero number.

Supplied

Definition 8.3. A system of linear equations with variables x_1, x_2, $\ldots x_n$ is said to be in **triangular form** provided all of the following conditions hold:

1. The subscripts of the variables in each equation are always increasing from left to right.
2. The leading variable in each equation has coefficient 1.
3. The subscript on the leading variable in a given equation is greater than the subscript on the leading variable in the equation above it.
4. Any equation without variables[a] cannot be placed above an equation with variables.

[a] necessarily an identity or contradiction

memorize

Definition: A system of equations is consistent if it has at least one solution.
A system of equations is inconsistent if it has no solution.
A system of equations is consistent dependent if it has at least one solution containing at least one free variable

Memorize

Definition: a matrix is a rectangular array of (usually) numbers.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2x 3
row column

$$2x - 5y = 1$$
$$x + 5y = 2$$

$$\left[\begin{array}{cc|c} 2 & -5 & 1 \\ 1 & 5 & 2 \end{array}\right]$$

Augmented matrix

$$\left[\begin{array}{cc|c} 2 & -5 & 1 \\ 1 & 5 & 2 \end{array}\right]$$ Augmented matrix

Memorize

Theorem 8.2. Row Operations: Given an augmented matrix for a system of linear equations, the following row operations produce an augmented matrix which corresponds to an equivalent system of linear equations.

- Interchange any two rows.
- Replace a row with a nonzero multiple of itself.[a]
- Replace a row with itself plus a nonzero multiple of another row.[b]

[a] That is, the row obtained by multiplying each entry in the row by the same nonzero number.
[b] Where we add entries in corresponding columns.

Goal: $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$

$$1\cdot x + 0\cdot y = a$$
$$0\cdot x + 1\cdot y = b$$

$$\boxed{\begin{array}{l} x = a \\ y = b \end{array}}$$

$$\left[\begin{array}{cc|c} 2 & -5 & 1 \\ 1 & 5 & 2 \end{array}\right] R1 - 2R_2$$

$$\left[\begin{array}{cc|c} 2-2(1) & -5-2(5) & 1-2(2) \\ 1 & 5 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & -15 & -3 \\ 1 & 5 & 2 \end{array}\right] \text{interchange } R1, R2$$

$$\left[\begin{array}{cc|c} 1 & 5 & 2 \\ 0 & -15 & -3 \end{array}\right] R2/-15$$

$$\left[\begin{array}{cc|c} 1 & 5 & 2 \\ 0 & 1 & \frac{1}{5} \end{array}\right] R1-5R2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{5} \end{array}\right]$$ rref
reduced row echelon form

$$\boxed{\begin{aligned} x &= 1 \\ y &= \frac{1}{5} \end{aligned}}$$

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rref $\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

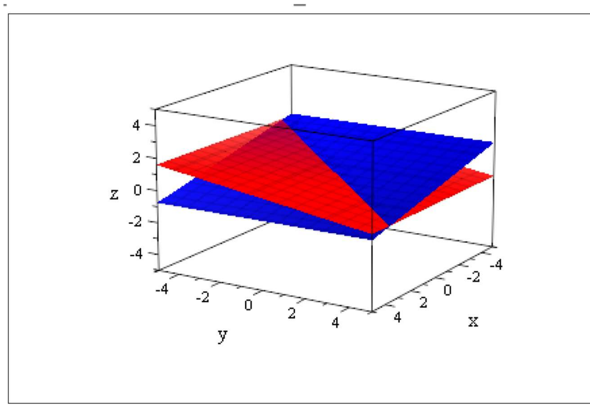
$x + 4z = 2$

$y + 5z = 3$

$0 \cdot x + 0 \cdot y + 0 \cdot z = 0$ (true for all x, y, z)

$$\boxed{\begin{aligned} x &= 2 - 4z \\ y &= 3 - 5z \\ z &= \text{free variable or parameter} \end{aligned}}$$

Parametric family of solutions: the set of all points on the intersection line of two planes in space.



rref $= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$z = 2$
$y = 3$
$0 \cdot x + 0 \cdot y + 0z = 1$
$0 = 1$ false

no solution
inconsistent system

Copilot

```
With systems that once seemed a chore,
We line up the rows to explore.
Swap, scale, and combine,
Step by step they align,
Till RREF opens the door!
```

Solve the system of equations by reducing the augmented matrix to rref, using elementary row operations. Then, check with calculator.

$$14. \begin{cases} x + y + z &= 3 \\ 2x - y + z &= 0 \\ -3x + 5y + 7z &= 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & -1 & 1 & 0 \\ -3 & 5 & 7 & 7 \end{array}\right] \begin{array}{l} \\ R2-2R1 \\ R3+3R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -3 & -1 & -6 \\ 0 & 8 & 10 & 16 \end{array}\right] \begin{array}{l} \\ \\ R3/2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -3 & -1 & -6 \end{array}\right. \begin{array}{l} \\ 4R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & \\ 0 & -3 & -1 & -6 \\ 0 & 4 & 5 & 8 \end{array}\right] \begin{array}{l} \\ 4R2 \\ 3R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -12 & -4 & -24 \\ 0 & 12 & 15 & 24 \end{array}\right] \begin{array}{l} \\ \\ R3+R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -12 & -4 & -24 \\ 0 & 0 & 11 & 0 \end{array}\right] \begin{array}{l} \\ R2/-4 \\ R3/11 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 6 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R1-R3 \\ R2-R3 \\ \end{array}$$

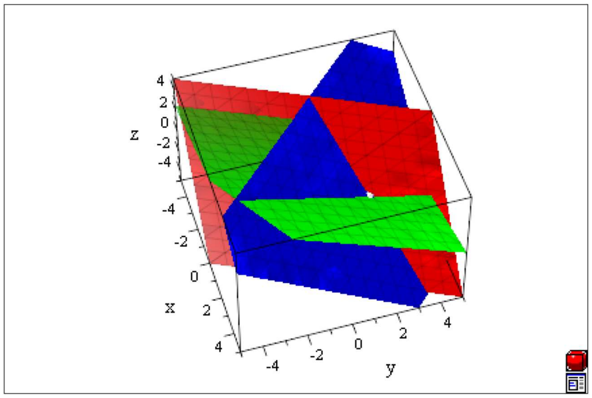
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ R2/3 \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R1-R2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x = 1$
$y = 2$

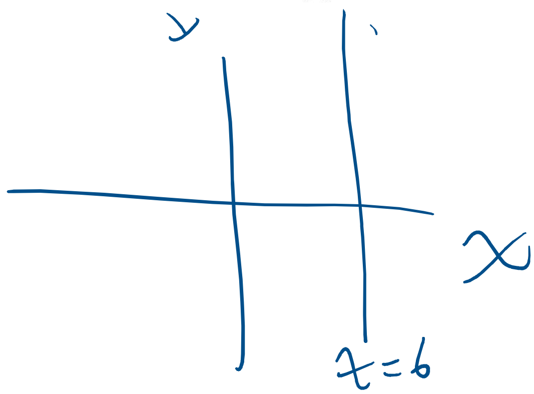
$$\boxed{\begin{aligned} y &= 2 \\ z &= 0 \end{aligned}}$$



11.4: 57

In Exercises 57 - 76, convert the equation from rectangular coordinates into polar coordinates. Solve for r in all but #60 through #63. In Exercises 60 - 63, you need to solve for θ

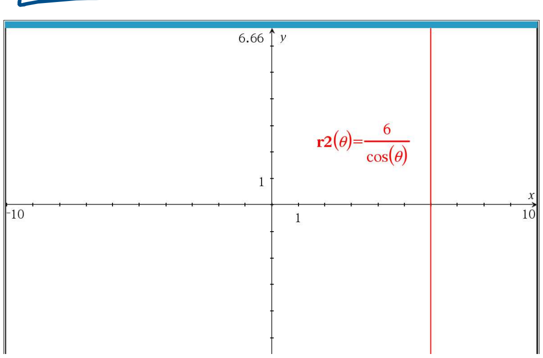
57. $x = 6$

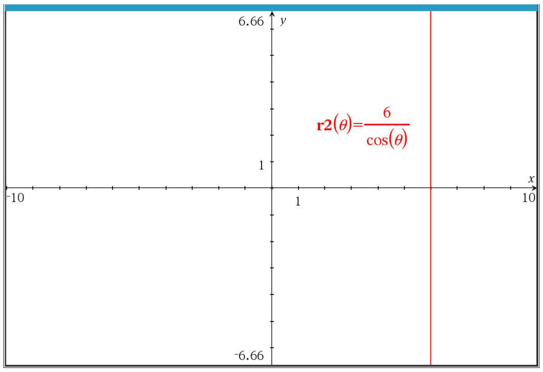


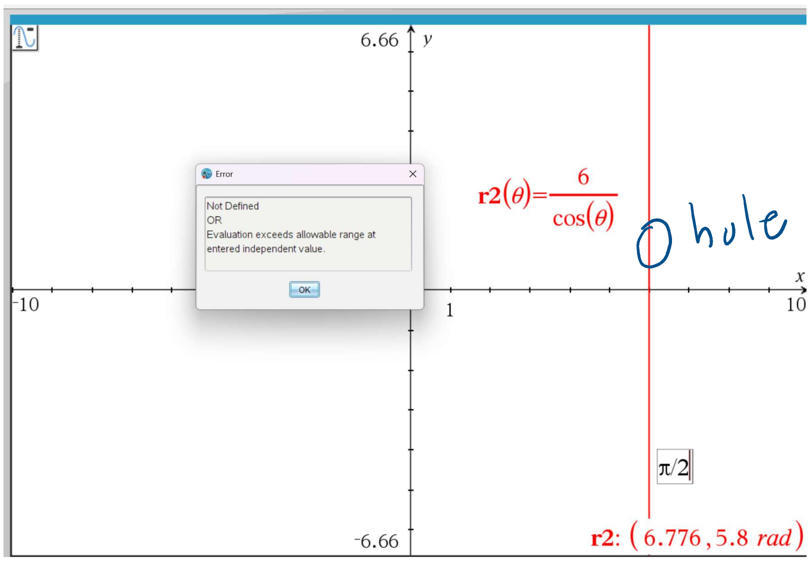
$$x = r\cos(\theta)$$

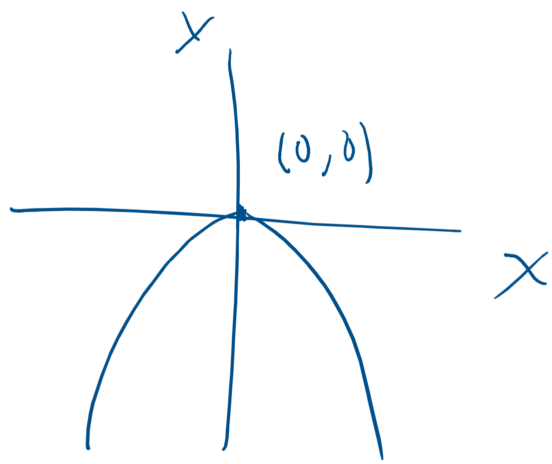
$$6 = r\cos(\theta)$$

$$\boxed{r = \frac{6}{\cos(\theta)}}$$









68. $y = -3x^2$

$$r\sin(\theta) = -3(r\cos\theta)^2$$

$$r\sin(\theta) = -3r^2\cos^2\theta$$

$$3r^2\cos^2(\theta) + r\sin(\theta) = 0$$

$$r(3r\cos^2(\theta) + \sin(\theta)) = 0$$

$$\boxed{r=0} \quad \text{or} \quad 3r\cos^2(\theta) + \sin(\theta) = 0$$

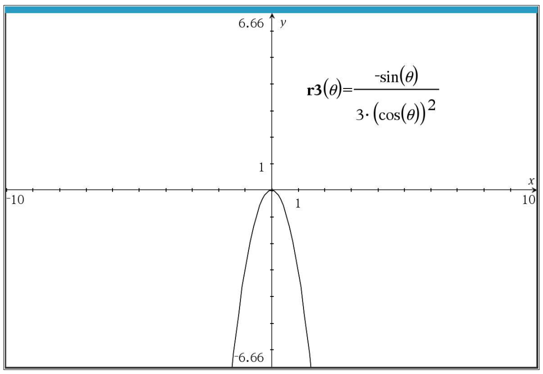
$$3r\cos^2(\theta) = -\sin(\theta)$$

$\boxed{r = 0}$

$$3r\cos^2(\theta) = -\sin(\theta)$$

$$\boxed{r = \frac{-\sin(\theta)}{3\cos^2(\theta)}} \checkmark$$

$$\theta = 0 \Rightarrow r = 0$$



Convert back to rectangular coordinates.

$$r = \frac{-\sin(\theta)}{3\cos^2(\theta)}$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$3r\cos^2(\theta) = -\sin\theta$$

$$3(r\cos\theta)(\cos\theta) = -\sin\theta$$

$$3x\cos\theta = -\sin\theta$$

$$3xr\cos\theta = -r\sin\theta$$

$$3x(x) = -y$$

$$3x^2 = -y$$

$$\boxed{y = -3x^2}$$