**10.7 Trigonometric Equations and Inequalities**
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**11 Applications of Trigonometry**
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**11.3 The Law of Cosines**
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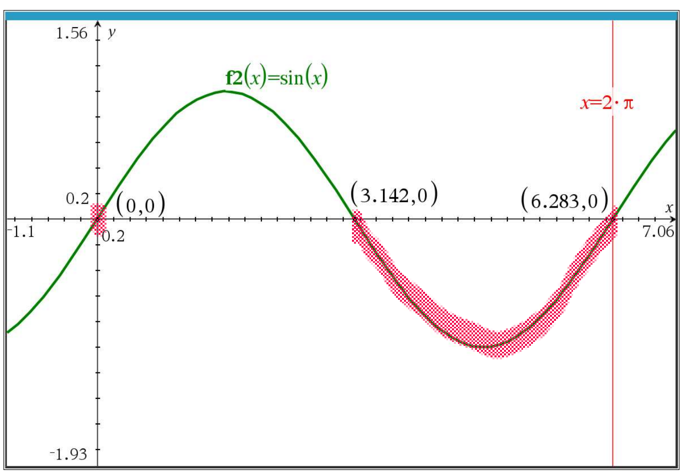
11.1:

In Exercises 69 - 80, solve the inequality. Express the exact answer in interval notation, restricting your attention to $0 \le x \le 2\pi$.

69. $\sin(x) \le 0$




solution set: $\{0\} \cup [\pi, 2\pi]$

Textbook answer:

69. $[\pi, 2\pi]$

10.7: 39

In Exercises 19 - 42, solve the equation, giving the exact solutions which lie in $[0, 2\pi)$

39. $\tan(2x) - 2\cos(x) = 0$

$$\frac{\sin(2x)}{\cos(2x)} - 2\cos(x) = 0 \quad , \text{ if } \cos(2x) \neq 0$$

$$\sin(2x) - 2(\cos(2x))(\cos x) = 0$$

$$2\sin(x)\cos(x) - 2(\cos^2(x) - \sin^2(x))(\cos(x)) = 0$$

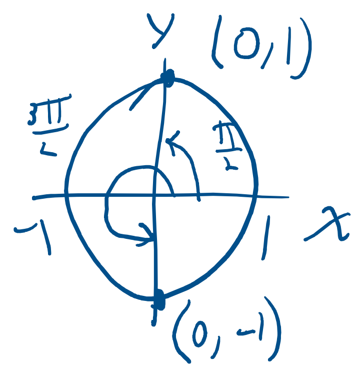
$$(\cos(x))((\sin(x)) - \cos^2(x) + \sin^2(x)) = 0$$

$$(\cos(x))(\sin(x) - (1 - \sin^2(x)) + \sin^2(x)) = 0$$

$$(\cos(x))(\sin(x) - 1 + \sin^2(x) + \sin^2(x)) = 0$$

$$(\cos(x))(2\sin^2(x) + \sin(x) - 1) = 0$$

$$\cos(x) = 0 \Rightarrow \boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$




Let $y = \sin(y)$

$$2y^2 + y - 1 = 0$$

$$y = \frac{-1 \pm \sqrt{1 - (4)(2)(-1)}}{4}$$

$$y = \frac{-1 \pm \sqrt{1 + 8}}{4}$$

$$y = \frac{-1 \pm 3}{4}$$

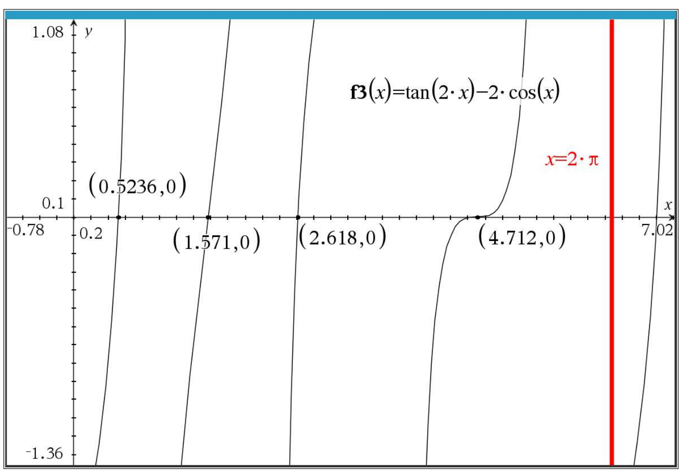
$$y = -\frac{4}{4}, \frac{2}{4}$$

$$\boxed{y = -1, \frac{1}{2}}$$

$\sin(x) = -1$ or $\sin(x) = \frac{1}{2}$

$$\boxed{x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{6}}$$




Pi/6=0.523598775598299
Pi/2=1.570796326794897
3*Pi/2=4.71238898038469
5*pi/6=2.617993877991493

11.3
Supplied

**Theorem 11.5. Law of Cosines:** Given a triangle with angle-side opposite pairs $(\alpha, a)$, $(\beta, b)$ and $(\gamma, c)$, the following equations hold

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha) \qquad b^2 = a^2 + c^2 - 2ac\cos(\beta) \qquad c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

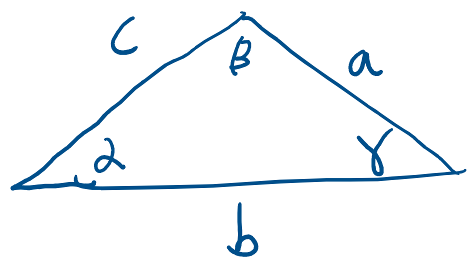
or, solving for the cosine in each equation, we have

$$\cos(\alpha) = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(\alpha) = \frac{b^2+c^2-a^2}{2bc} \qquad \cos(\beta) = \frac{a^2+c^2-b^2}{2ac} \qquad \cos(\gamma) = \frac{a^2+b^2-c^2}{2ab}$$

Let $\gamma = \frac{\pi}{2}$

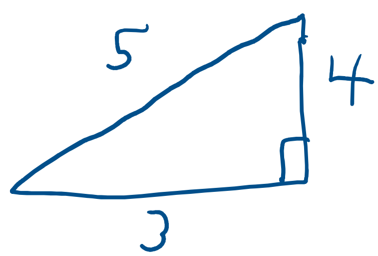
$$\Rightarrow \quad c^2 = a^2 + b^2 - 2ab\cos\left(\frac{\pi}{2}\right)$$
$$c^2 = a^2 + b^2 - 0$$
$$c^2 = a^2 + b^2$$




Supplied

**Theorem 11.6. Heron's Formula:** Suppose $a$, $b$ and $c$ denote the lengths of the three sides of a triangle. Let $s$ be the semiperimeter of the triangle, that is, let $s = \frac{1}{2}(a+b+c)$. Then the area $A$ enclosed by the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$




Area of $\Delta = \left(\frac{1}{2}\right)(3)(4) = \boxed{6}$

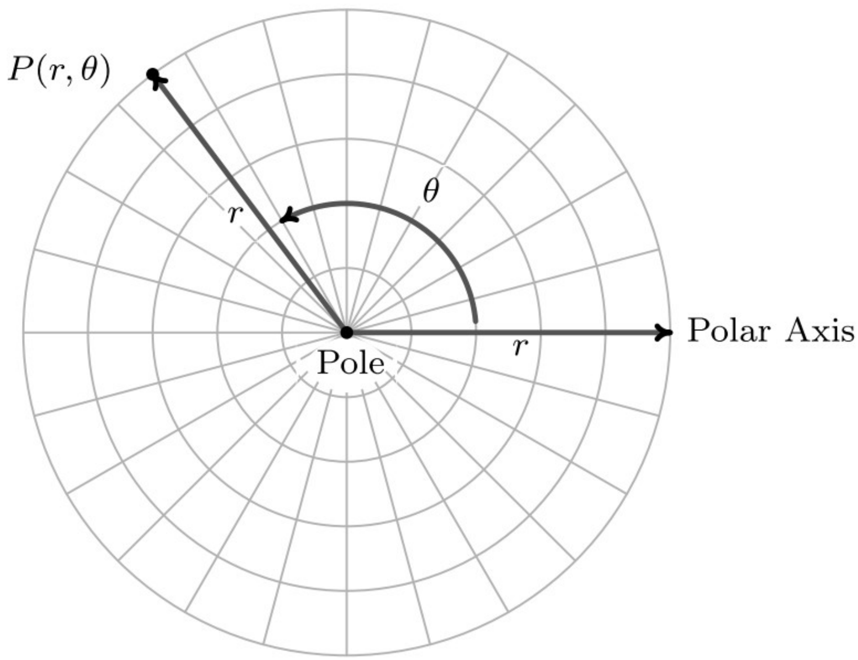
$$s = \left(\frac{1}{2}\right)(3+4+5) = \frac{1}{2}(12)$$

$$\boxed{s = 6}$$

$$A = \sqrt{6(6-3)(6-4)(6-5)}$$
$$= \sqrt{6(3)(2)(1)}$$
$$= \sqrt{6^2}$$
$$= \boxed{6}$$

$= \boxed{6}$

11.4
Memorize




$r$ = radial distance from the origin
$\theta$ = the angle from the positive x-axis

**Equivalent Representations of Points in Polar Coordinates**

Suppose $(r, \theta)$ and $(r', \theta')$ are polar coordinates where $r \neq 0$, $r' \neq 0$ and the angles are measured in radians. Then $(r, \theta)$ and $(r', \theta')$ determine the same point $P$ if and only if one of the following is true:

- $r' = r$ and $\theta' = \theta + 2\pi k$ for some integer $k$ (memorize)
- $r' = -r$ and $\theta' = \theta + (2k+1)\pi$ for some integer $k$ (supplied)

All polar coordinates of the form $(0, \theta)$ represent the pole regardless of the value of $\theta$.

Memorize

**Theorem 11.7. Conversion Between Rectangular and Polar Coordinates:** Suppose $P$ is represented in rectangular coordinates as $(x, y)$ and in polar coordinates as $(r, \theta)$. Then

- $x = r\cos(\theta)$ and $y = r\sin(\theta)$
- $x^2 + y^2 = r^2$ and $\tan(\theta) = \dfrac{y}{x}$ (provided $x \neq 0$)

11.4

In Exercises 57 - 76, convert the equation from rectangular coordinates into polar coordinates. Solve for $r$ in all but #60 through #63. In Exercises 60 - 63, you need to solve for $\theta$

63. $y = 2x$

$r\sin(\theta) = 2r\cos(\theta)$

$r\sin(\theta) - 2r\cos(\theta) = 0$

$r(\sin(\theta) - 2\cos(\theta)) = 0$

$\boxed{r=0}$ or $\sin(\theta) - 2\cos(\theta) = 0$

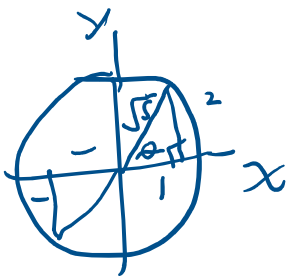
$\sin(\theta) = 2\cos(\theta)$

$\dfrac{\sin(\theta)}{\cos(\theta)} = 2$

$\tan(\theta) = 2$

$\boxed{\arctan(2),\ \arctan(2)+\pi}$

$\theta = \arctan(2)$




Copilot

Find the angle that the line makes with the positive x-axis.

To find the angle $\theta$ that the line $f(x) = 2x$ makes with the positive x-axis, we use the relationship between the slope $m$ of the line and the angle:

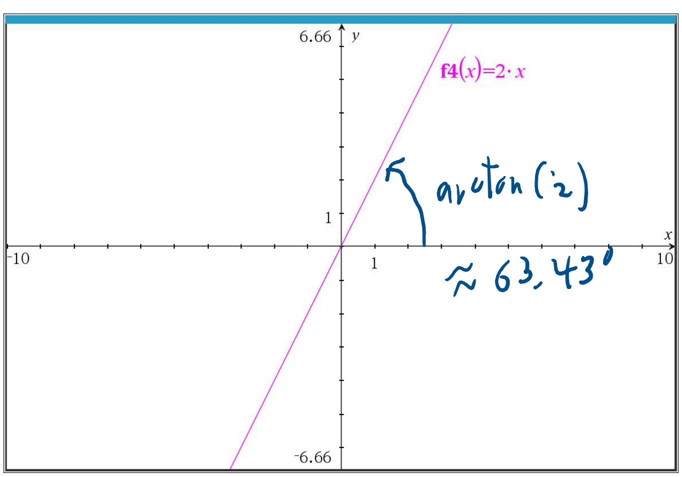
$$\theta = \tan^{-1}(m)$$

Since the slope $m = 2$, we compute:

$$\theta = \tan^{-1}(2) \approx 63.43^\circ$$

✅ So, the line $f(x) = 2x$ makes an angle of approximately **63.43 degrees** with the positive x-axis.

Would you like a quick mnemonic or visual aid to help students remember this connection between slope and angle?




73. $y^2 - 7y = x^2$ $\Rightarrow x^2 + (y^2 - 7y) = 0$

73. $y^2 = 7y - x^2$ $\Rightarrow$ $x^2+(y^2-7y)=0$

$$x^2+\left(y^2-7y+\frac{49}{4}\right)=\frac{49}{4}$$

$$x^2+\left(y-\frac{7}{2}\right)^2=\left(\frac{7}{2}\right)^2$$ $\Leftarrow$ circle with center $\left(0,\frac{7}{2}\right)$ radius $\frac{7}{2}$

$$r^2\sin^2(\theta) = 7r\sin(\theta) - r^2\cos^2(\theta)$$

$$r^2\sin^2(\theta) + r^2\cos^2(\theta) = 7r\sin(\theta)$$

$$r^2\left(\sin^2(\theta)+\cos^2(\theta)\right) = 7r\sin(\theta)$$

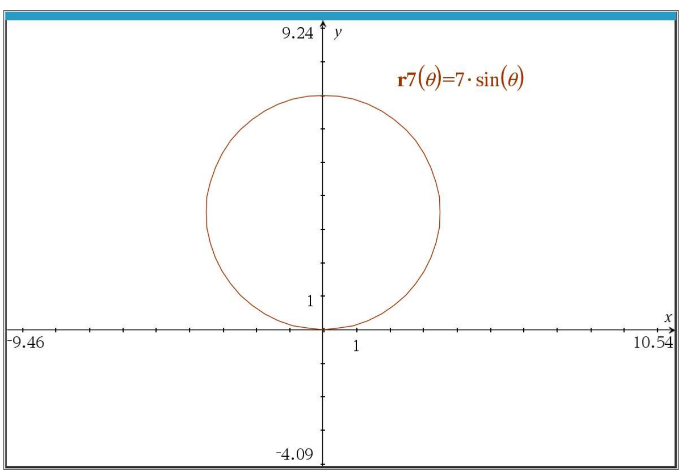
$$r^2 = 7r\sin(\theta)$$

$$r^2 - 7r\sin(\theta) = 0$$

$$r(r-7\sin\theta)=0$$

$\boxed{r=0}$ $\quad$ $\boxed{r=7\sin\theta}$ $\quad$ ($r=0$ is included in $r=7\sin\theta$

$r(0)=7\sin(0)$
$=0$




11.5

Memorize

**The Fundamental Graphing Principle for Polar Equations**

The graph of an equation in polar coordinates is the set of points which satisfy the equation. That is, a point $P(r,\theta)$ is on the graph of an equation if and only if there is a representation of $P$, say $(r',\theta')$, such that $r'$ and $\theta'$ satisfy the equation.

Memorize

**Theorem 11.8. Graphs of Constant $r$ and $\theta$:** Suppose $a$ and $\alpha$ are constants, $a \neq 0$.

- The graph of the polar equation $r = a$ on the Cartesian plane is a circle centered at the origin of radius $|a|$.
- The graph of the polar equation $\theta = \alpha$ on the Cartesian plane is the line containing the terminal side of $\alpha$ when plotted in standard position.

Supplied

**Guidelines for Finding Points of Intersection of Graphs of Polar Equations**
To find the points of intersection of the graphs of two polar equations $E_1$ and $E_2$:

- Sketch the graphs of $E_1$ and $E_2$. Check to see if the curves intersect at the origin (pole).
- Solve for pairs $(r, \theta)$ which satisfy both $E_1$ and $E_2$.
- Substitute $(\theta + 2\pi k)$ for $\theta$ in either one of $E_1$ or $E_2$ (but not both) and solve for pairs $(r, \theta)$ which satisfy both equations. Keep in mind that $k$ is an integer.
- Substitute $(-r)$ for $r$ and $(\theta + (2k+1)\pi)$ for $\theta$ in either one of $E_1$ or $E_2$ (but not both) and solve for pairs $(r, \theta)$ which satisfy both equations. Keep in mind that $k$ is an integer.

Quiz 7 open homework

10.7.1 Exercises

In Exercises 1 - 18, find all of the exact solutions of the equation and then list those solutions which are in the interval $[0, 2\pi)$.

1. $\sin(5x) = 0$

Make a labeled sketch from your calculator.

Let $u = 5x$

$\sin(u) = 0$

$\Rightarrow u = k\pi$

$\Rightarrow 5x = k\pi$

$\Rightarrow \boxed{x = \frac{k\pi}{5}, \; k \in \mathbb{Z}}$ all solutions

$\boxed{x = \frac{k\pi}{5}, \; k = 0, 1, 2, \ldots 9}$

solutions in $[0, 2\pi)$

| $k$ | $x$ |
|---|---|
| 0 | 0 |
| 1 | $\frac{\pi}{5}$ |
| 2 | $\frac{2\pi}{5}$ |
| 3 | $\frac{3\pi}{5}$ |
| 4 | $4\pi/5$ |
| 5 | $\pi$ |
| 6 | $6\pi/5$ |
| 7 | $7\pi/5$ |
| 8 | $8\pi/5$ |
| 9 | [illegible] |

Pi/5=0.628318530717959
2*Pi/5=1.256637061435917
⋮
9*Pi/5=5.654866776461628

| | |
|---|---|
| 7 | $7\pi/5$ |
| 8 | $8\pi/5$ |
| 9 | $9\pi/5$ |
| 10 | $2\pi \notin [0, 2\pi)$ |