10.5 Graphs of the Trigonometric Functions
10.5.4 Exercises
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10.6 The Inverse Trigonometric Functions
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11 Applications of Trigonometry
11.1 Applications of Sinusoids
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10 textbook sections
5 class meetings before final exam
2-3 sections/class + 1 day of review

10.5: 25

In Exercises 25 - 34, use Example 10.5.3 as a guide to show that the function is a sinusoid by rewriting it in the forms $C(x) = A\cos(\omega x + \phi) + B$ and $S(x) = A\sin(\omega x + \phi) + B$ for $\omega > 0$ and $0 \le \phi < 2\pi$.

25. $f(x) = \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1$

Properties of the Sinusoid $S(t) = A\sin(\omega t + \phi) + B$

- The **amplitude** is $|A|$
- The **angular frequency** is ω and the **ordinary frequency** is $f = \dfrac{\omega}{2\pi}$
- The **period** is $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- The **phase** is ϕ and the **phase shift** is $-\dfrac{\phi}{\omega}$
- The **vertical shift** or **baseline** is B

Find A, ω, ϕ, B

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$f(x) = A\sin(\omega x + \varphi) + B$

$f(x) = A[\sin(\omega x)\cos\varphi + \cos(\omega x)\sin\varphi] + B$

$f(x) = A\sin(\omega x)\cos\varphi + A\cos(\omega x)\sin\varphi + B$

$$\boxed{\begin{aligned} f(x) &= (A\cos\varphi)\sin(\omega x) + (A\sin\varphi)\cos(\omega x) + B \\ f(x) &= \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1 \end{aligned}}$$

$$f(x) = \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1$$

$B = 1$ $w = 1$	$A\cos\varphi = \sqrt{2}$	$A\sin\varphi = \sqrt{2}$

$$\Rightarrow A\cos\varphi = A\sin\varphi$$

$$\Rightarrow \cos\varphi = \sin\varphi$$

$$\Rightarrow \frac{\sin\varphi}{\cos\varphi} = 1$$

$$\Rightarrow \tan\varphi = 1$$

$$\boxed{\varphi = \frac{\pi}{4}}$$

$$A\sin\varphi = \sqrt{2}$$

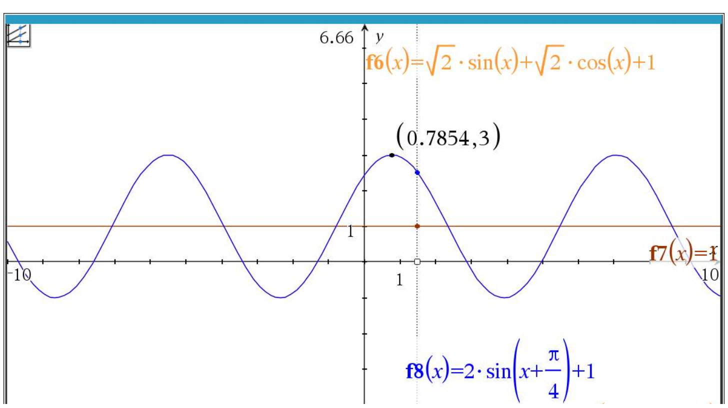
$$A\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$$

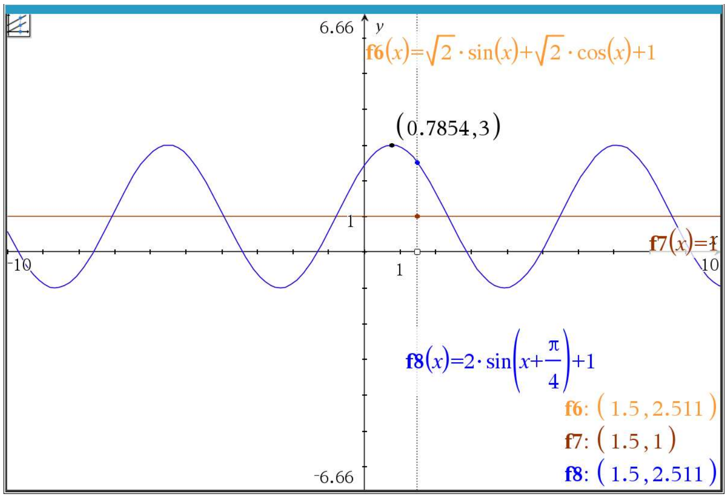
$$A = \frac{\sqrt{2}}{\sin\left(\frac{\pi}{4}\right)}$$

$$A = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}}$$

$$\boxed{A = 2}$$

$$\boxed{f(x) = 2\sin\left(x + \frac{\pi}{4}\right) + 1}$$





We graphed both forms of the function, and the graphs are identical

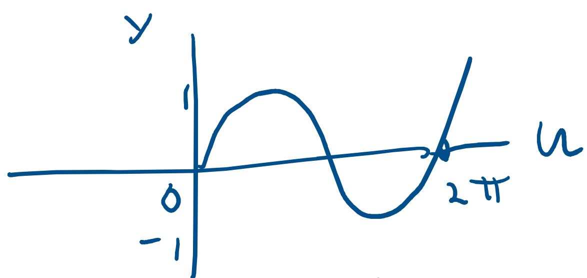
10.5.4 Exercises

In Exercises 1 - 12, graph one cycle of the given function. State the period, amplitude, phase shift and vertical shift of the function.

2. $y = \sin(3x)$

Let $u = 3x$

graph $y = \sin(u)$



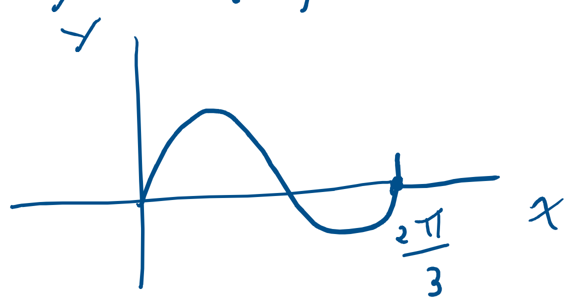
amplitude = 1

period = 2π

$-\frac{\varphi}{w} = 0$

$B = 0$

$y = \sin(3x)$

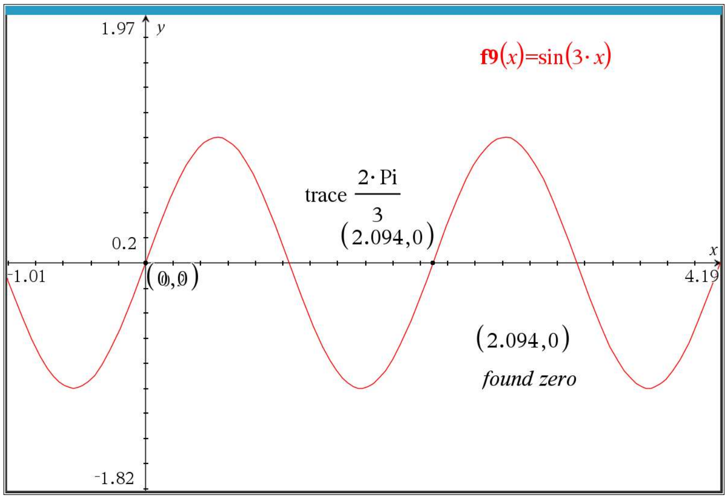


$B = 0$ $\quad w = 3$

$-\frac{\varphi}{w} = 0$ $\quad T = \frac{2\pi}{3}$

$A = 1$

$\frac{2\cdot\pi}{3}$ ▶Decimal $\qquad$ 2.0944



10.7 supplied

Strategies for Solving Basic Equations Involving Trigonometric Functions

- To solve $\cos(u) = c$ or $\sin(u) = c$ for $-1 \leq c \leq 1$, first solve for u in the interval $[0, 2\pi)$ and add integer multiples of the period 2π. If $c < -1$ or of $c > 1$, there are no real solutions.
- To solve $\sec(u) = c$ or $\csc(u) = c$ for $c \leq -1$ or $c \geq 1$, convert to cosine or sine, respectively, and solve as above. If $-1 < c < 1$, there are no real solutions.
- To solve $\tan(u) = c$ for any real number c, first solve for u in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and add integer multiples of the period π.
- To solve $\cot(u) = c$ for $c \neq 0$, convert to tangent and solve as above. If $c = 0$, the solution to $\cot(u) = 0$ is $u = \frac{\pi}{2} + \pi k$ for integers k.

solve $\sin(\theta) = 10$

$1 \leq \sin(\theta) \leq 1$

all θ

$\therefore$ No solution

10.7.1 Exercises

In Exercises 1 - 18, find all of the exact solutions of the equation and then list those solutions which are in the interval $[0, 2\pi)$.

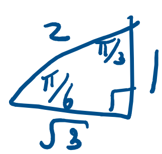
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

2. $\cos(3x) = \frac{1}{2}$

Let $u = 3x$

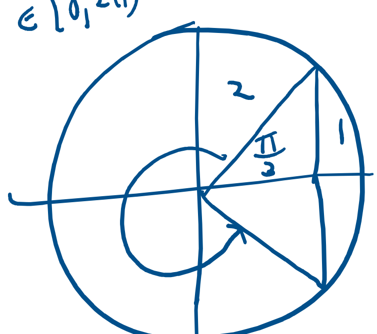
$\cos(u) = \frac{1}{2}$

$u = \frac{\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi)$



All solutions

$$\boxed{u = \frac{\pi}{3} + 2\pi k \qquad u = \frac{5\pi}{3} + 2\pi k}$$



$2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$

$3x = \frac{\pi}{3} + 2\pi k$

$$2\pi - \frac{\pi}{3} = \frac{6\pi-\pi}{3} = \frac{5\pi}{3}$$

$$3x = \frac{\pi}{3} + 2\pi k$$

$$3x = \frac{5\pi}{3} + 2\pi k$$

$$\boxed{\begin{aligned} x &= \frac{\pi}{9} + \frac{2\pi k}{3} \\ x &= \frac{5\pi}{9} + \frac{2\pi k}{3} \end{aligned}}$$ all solutions

$k=0$ $\quad x = \boxed{\frac{\pi}{9}, \frac{5\pi}{9}} \in (0, 2\pi)$

$k=-1$ $\quad x = \frac{\pi}{9} - \frac{2\pi}{3}, \quad \frac{5\pi}{9} - \frac{2\pi}{3}$

$x = \frac{\pi}{9} - \frac{6\pi}{9}, \quad \frac{5\pi}{9} - \frac{6\pi}{9}$

$x = -\frac{5\pi}{9}, \quad -\frac{\pi}{9} \notin (0, 2\pi)$

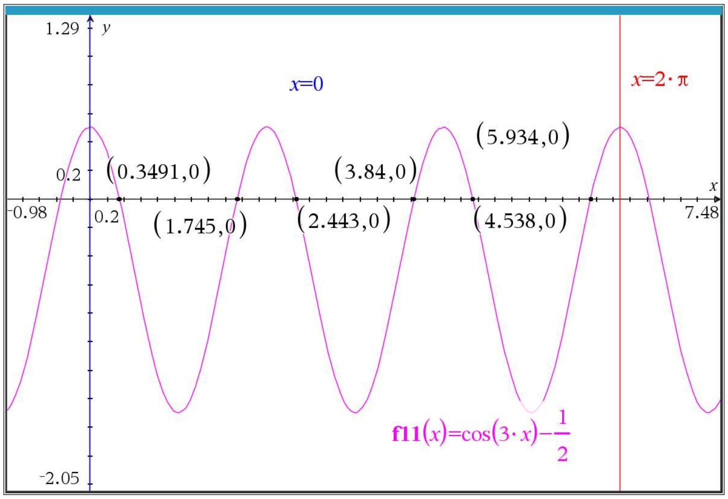
$k=1$ $\quad x = \frac{\pi}{9} + \frac{6\pi}{9}, \quad \frac{5\pi}{9} + \frac{6\pi}{9}$

$x = \boxed{\frac{7\pi}{9}, \frac{11\pi}{9}} \in (0, 2\pi)$

$k=2$ $\quad x = \frac{\pi}{9} + \frac{4\pi}{3}, \quad \frac{5\pi}{9} + \frac{4\pi}{3}$

$= \frac{\pi}{9} + \frac{12\pi}{9}, \quad \frac{5\pi}{9} + \frac{12\pi}{9}$

$= \boxed{\frac{13\pi}{9}, \frac{17\pi}{9}} \in [0, 2\pi)$



Pi/9=0.349065850398866

10.7

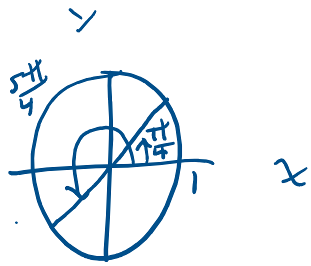
In Exercises 19 - 42, solve the equation, giving the exact solutions which lie in $[0, 2\pi)$

19. $\sin(x) = \cos(x)$

$\frac{\sin(x)}{\cos(x)} = 1$

$\tan(x) = 1$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$



26. $\cos(2x) = 5\sin(x) - 2$

$\cos^2(x) - \sin^2(x) = 5\sin(x) - 2$

$(1 - \sin^2(x)) - \sin^2(x) = 5\sin(x) - 2$

$-2\sin^2(x) - 5\sin(x) + 3 = 0$

Let $u = \sin(x)$

$-2u^2 - 5u + 3 = 0$

$2u^2 + 5u - 3 = 0$

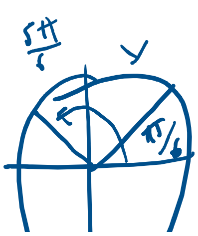
$(2u - 1)(u + 3) = 0$

$2u - 1 = 0$ or $u + 3 = 0$

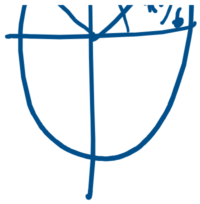
$2u = 1$

$\boxed{u = \frac{1}{2}}$ or $\boxed{u = -3}$

$\sin(x) = \frac{1}{2}$ or $\sin(x) = -3$
no solution



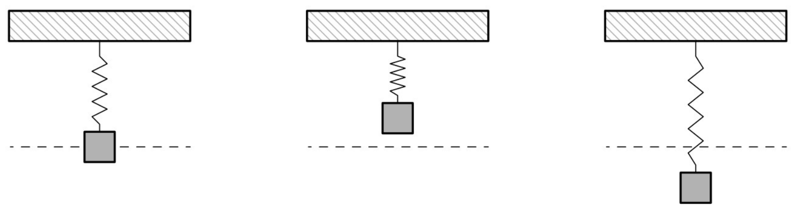
$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$



x $\boxed{t = \frac{\pi}{6}, \frac{5\pi}{6}}$

Theorem 11.1. Equation for Free Undamped Harmonic Motion: Suppose an object of mass m is suspended from a spring with spring constant k. If the initial displacement from the equilibrium position is x_0 and the initial velocity of the object is v_0, then the displacement x from the equilibrium position at time t is given by $x(t) = A\sin(\omega t + \phi)$ where

- $\omega = \sqrt{\frac{k}{m}}$ and $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$
- $A\sin(\phi) = x_0$ and $A\omega\cos(\phi) = v_0$.



$x(t) = 0$ at the equilibrium position | $x(t) < 0$ above the equilibrium position | $x(t) > 0$ below the equilibrium position

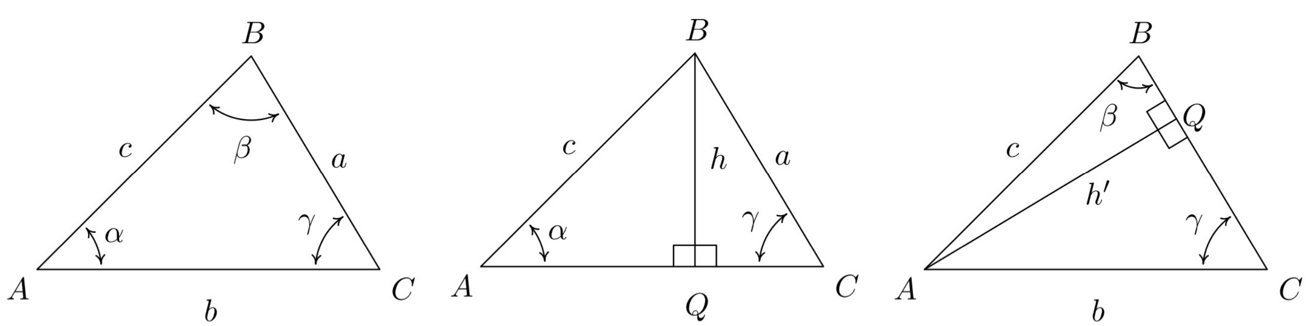
11.2
Memorize

Theorem 11.2. The Law of Sines: Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following ratios hold

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

or, equivalently,

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$



$\sin(\alpha) = \frac{\text{opp}}{\text{hyp}} = \frac{h}{c} \Rightarrow h = c\sin(\alpha)$

$$\sin(\alpha) = \frac{opp}{hyp} = \frac{h}{c} \Rightarrow h = c\sin(\alpha)$$

ΔAQB

$$\sin(\gamma) = \frac{opp}{hyp} = \frac{h}{a} \Rightarrow h = a\sin(\gamma)$$

ΔQCB

$$\Rightarrow c\sin(\alpha) = a\sin(\gamma)$$

$$\Rightarrow \frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c}$$

Supplied

Theorem 11.3. Suppose (α, a) and (γ, c) are intended to be angle-side pairs in a triangle where α, a and c are given. Let $h = c\sin(\alpha)$

- If $a < h$, then no triangle exists which satisfies the given criteria.
- If $a = h$, then $\gamma = 90°$ so exactly one (right) triangle exists which satisfies the criteria.
- If $h < a < c$, then two distinct triangles exist which satisfy the given criteria.
- If $a \geq c$, then γ is acute and exactly one triangle exists which satisfies the given criteria