11-06-25 MTH 167-004N

6.5 Applications of Exponential and Logarithmic Functions
  6.5.3 Exercises
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7 Hooked on Conics
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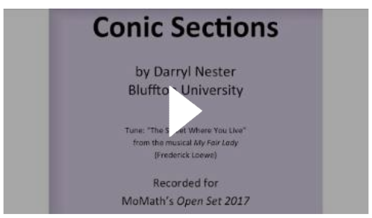
"Conic sections" Song (with vocals)



6.5: 27

27. The population of Sasquatch in Bigfoot county is modeled by

$$P(t) = \frac{120}{1 + 3.167e^{-0.05t}}$$

where $P(t)$ is the population of Sasquatch t years after 2010.

(a) Find and interpret $P(0)$.
(b) Find the population of Sasquatch in Bigfoot county in 2013. Round your answer to the nearest Sasquatch.
(c) When will the population of Sasquatch in Bigfoot county reach 60? Round your answer to the nearest year.
(d) Find and interpret the end behavior of the graph of $y = P(t)$. Check your answer using a graphing utility.

$$P(0) = \frac{120}{1 + 3.167 e^{-(0.05)(0)}}$$

$$= \frac{120}{1 + 3.167} = \frac{120}{4.167} \approx 29$$

120/4.167=28.7977

Interpretation: The initial population is 29

120/4.167=28.7977

Interpretation: The initial population is 29

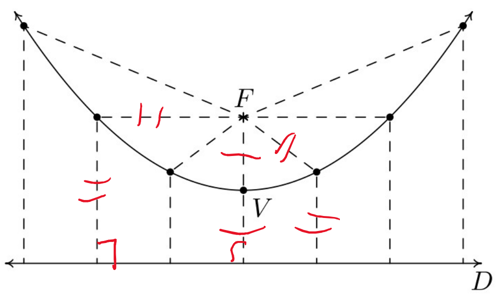
(b) Find P(3)

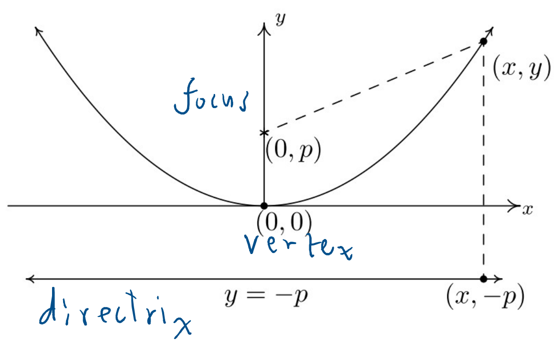
(c) Find t such that P(t) = 60

7.3

Supplied locus definition

Definition 7.3. Let F be a point in the plane and D be a line not containing F. A **parabola** is the set of all points equidistant from F and D. The point F is called the **focus** of the parabola and the line D is called the **directrix** of the parabola.





Supplied

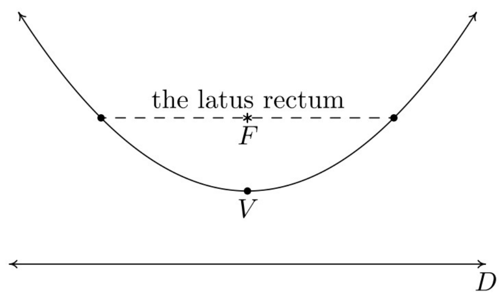
Equation 7.2. The Standard Equation of a Vertical[a] Parabola: The equation of a (vertical) parabola with vertex (h, k) and focal length $|p|$ is

$$(x-h)^2 = 4p(y-k)$$

If $p > 0$, the parabola opens upwards; if $p < 0$, it opens downwards.

[a]That is, a parabola which opens either upwards or downwards.

The latus rectum of a parabola is the line segment parallel to the directrix which contains the focus. The endpoints of the latus rectum are, then, two points on ‘opposite’ sides of the parabola.



Supplied

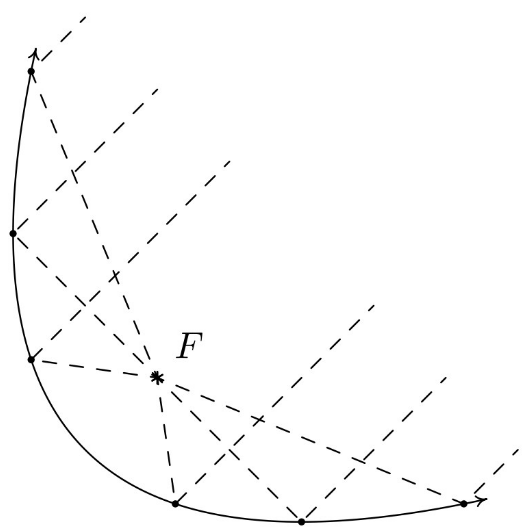
Equation 7.3. The Standard Equation of a Horizontal Parabola: The equation of a (horizontal) parabola with vertex (h, k) and focal length $|p|$ is

$$(y-k)^2 = 4p(x-h)$$

If $p > 0$, the parabola opens to the right; if $p < 0$, it opens to the left.

To Write the Equation of a Parabola in Standard Form

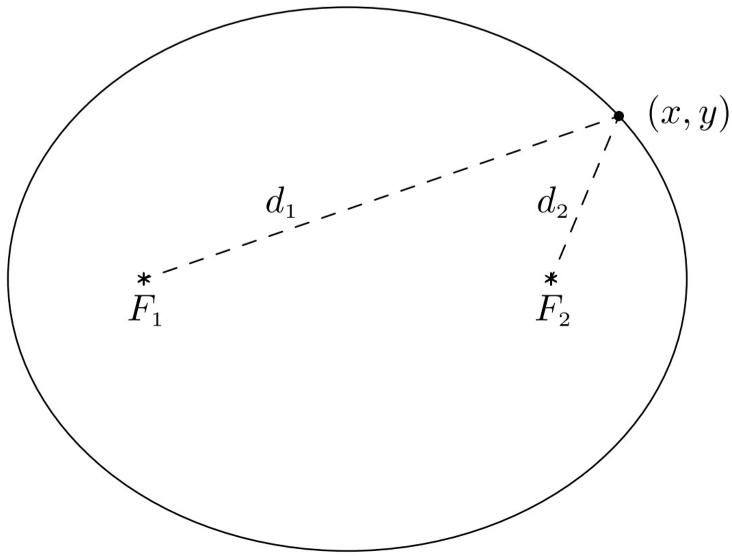
1. Group the variable which is squared on one side of the equation and position the non-squared variable and the constant on the other side.
2. Complete the square if necessary and divide by the coefficient of the perfect square.
3. Factor out the coefficient of the non-squared variable from it and the constant.



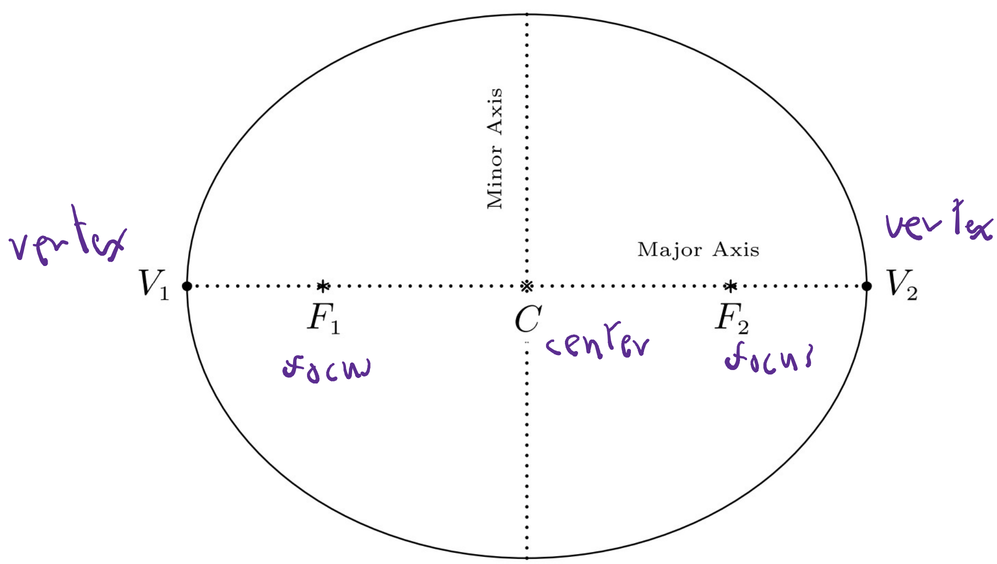
7.4
Supplied

Definition 7.4. Given two distinct points F_1 and F_2 in the plane and a fixed distance d, an **ellipse** is the set of all points (x, y) in the plane such that the sum of each of the distances from F_1 and F_2 to (x, y) is d. The points F_1 and F_2 are called the **foci**[a] of the ellipse.

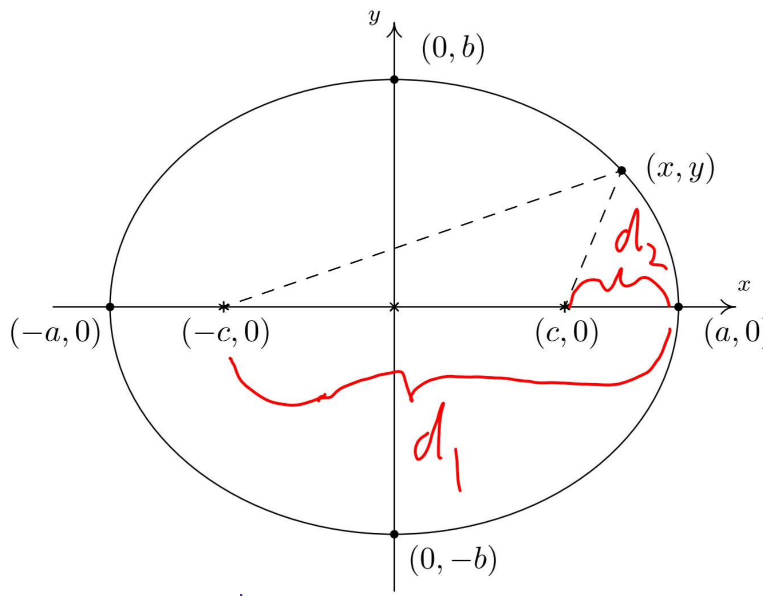
[a]the plural of 'focus'



$d_1 + d_2 = d$ for all (x, y) on the ellipse



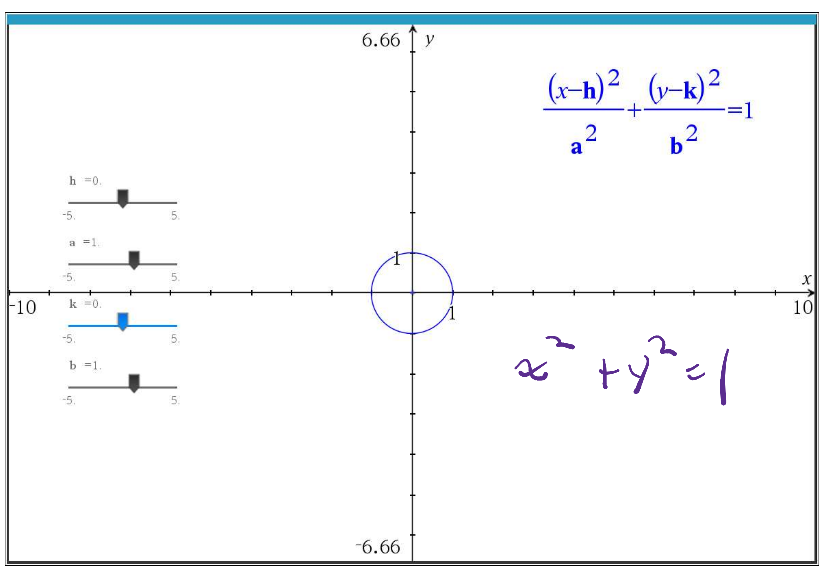
An ellipse with center C; foci F_1, F_2; and vertices V_1, V_2

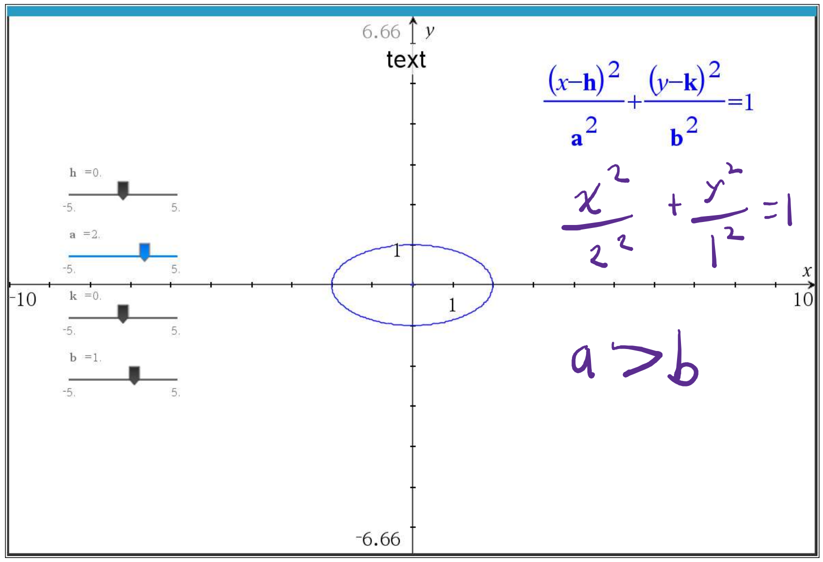


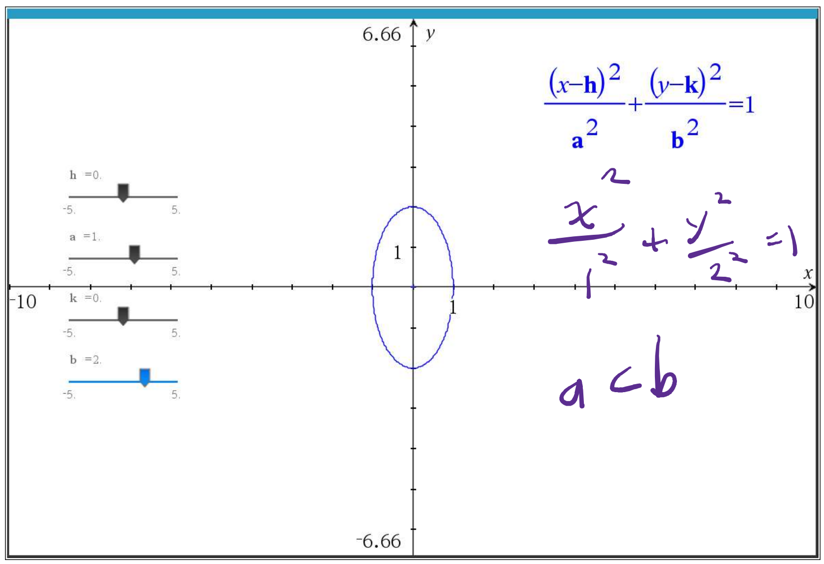
$$d_1 + d_2 = (a+c) + (a-c)$$
$$= 2a = d$$

Supplied

> **Equation 7.4. The Standard Equation of an Ellipse:** For positive unequal numbers a and b, the equation of an ellipse with center (h, k) is
>
> $$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$







Equation 7.5. The Alternate Standard Equation of a Circle: The equation of a circle with center (h, k) and radius $r > 0$ is

$$\frac{(x-h)^2}{r^2}+\frac{(y-k)^2}{r^2}=1$$

To Write the Equation of an Ellipse in Standard Form

1. Group the same variables together on one side of the equation and position the constant on the other side.
2. Complete the square in both variables as needed.
3. Divide both sides by the constant term so that the constant on the other side of the equation becomes 1.

Supplied

Definition 7.5. The **eccentricity** of an ellipse, denoted e, is the following ratio:

$$e = \frac{\text{distance from the center to a focus}}{\text{distance from the center to a vertex}}$$

7.3

$e = \frac{c}{a}$

In Exercises 9 - 14, put the equation into standard form and identify the vertex, focus and directrix.

12. $2y^2 + 4y + x - 8 = 0$

Equation 7.3. The Standard Equation of a Horizontal Parabola: The equation of a (horizontal) parabola with vertex (h, k) and focal length $|p|$ is

$$(y - k)^2 = 4p(x - h)$$

If $p > 0$, the parabola opens to the right; if $p < 0$, it opens to the left.

$$(2y^2 + 4y) + x - 8 = 0$$

$$2(y^2 + 2y) + x - 8 = 0$$

$$2(y^2 + 2y + 1 - 1) + x - 8 = 0$$

$$2(y^2 + 2y + 1) - 2 + x - 8 = 0$$

$$2(y + 1)^2 = -x + 10$$

$$(y + 1)^2 = \frac{-x + 10}{2}$$

$$(y - (-1))^2 = \left(-\frac{1}{2}\right)(x - 10)$$

$$\boxed{(y - (-1))^2 = (4)\left(-\frac{1}{8}\right)(x - 10)}$$

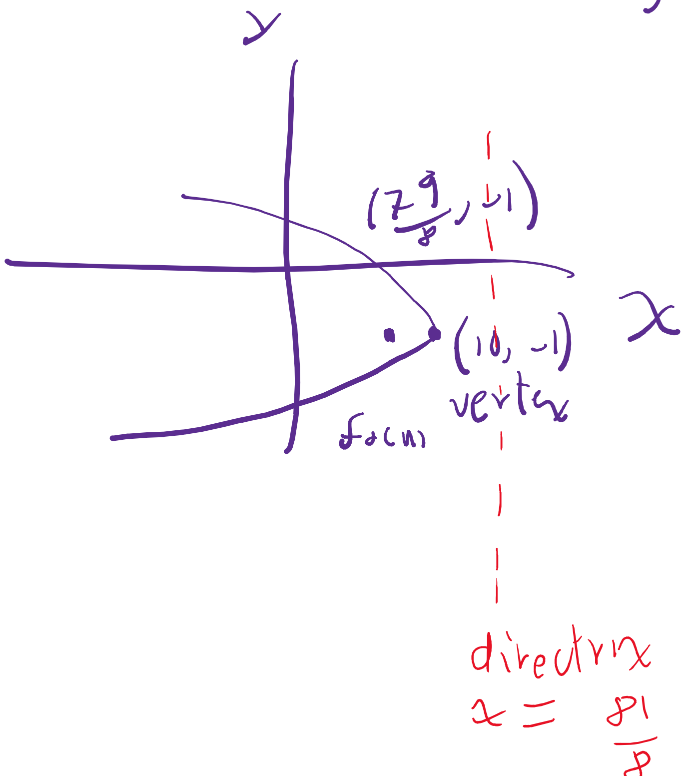
$$(y - k)^2 = 4p(x - h)$$

$-\frac{1}{2} = 4p$

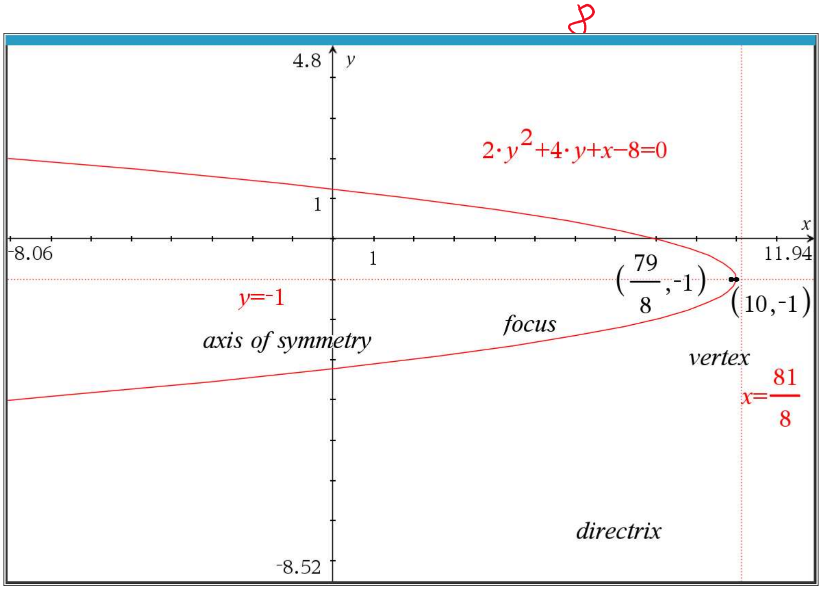
$p = -\frac{1}{8}$

vertex $= (h, k)$

vertex $= (h, k)$
$= (10, -1)$



$10 - \frac{1}{8}$
$= \frac{80 - 1}{8} = \frac{79}{8}$



Your Name MTH 167-004N Bonus quiz 2

(1) In Exercises 16 - 29, use the properties of logarithms to write the expression as a single logarithm.

$\log_5(x) - 3$

$3 = \log_5(p)$

$\log_5(x)$

$\log_5(x) - \log_5(125)$

$= \boxed{\log_5\left(\frac{x}{125}\right)}$

$3 = \log_5(P)$

$5^3 = P = 125$

(2) Solve analytically.

29. $e^{2x} = e^x + 6$

Let $y = e^x$

$\Rightarrow y^2 = e^{2x}$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

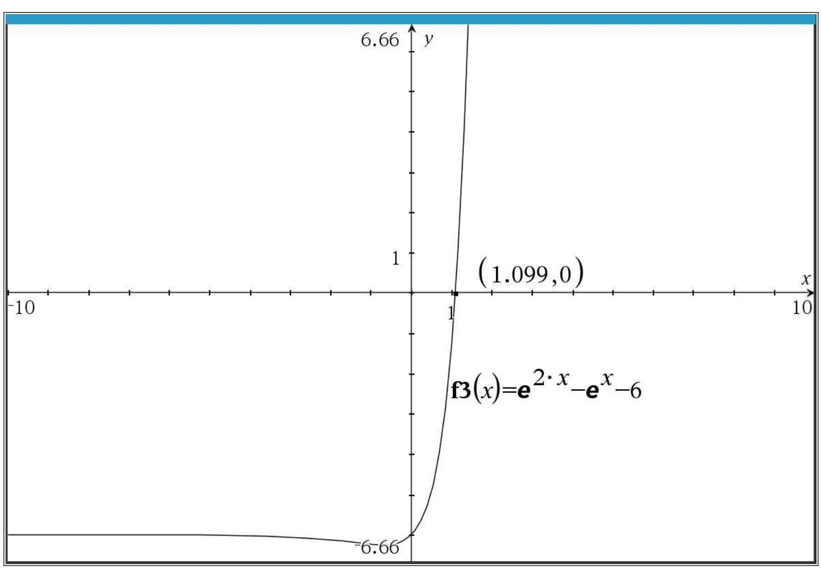
$\boxed{y = 3, -2}$

$e^x = 3$ or $e^x = -2$ (no solution)

$\ln(e^x) = \ln(3)$ $\quad e^x > 0$ all x

$\boxed{x = \ln(3)}$

(3) Solve #2 graphically. Sketch a labeled graph.



(ln(3))▸Decimal 1.09861