11-04-25 MTH 167-004N

6.3 Exponential Equations and Inequalities
6.3.1 Exercises
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6.4 Logarithmic Equations and Inequalities
6.4.1 Exercises
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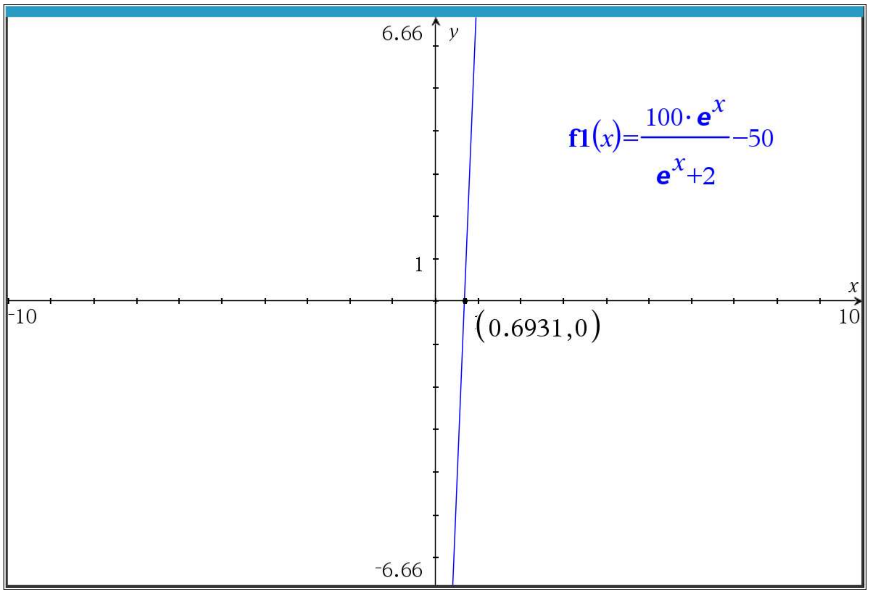
6.3:19

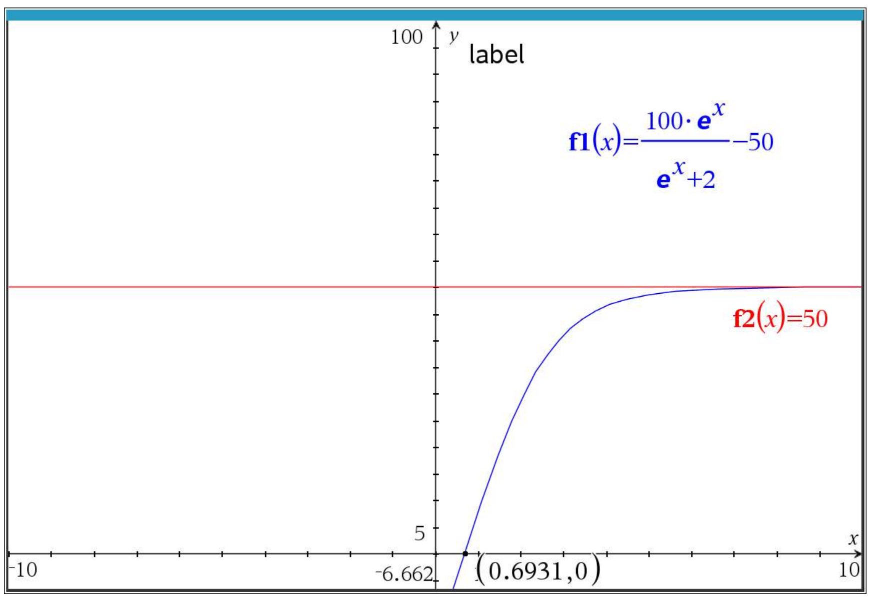
In Exercises 1 - 33, solve the equation analytically.

19. $\dfrac{100e^x}{e^x+2} = 50$

$$100e^x = 50e^x + 100$$
$$2e^x = e^x + 2$$
$$e^x = 2$$
$$\ln(e^x) = \ln(2)$$
$$\boxed{x = \ln(2)}$$

$(\ln(2))$ ▸ Decimal 0.693147





6.3: 36

In Exercises 34 - 39, solve the inequality analytically.

36. $2^{(x^3-x)} < 1$

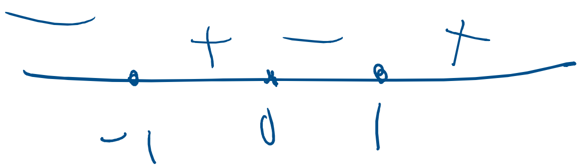
$2^{(x^3-x)} < 2^0$

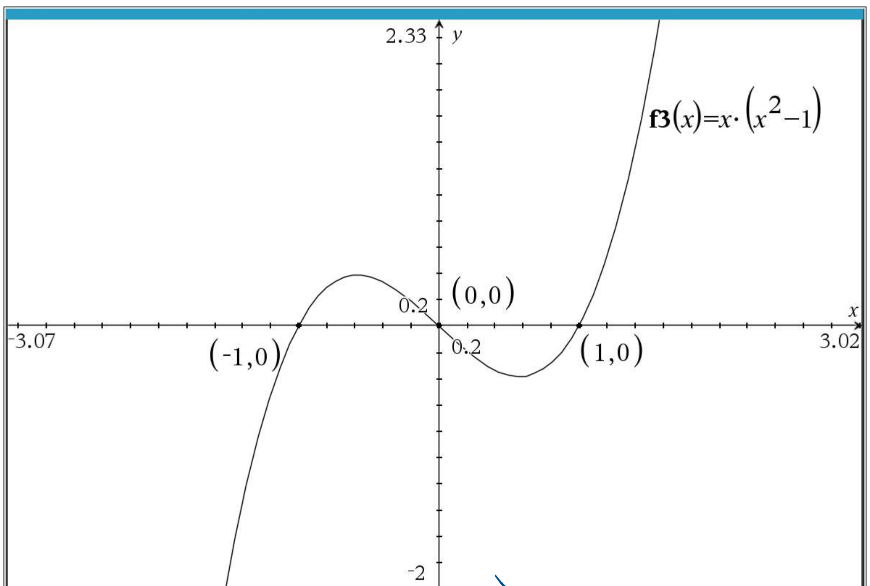
$x^3 - x < 0$

$x(x^2-1)<0$

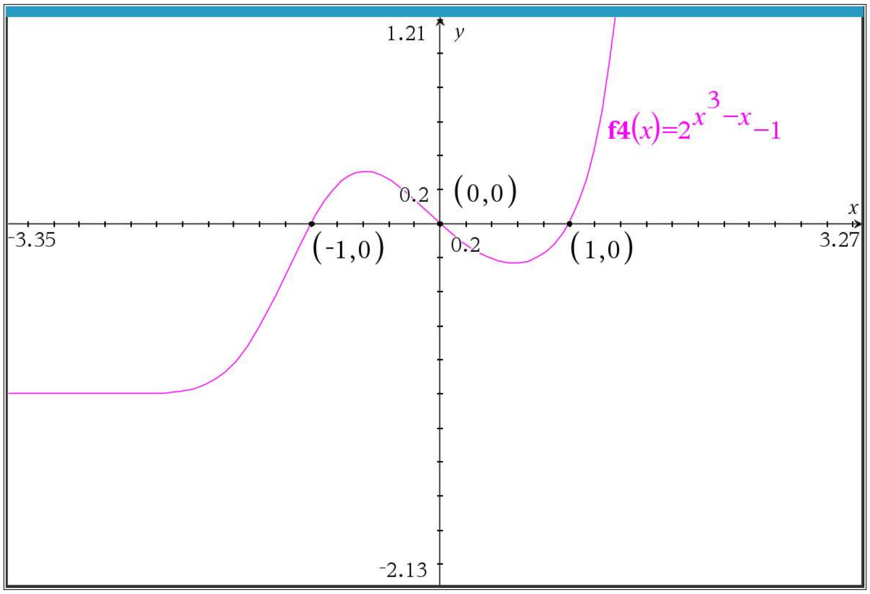
solve $x(x^2-1)=0$

$\boxed{x=0,\ \pm 1}$





$(-\infty,-1)\cup(0,1)$



6.3: 44

In Exercises 40 - 45, use your calculator to help you solve the equation or inequality.

44. $3^{(x-1)} < 2^x$

$$\ln(3^{x-1}) < \ln(2^x)$$

$$(x-1)\ln(3) < x\ln(2)$$

$$(x-1)\ln(3) - x\ln(2) < 0$$

$$x\ln(3) - \ln 3 - x\ln(2) < 0$$

$$(x\ln(3) - x\ln(2)) - \ln 3 < 0$$

$$x(\ln(3) - \ln(2)) < \ln 3$$

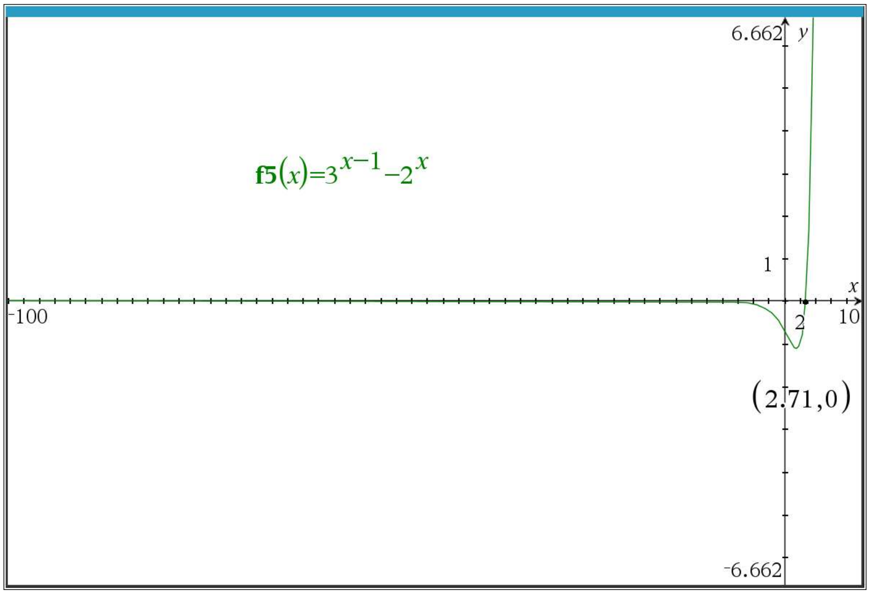
$$\boxed{x < \frac{\ln 3}{\ln 3 - \ln 2}}$$

$$\left\{x \,\middle|\, x < \frac{\ln 3}{\ln 3 - \ln 2}\right\}$$

$$= \left(-\infty, \frac{\ln 3}{}\right)$$

$$= \left(-\infty, \frac{\ln 3}{\ln 3 - \ln 2}\right)$$

$\frac{\ln(3)}{\ln(3)-\ln(2)}$	$\frac{\ln(3)}{\ln\left(\frac{3}{2}\right)}$
$\frac{\ln(3)}{\ln\left(\frac{3}{2}\right)}$ ▸Decimal	2.70951



6.4: 22

In Exercises 1 - 24, solve the equation analytically.

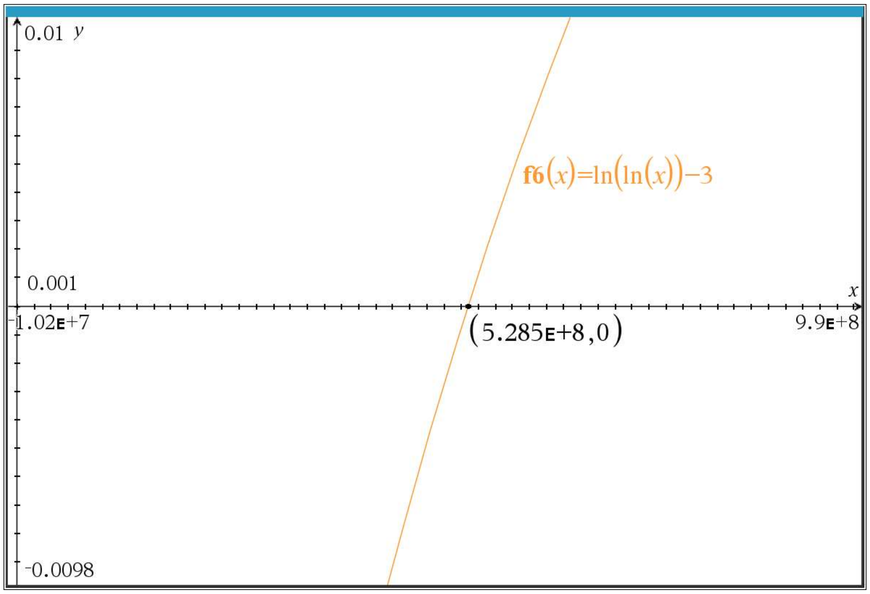
22. $\ln(\ln(x)) = 3$

$$e^3 = \ln(x)$$

$$e^{e^3} = e^{\ln(x)}$$

$$\boxed{x = e^{e^3}}$$

$\left(e^{e^{3}}\right)$▶Decimal 5.28491E8



6.5 supplied

Equation 6.1. Simple Interest The amount of interest I accrued at an annual rate r on an investment[a] P after t years is

$$I = Prt$$

The amount A in the account after t years is given by

$$A = P + I = P + Prt = P(1 + rt)$$

[a]Called the **principal**

supplied

Equation 6.2. Compounded Interest: If an initial principal P is invested at an annual rate r and the interest is compounded n times per year, the amount A in the account after t years is

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

Supplied

Equation 6.3. Continuously Compounded Interest: If an initial principal P is invested at an annual rate r and the interest is compounded continuously, the amount A in the account after t years is

$$A(t) = Pe^{rt}$$

Supplied

Equation 6.4. Uninhibited Growth: If a population increases according to The Law of Uninhibited Growth, the number of organisms N at time t is given by the formula

$$N(t) = N_0 e^{kt},$$

where $N(0) = N_0$ (read 'N nought') is the initial number of organisms and $k > 0$ is the constant of proportionality which satisfies the equation

$$(\text{instantaneous rate of change of } N(t) \text{ at time } t) = k\, N(t)$$

$$N(t) = N_0 e^{kt}$$
$$N(0) = N_0 e^{(k)(0)}$$
$$= N_0 e^0$$
$$= (N_0)(1)$$
$$N(0) = N_0 = \text{initial population}$$

Supplied

Equation 6.5. Radioactive Decay The amount of a radioactive element A at time t is given by the formula

$$A(t) = A_0 e^{kt},$$

where $A(0) = A_0$ is the initial amount of the element and $k < 0$ is the constant of proportionality which satisfies the equation

$$(\text{instantaneous rate of change of } A(t) \text{ at time } t) = k\, A(t)$$

Supplied

Equation 6.6. Newton's Law of Cooling (Warming): The temperature T of an object at time t is given by the formula

$$T(t) = T_a + (T_0 - T_a)\, e^{-kt},$$

where $T(0) = T_0$ is the initial temperature of the object, T_a is the ambient temperature[a] and $k > 0$ is the constant of proportionality which satisfies the equation

$$(\text{instantaneous rate of change of } T(t) \text{ at time } t) = k\, (T(t) - T_a)$$

[a] That is, the temperature of the surroundings.

Supplied

Equation 6.7. Logistic Growth: If a population behaves according to the assumptions of logistic growth, the number of organisms N at time t is given by the equation

$$N(t) = \frac{L}{1 + Ce^{-kLt}},$$

where $N(0) = N_0$ is the initial population, L is the limiting population,[a] C is a measure of how much room there is to grow given by

$$C = \frac{L}{N_0} - 1.$$

and $k > 0$ is the constant of proportionality which satisfies the equation

$$\text{(instantaneous rate of change of } N(t) \text{ at time } t) = k\,N(t)\,(L - N(t))$$

[a] That is, as $t \to \infty$, $N(t) \to L$

6.5

In Exercises 14 - 18, we list some radioactive isotopes and their associated half-lives. Assume that each decays according to the formula $A(t) = A_0 e^{kt}$ where A_0 is the initial amount of the material and k is the decay constant. For each isotope:

- Find the decay constant k. Round your answer to four decimal places.
- Find a function which gives the amount of isotope A which remains after time t. (Keep the units of A and t the same as the given data.)
- Determine how long it takes for 90% of the material to decay. Round your answer to two decimal places. (HINT: If 90% of the material decays, how much is left?)

15. Phosphorus 32, used in agriculture, initial amount 2 milligrams, half-life 14 days.

$A(t) = A_0 e^{kt}$

$A_0 = 2$ mg

$A(t) = 2e^{kt}$

$A(14) = \boxed{2e^{(k)(14)} = 1}$

$2e^{14k} = 1$

$e^{14k} = \frac{1}{2}$

$\ln(e^{14k}) = \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2)$

$$\ln\left(e^{14k}\right) = \ln\left(\tfrac{1}{2}\right) = \ln(1) - \ln(2)$$
$$= 0 - \ln(2)$$
$$= -\ln(2)$$

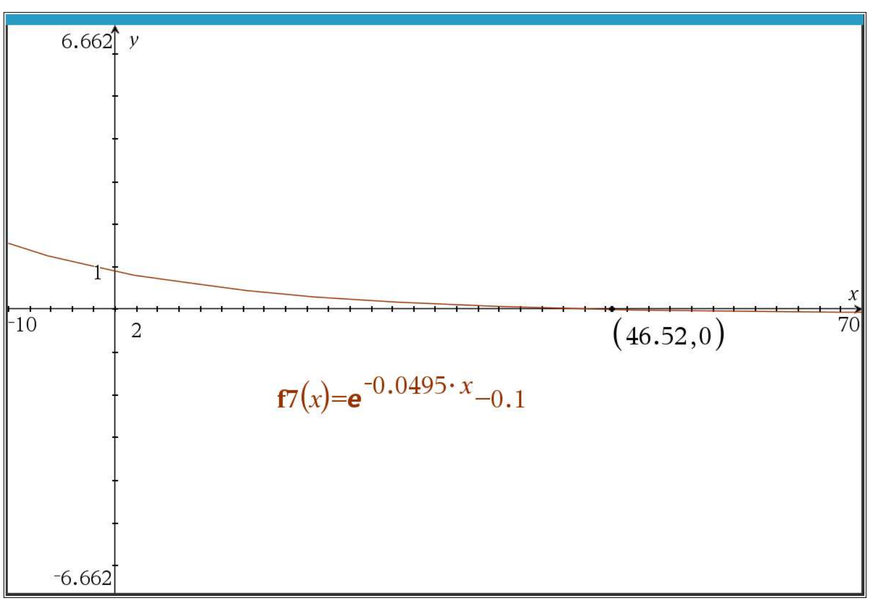
$$14k = \ln\left(\tfrac{1}{2}\right)$$
$$\boxed{k = \frac{\ln\left(\frac{1}{2}\right)}{14} \approx -0.0495}$$

$\frac{\ln(0.5)}{14}$	$^-0.049511$
$\frac{^-\ln(2.)}{14}$	$^-0.049511$

$$\boxed{A(t) = 2e^{-0.0495t}}$$

Find t such that $A(t) = 10\%$ of 2 mg
$= 0.2$

$$\boxed{2e^{-0.0495t} = 0.2}$$
$$e^{-0.0495t} = 0.1$$



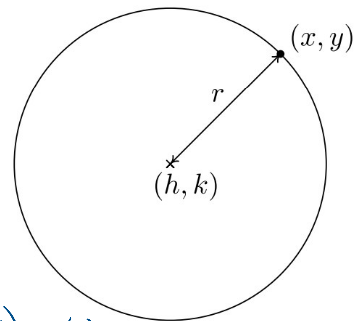
90% of the P32 will decay in about 47 days.

7.1 discussed in class

Memorize

locus definition

Definition 7.1. A **circle** with center (h, k) and radius $r > 0$ is the set of all points (x, y) in the plane whose distance to (h, k) is r.



$$\text{dist}((x,y),(h,k)) = r$$

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

equation of a circle with center (h,k) and radius $= r$

Memorize

Definition 7.2. The **Unit Circle** is the circle centered at $(0, 0)$ with a radius of 1. The standard equation of the Unit Circle is $x^2 + y^2 = 1$.

Trig preview

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$