10-28-25 MTH 167-004N

5.2 Inverse Functions

5.2.1 Exercises

page 391 (403): 1, 5, 11, 17, 23

---

6 Exponential and Logarithmic Functions

6.1 Introduction to Exponential and Logarithmic Functions

6.1.1 Exercises

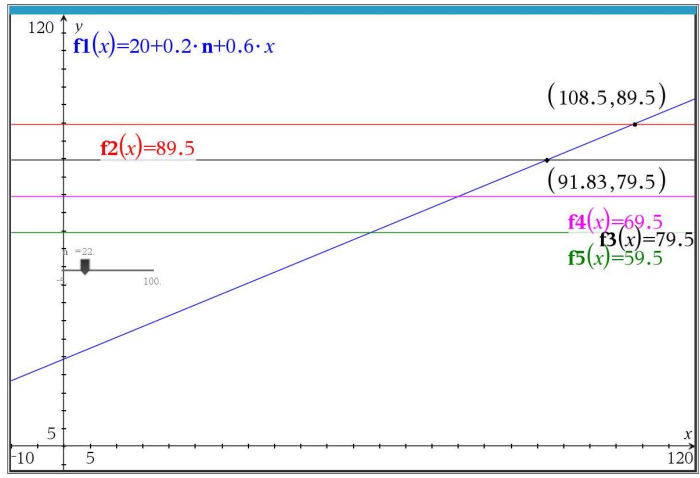
page 429 (441): 1, 5, 16, 26, 45, 56, 71, 75

6.2 Properties of Logarithms

6.2.1 Exercises

page 445 (457): 1, 5, 10, 16, 23, 30, 42

Substitute your high score for exam 1 and exam 2 for n. Find intersection for each grade.



Exam 2		stem & leaf		
43.14815	mean			A-1
40	median	9	6	B-2
24.938	st. dev	8	34	C-2
5	min	7	99	D-2
96	max	6	27	F- 20
27	count	5	28	
		4	02556	
		3	149	
		2	2379	
		1	12789	
		0	5	

Exam 1		stem & leaf		
49.82759	mean	10	0	A-1
49	median	9		B-2
19.64525	st. dev	8	25	C-2
14	min	7	11	D-3
100	max	6	067	F- 21
29	count	5	011556	
		4	01899	
		3	0235559	
		2	6	
		1	49	

5.2: 5

In Exercises 1 - 20, show that the given function is one-to-one and find its inverse. Check your answers algebraically and graphically. Verify that the range of f is the domain of f^{-1} and vice-versa.

5. $f(x) = \sqrt{3x-1} + 5$

f is 1-1 $\Rightarrow f^{-1}$ exists

replace $f(x)$ with y

$y = \sqrt{3x-1} + 5$

interchange x, y

$x = \sqrt{3y-1} + 5$

solve for y

$\sqrt{3y-1} = x-5$

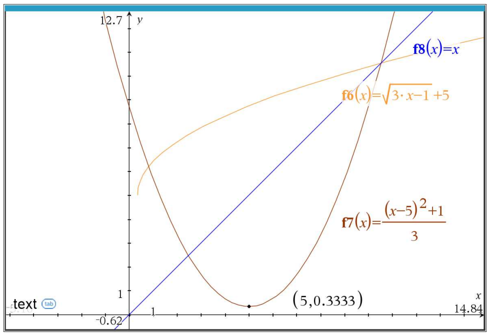
$3y-1 = (x-5)^2$

$3y = (x-5)^2 + 1$

$y = \dfrac{(x-5)^2+1}{3}$

Replace y with $f^{-1}(x)$

$\boxed{f^{-1}(x) = \dfrac{(x-5)^2+1}{3}}$ $\boxed{x \geq 5}$



$(f\circ f^{-1})(x) = f(f^{-1}(x))$

$= f\left(\frac{(x-5)^2+1}{3}\right)$ $\boxed{x \geq 5}$

$= \sqrt{3\left(\frac{(x-5)^2+1}{3}\right)-1} + 5$

$= \sqrt{(x-5)^2+1-1} + 5$

$= \sqrt{(x-5)^2} + 5$

$= |x-5| + 5$

$= x-5+5$ because $x-5 \geq 0$

$= x$ ✓

$\sqrt{x^2} = |x|$

$\sqrt{3^2} = \sqrt{9} = 3$

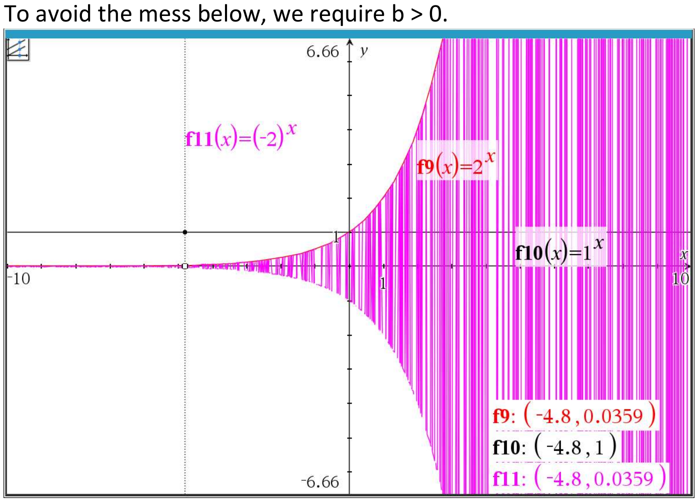
$\sqrt{(-3)^2} = \sqrt{9} = 3$

$= |-3|$

6.1

Definition 6.1. A function of the form $f(x) = b^x$ where b is a fixed real number, $b > 0$, $b \neq 1$ is called a **base b exponential function**.

To avoid the mess below, we require b > 0.

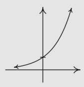


Memorize

Theorem 6.1. Properties of Exponential Functions: Suppose $f(x) = b^x$.

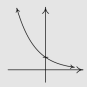
- The domain of f is $(-\infty, \infty)$ and the range of f is $(0, \infty)$.
- $(0, 1)$ is on the graph of f and $y = 0$ is a horizontal asymptote to the graph of f.
- f is one-to-one, continuous and smooth[a]

- If $b > 1$:
  - f is always increasing
  - As $x \to -\infty$, $f(x) \to 0^+$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



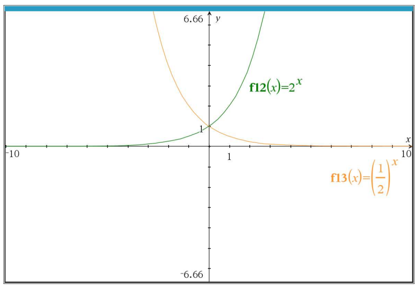
$y = b^x$, $b > 1$

- If $0 < b < 1$:
  - f is always decreasing
  - As $x \to -\infty$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to 0^+$
  - The graph of f resembles:



$y = b^x$, $0 < b < 1$

[a] Recall that this means the graph of f has no sharp turns or corners.



$$e^{\text{very big } (-)} = \frac{1}{e^{\text{very big } (+)}} \approx \frac{1}{\text{very big } (+)} \approx \text{very small } (+)$$

Memorize

Definition 6.2. The inverse of the exponential function $f(x) = b^x$ is called the **base b logarithm function**, and is denoted $f^{-1}(x) = \log_b(x)$ We read '$\log_b(x)$' as 'log base b of x.'

Memorize

Definition 6.3. The **common logarithm** of a real number x is $\log_{10}(x)$ and is usually written $\log(x)$. The **natural logarithm** of a real number x is $\log_e(x)$ and is usually written $\ln(x)$.

$\log(100) = \ln 100 = 4.6052$

Scientific Notebook inteprets log as natural log

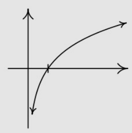
$\log_{10}(100) = 2$

T

Theorem 6.2. Properties of Logarithmic Functions: Suppose $f(x) = \log_b(x)$.

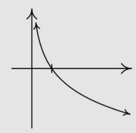
- The domain of f is $(0, \infty)$ and the range of f is $(-\infty, \infty)$.
- $(1, 0)$ is on the graph of f and $x = 0$ is a vertical asymptote of the graph of f.
- f is one-to-one, continuous and smooth
- $b^a = c$ if and only if $\log_b(c) = a$. That is, $\log_b(c)$ is the exponent you put on b to obtain c.
- $\log_b(b^x) = x$ for all x and $b^{\log_b(x)} = x$ for all $x > 0$

- If $b > 1$:
  - f is always increasing
  - As $x \to 0^+$, $f(x) \to -\infty$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



$y = \log_b(x)$, $b > 1$

- If $0 < b < 1$:
  - f is always decreasing
  - As $x \to 0^+$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to -\infty$
  - The graph of f resembles:



$y = \log_b(x)$, $0 < b < 1$

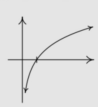
TI-nspire interprets log as common log

memorize

Theorem 6.2. Properties of Logarithmic Functions: Suppose $f(x) = \log_b(x)$.

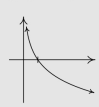
- The domain of f is $(0, \infty)$ and the range of f is $(-\infty, \infty)$.
- $(1, 0)$ is on the graph of f and $x = 0$ is a vertical asymptote of the graph of f.
- f is one-to-one, continuous and smooth
- $b^a = c$ if and only if $\log_b(c) = a$. That is, $\log_b(c)$ is the exponent you put on b to obtain c.
- $\log_b(b^x) = x$ for all x and $b^{\log_b(x)} = x$ for all $x > 0$

- If $b > 1$:
  - f is always increasing
  - As $x \to 0^+$, $f(x) \to -\infty$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



$y = \log_b(x)$, $b > 1$

- If $0 < b < 1$:
  - f is always decreasing
  - As $x \to 0^+$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to -\infty$
  - The graph of f resembles:



$y = \log_b(x)$, $0 < b < 1$

6.1

In Exercises 1 - 15, use the property: $b^a = c$ if and only if $\log_b(c) = a$ from Theorem 6.2 to rewrite the given equation in the other form. That is, rewrite the exponential equations as logarithmic equations and rewrite the logarithmic equations as exponential equations.

exponent

number

$$2^3 = 8 \iff \log_2 8 = 3$$

base

$$\log_{10}(1000) = 3$$

$$\log_{10}(1000) = 3$$
$$\Leftrightarrow 10^3 = 1000$$
$$\log_{10}(10) = 1 \Leftarrow 10^1 = 10$$
$$\log_{10}(1,000,000) = 6$$

5. $\left(\frac{4}{25}\right)^{-1/2} = \frac{5}{2}$ $\Leftrightarrow$ $\log_{\frac{4}{25}}\left(\frac{5}{2}\right) = -\frac{1}{2}$

$$\frac{1}{\sqrt{\frac{4}{25}}} = \frac{1}{\frac{\sqrt{4}}{\sqrt{25}}} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$

$\log_{\frac{4}{25}}\left(\frac{5}{2}\right)$	$\frac{-1}{2}$

6.1

In Exercises 16 - 42, evaluate the expression.

27. $\ln\left(e^3\right) = 3$

6.2

Memorize

Theorem 6.3. (Inverse Properties of Exponential and Logarithmic Functions) Let $b > 0$, $b \neq 1$.

- $b^a = c$ if and only if $\log_b(c) = a$
- $\log_b\left(b^x\right) = x$ for all x and $b^{\log_b(x)} = x$ for all $x > 0$

Memorize

Theorem 6.4. (One-to-one Properties of Exponential and Logarithmic Functions) Let $f(x) = b^x$ and $g(x) = \log_b(x)$ where $b > 0$, $b \neq 1$. Then f and g are one-to-one and

- $b^u = b^w$ if and only if $u = w$ for all real numbers u and w.
- $\log_b(u) = \log_b(w)$ if and only if $u = w$ for all real numbers $u > 0$, $w > 0$.

solve $5^x = 25$

$$5^x = 5^2$$
$$\Rightarrow \boxed{x = 2}$$

check $5^2 \stackrel{?}{=} 25$

$$25 = 25 \checkmark$$

Memorize

Theorem 6.5. (Algebraic Properties of Exponential Functions) Let $f(x) = b^x$ be an exponential function ($b > 0$, $b \neq 1$) and let u and w be real numbers.

- **Product Rule:** $f(u + w) = f(u)f(w)$. In other words, $b^{u+w} = b^u b^w$
- **Quotient Rule:** $f(u - w) = \dfrac{f(u)}{f(w)}$. In other words, $b^{u-w} = \dfrac{b^u}{b^w}$
- **Power Rule:** $(f(u))^w = f(uw)$. In other words, $(b^u)^w = b^{uw}$

Memorize

Theorem 6.6. (Algebraic Properties of Logarithmic Functions) Let $g(x) = \log_b(x)$ be a logarithmic function ($b > 0$, $b \neq 1$) and let $u > 0$ and $w > 0$ be real numbers.

- **Product Rule:** $g(uw) = g(u) + g(w)$. In other words, $\log_b(uw) = \log_b(u) + \log_b(w)$
- **Quotient Rule:** $g\left(\frac{u}{w}\right) = g(u) - g(w)$. In other words, $\log_b\left(\frac{u}{w}\right) = \log_b(u) - \log_b(w)$
- **Power Rule:** $g(u^w) = wg(u)$. In other words, $\log_b(u^w) = w\log_b(u)$

Product rule: the log of a product is the sum of logs

Proof:

Let $A = \log_b u$, $B = \log_b w$

$b^A = u$, $b^B = w$

left side $\log_b(u\,w)$

$= \log_b(b^A b^B)$

$= \log_b(b^{A+B})$ right side

$= A + B = \log_b(u) + \log_b(w)$

power rule

$\log_b(u^w) = w\log_b u$

$\log_{10}(2^3) \stackrel{?}{=} 3\log_{10} 2$

$\log_{10}(2 \cdot 2 \cdot 2) \stackrel{?}{=} 3\log_2 2$

$\log_{10}(2) + \log_{10}(2 \cdot 2) \stackrel{?}{=} 3\log_2 2$

$\log_{10}(2) + \log_{10}(2) + \log_{10}(2) \stackrel{?}{=} 3\log_{10}(2)$

$3\log_{10}(2) = 3\log_{10}(2)$

Theorem 6.7. (Change of Base Formulas) Let $a, b > 0$, $a, b \neq 1$.

- $a^x = b^{x\log_b(a)}$ for all real numbers x.
- $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$ for all real numbers $x > 0$.

convert $\log_2(x)$ to base e

Let $y = \log_2(x)$

$2^y = x$

$\ln(2^y) = \ln(x)$

$y\ln(2) = \ln(x)$

$y = \dfrac{\ln(x)}{\ln(2)}$

$\boxed{\log_2(x) = \dfrac{\ln(x)}{\ln 2}} = \dfrac{\log(x)}{\log(2)}$