5.2 Inverse Functions
5.2.1 Exercises
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6 Exponential and Logarithmic Functions
6.1 Introduction to Exponential and Logarithmic Functions
6.1.1 Exercises
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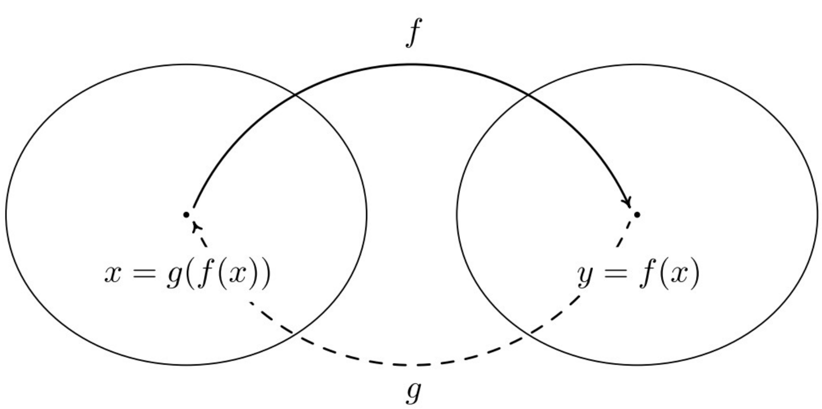
Took exam 2

Memorize

Definition 5.2. Suppose f and g are two functions such that

1. $(g \circ f)(x) = x$ for all x in the domain of f **and**
2. $(f \circ g)(x) = x$ for all x in the domain of g

then f and g are **inverses** of each other and the functions f and g are said to be **invertible**.



memorize

Theorem 5.2. Properties of Inverse Functions: Suppose f and g are inverse functions.

- The range[a] of f is the domain of g and the domain of f is the range of g
- $f(a) = b$ if and only if $g(b) = a$
- (a, b) is on the graph of f if and only if (b, a) is on the graph of g

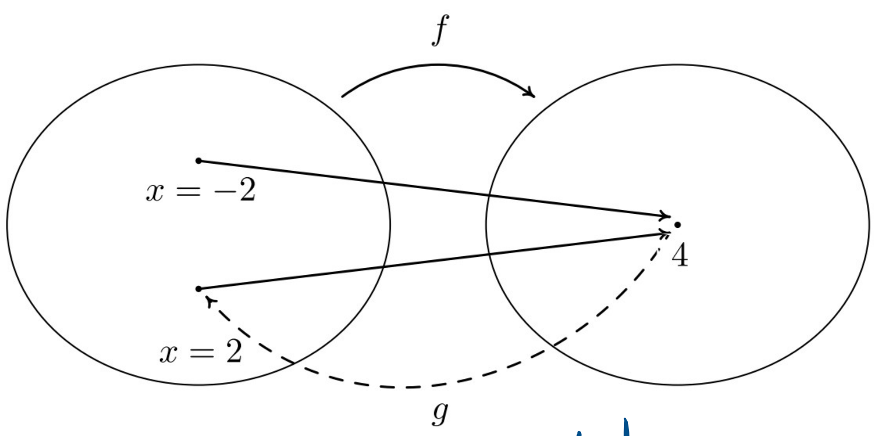
[a]Recall this is the set of all outputs of a function.

Memorize

Theorem 5.3. Uniqueness of Inverse Functions and Their Graphs : Suppose f is an invertible function.

- There is exactly one inverse function for f, denoted f^{-1} (read f-inverse)
- The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ across the line $y = x$.

Let $f(x) = x^2$



1-1

Memorize

Definition 5.3. A function f is said to be **one-to-one** if f matches different inputs to different outputs. Equivalently, f is one-to-one if and only if whenever $f(c) = f(d)$, then $c = d$.

Memorize

Theorem 5.4. The Horizontal Line Test: A function f is one-to-one if and only if no horizontal line intersects the graph of f more than once.

Memorize

Theorem 5.5. Equivalent Conditions for Invertibility: Suppose f is a function. The following statements are equivalent.

- f is invertible
- f is one-to-one
- The graph of f passes the Horizontal Line Test

Let $f(x) = 2x - 3$

is f 1-1 ?

Assume $f(c) = f(d)$

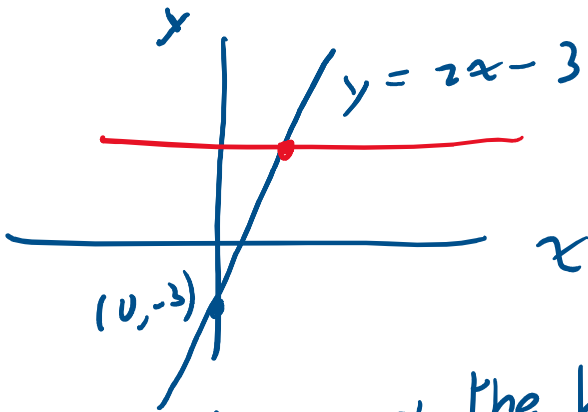
$$2c - 3 = 2d - 3$$

$$2c = 2d$$

$$c = d \quad \checkmark$$

$c = d$ ✓

$\therefore f$ is 1-1



graph passes the horizontal line test

i.e no horizontal line crosses the graph in more than 1 point

$\therefore f$ is 1-1

Let $f(x) = x^2$

is f 1-1 ?

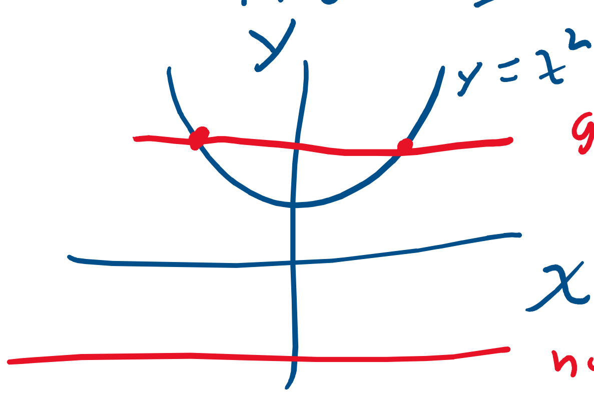
Assume $f(c) = f(d)$

$$c^2 = d^2$$

$$c = \pm d$$

It is not necessary that $c = d$

$\therefore f$ is <u>not</u> 1-1



graph fails the horizontal line test

$\therefore f$ is <u>not</u> 1-1

not relevant

Memorize

Steps for finding the Inverse of a One-to-one Function

1. Write $y = f(x)$
2. Interchange x and y
3. Solve $x = f(y)$ for y to obtain $y = f^{-1}(x)$

$f(x) = 2x - 3$

Find $f^{-1}(x)$

We already know that f(x) is 1-1, and so its inverse exists.

$y = 2x - 3$ interchange x, y

$x = 2y - 3$ solve for y

$2y = x + 3$

$y = \frac{x+3}{2}$

$$\boxed{f^{-1}(x) = \frac{x+3}{2}}$$

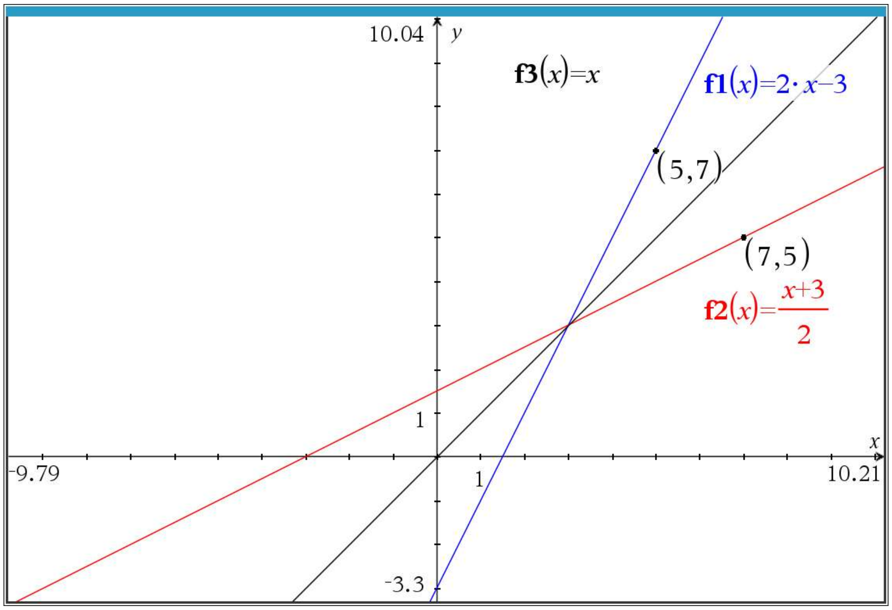
$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f\left(\frac{x+3}{2}\right)$

$= 2\left(\frac{x+3}{2}\right) - 3$

$= x + 3 - 3$

$= x$ ✓



5.2

In Exercises 1 - 20, show that the given function is one-to-one and find its inverse. Check your answers algebraically and graphically. Verify that the range of f is the domain of f^{-1} and vice-versa.

4. $f(x) = 1 - \frac{4+3x}{5}$

Given $f(c) = f(d)$
Prove $c = d$
Then f is 1-1

Assume $f(c) = f(d)$

$$1 - \frac{4+3c}{5} = 1 - \frac{4+3d}{5}$$

$$-\frac{4+3c}{5} = -\frac{4+3d}{5}$$

$$-4+3c = -4+3d$$

$$3c = 3d$$

$$c = d \checkmark$$

$\therefore \exists f^{-1}$
there exists

$$f(x) = 1 - \frac{4+3x}{5}$$

$$y = 1 - \frac{4+3x}{}$$

$$y = 1 - \frac{4+3x}{5}$$

$$x = 1 - \frac{4+3y}{5}$$

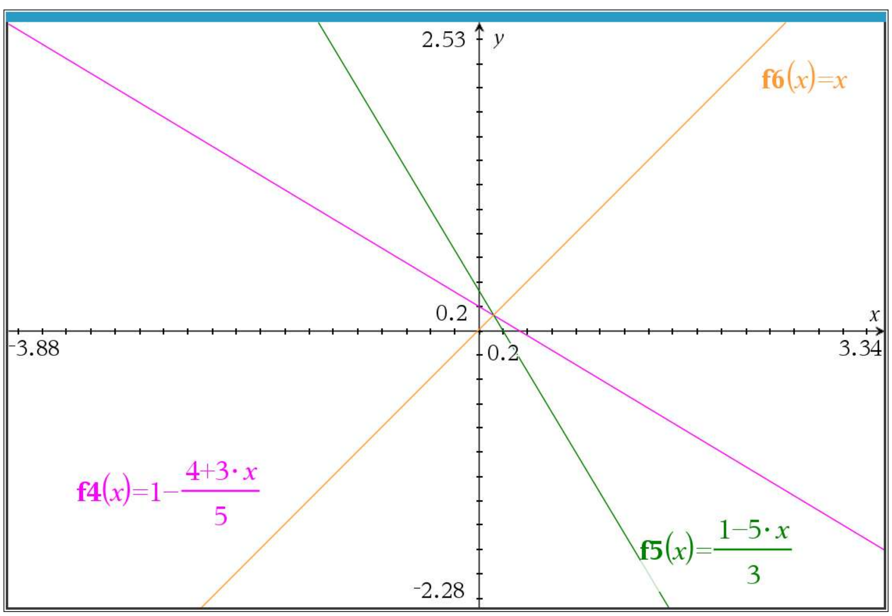
$$5x = 5 - 4 - 3y$$

$$5x = 1 - 3y$$

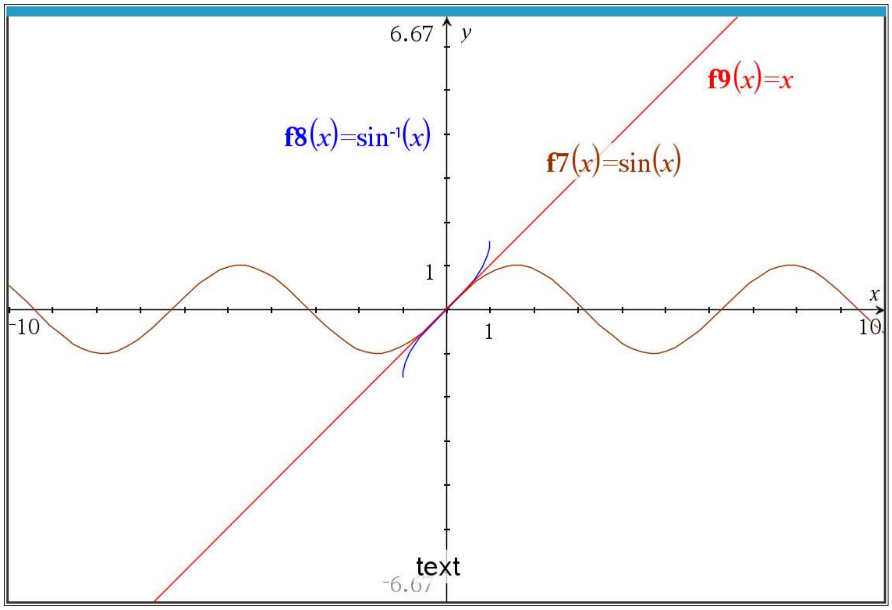
$$3y = 1 - 5x$$

$$y = \frac{1-5x}{3}$$

$$\boxed{f^{-1}(x) = \frac{1-5x}{3}}$$



Trigonometry preview


6.67
y
f9(x)=x
f8(x)=sin-1(x)
f7(x)=sin(x)
1
x
-10
1
10
text
-6.67