4.2 Graphs of Rational Functions
4.2.1 Exercises
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4.3 Rational Inequalities and Applications
4.3.2 Exercises
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5 Further Topics in Functions
5.1 Function Composition
5.1.1 Exercises
page 369 (381): 1, 9, 13, 17, 22, 31, 39, 62

Exam 2, Thursday, 10/23/25, 2.2 - 2.4, 3.1 - 3.4, 4.1 - 4.3, 5.1

4.2: 5

4.2.1 EXERCISES

In Exercises 1 - 16, use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

5. $f(x) = \dfrac{2x-1}{-2x^2-5x+3}$

Find domain
check where $-2x^2-5x+3=0$
and exclude those values

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$x=\frac{1}{2}, -3$

$f(x) = \dfrac{2x-1}{-(2x-1)(x+3)} = \dfrac{-1}{x+3}$ if $x \neq \frac{1}{2}$

hole at $x=\frac{1}{2}$ ($2x-1$ is cancelled in the numerator)
vert asymptote at $x=-3$ ($x+3$ is not cancelled)

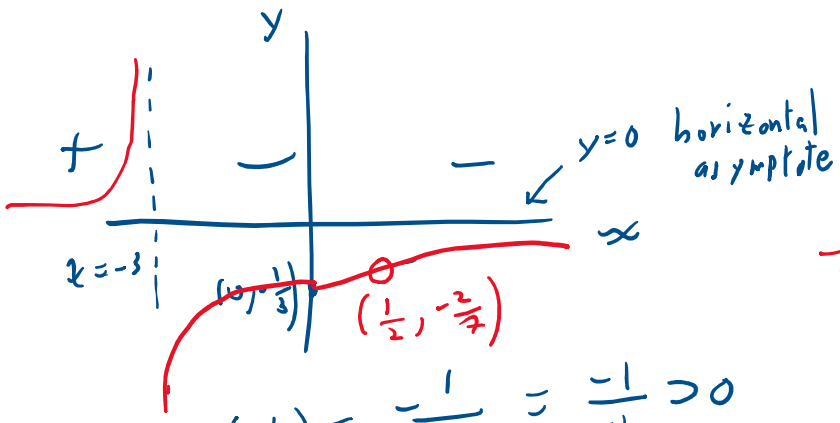
y-intercept $\Rightarrow f(0) = \dfrac{2(0)-1}{-2(0^2)-5(0)+3}$

$f(0) = \dfrac{-1}{3}$

No zero

deg of num < deg denom
horizontal asymptote

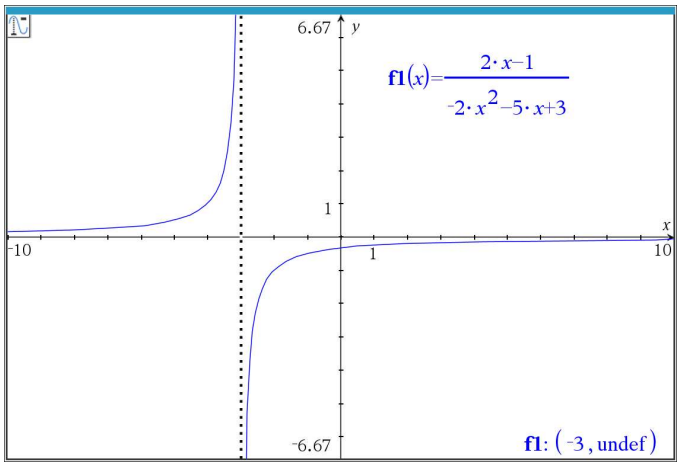
deg of num < deg denom
$\Rightarrow$ $y=0$ horizontal asymptote
$\Rightarrow$ no slant asymptote

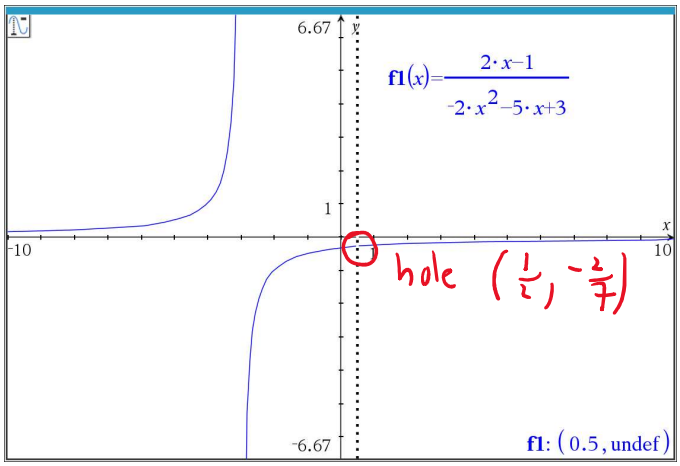


$\frac{-1}{\frac{1}{2}+3} = \frac{-1}{\frac{7}{2}} = -\frac{2}{7}$

$f(-4) = \frac{-1}{-4+3} = \frac{-1}{-1} > 0$

$f(1) = \frac{-1}{1+3} = \frac{-1}{4} < 0$





4.3: 13

In Exercises 7 - 20, solve the rational inequality. Express your answer using interval notation.

13. $\dfrac{x^3+2x^2+x}{x^2-x-2} \geq 0$

holes and asymptotes

Let $f(x) = \dfrac{x^3+2x^2+x}{x^2-x-2}$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2-(4)(1)(-2)}}{2(1)}$

$x = \dfrac{1 \pm \sqrt{1+8}}{2}$

$x = \dfrac{1 \pm 3}{2}$

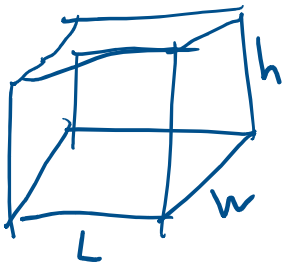
$x = -\dfrac{2}{2}, \dfrac{4}{2}$

$x = -1, 2$

4.3: 28

28. The box for the new Sasquatch-themed cereal, 'Crypt-Os', is to have a volume of 140 cubic inches. For aesthetic reasons, the height of the box needs to be 1.62 times the width of the base of the box.[13] Find the dimensions of the box which will minimize the surface area of the box. What is the minimum surface area? Round your answers to two decimal places.

Because the answer is only required to 2 decimal places, we can use a calculator.



Let V = volume of box $= 140\ in^3$

L = length of base

w = width of base

h = height of box

Let A = surface area

minimize $A(L, w, h)$

$h = 1.62\,w$

$h = 1.62\,w$

$A(L, w, h) = A(L, w)$

$V = Lwh = Lw(1.62\,w)$

$\boxed{V = 1.62\,w^2 L}$

$A = 2(Lw + wh + Lh)$

$A = 2(Lw + (w + L)\,1.62\,w)$

$V = 140\,\text{in}^3 = 1.62\,w^2 L$

$\boxed{L = \dfrac{140\,\text{in}^3}{1.62\,w^2}}$

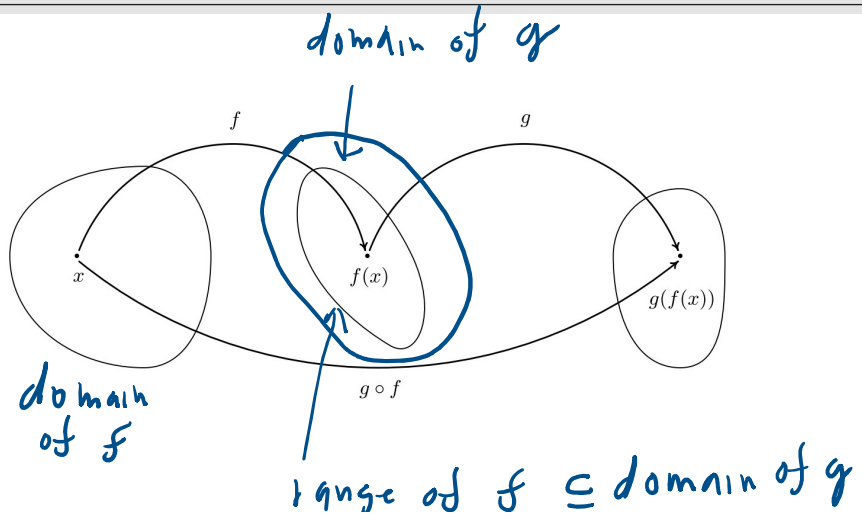
$A = 2\left(\dfrac{140\,\text{in}^3}{1.62\,w^2}\,w + \left(w + \dfrac{140\,\text{in}^3}{1.62\,w^2}\right)1.62\,w\right)$

Graph A(w) on a calculator and find the global minimum.

Finish after class

5.1
Memorize

Definition 5.1. Suppose f and g are two functions. The **composite** of g with f, denoted $g \circ f$, is defined by the formula $(g \circ f)(x) = g(f(x))$, provided x is an element of the domain of f and $f(x)$ is an element of the domain of g.



of f | range of $f \subseteq$ domain of g

Note: when finding the domain of a composite function, consider the unsimplified form.

Let $f(x) = \frac{x-4}{x-4} = 1$ if $x \neq 4$

domain of $f = \{x \mid x \neq 4\}$

domain of $g(x) = 1 = \mathbb{R} = (-\infty, \infty)$

Memorize

Theorem 5.1. Properties of Function Composition: Suppose f, g, and h are functions.

- $h \circ (g \circ f) = (h \circ g) \circ f$, provided the composite functions are defined.
- If I is defined as $I(x) = x$ for all real numbers x, then $I \circ f = f \circ I = f$.

In general is function composition commutative?

Let $f(x) = 3 + x$

$g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x})$

$(f \circ g)(x) = 3 + \sqrt{x}$

$(g \circ f)(x) = g(f(x))$

$= g(3 + x)$

$= \sqrt{3 + x}$

$3 + \sqrt{x} \stackrel{?}{=} \sqrt{3 + x}$

let $x = 0$

$3 + \sqrt{0} \stackrel{?}{=} \sqrt{3 + 0}$

$3 + 0 \stackrel{?}{=} \sqrt{3}$

$3 \neq \sqrt{3}$

$\therefore$ in general $3 + \sqrt{x} \neq \sqrt{3 + x}$

$\therefore$ composition is not commutative in general

In Exercises 13 - 24, use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

- $(g \circ f)(x)$
- $(f \circ g)(x)$
- $(f \circ f)(x)$

21. $f(x) = 3x - 1$, $g(x) = \dfrac{1}{x+3}$

$(g \circ f)(x) = g(f(x))$

$g(3x-1) = \dfrac{1}{(3x-1)+3}$

$= \dfrac{1}{3x+2}$

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$ exclude from domain

23. $f(x) = \dfrac{x}{2x+1}$, $g(x) = \dfrac{2x+1}{x}$

$(f \circ g)(x) = f(g(x))$

$= f\left(\dfrac{2x+1}{x}\right)$

$= \dfrac{\frac{2x+1}{x}}{2\left(\frac{2x+1}{x}\right)+1}$ exclude $x = 0$ $= ?$

$2\left(\dfrac{2x+1}{x}\right)+1 = 0$

$\dfrac{4x+2}{x} + \dfrac{x}{x} = 0$

$\dfrac{5x+2}{x} = 0$

$5x + 2 = 0$

$x = -\frac{2}{5}$ exclude from domain

Your name MTH 167-004N quiz 4

1. Let $f(x) = x^2 + 5x + 2$.
   Write in vertex form by completing the square.
   Find and plot the vertex.
   Calculate and plot the y-intercept.
   Find and plot a 3rd point by symmetry with respect to the y-intercept.
   Connect the points to graph the parabola.

$f(x) = (x^2 + 5x) + 2$

$= (x^2 + 5x + \frac{25}{4} - \frac{25}{4}) + 2$

$= (x^2 + 5x + \frac{25}{4}) - \frac{25}{4} + 2$

$= (x + \frac{5}{2})^2 - \frac{25}{4} + \frac{8}{4}$

$f(x) = (x + \frac{5}{2})^2 - \frac{17}{4}$

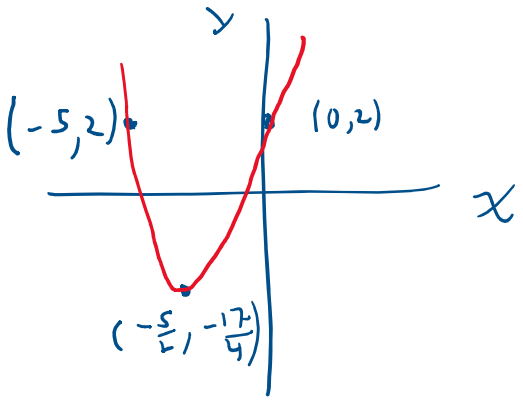
$f(x) = (x - (-\frac{5}{2}))^2 - \frac{17}{4}$

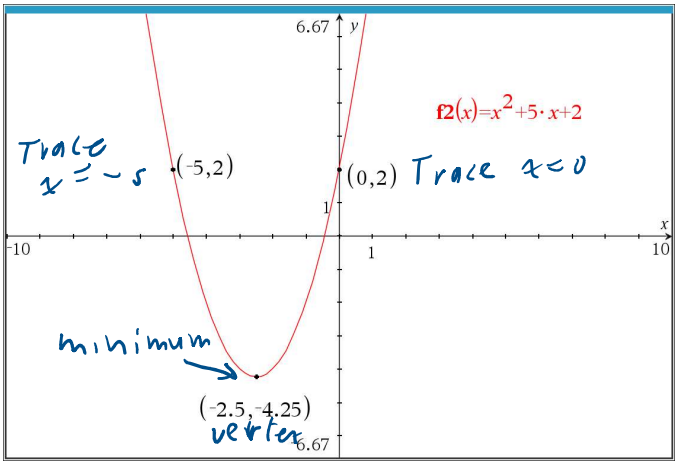
vertex $= (-\frac{5}{2}, -\frac{17}{4})$

$(\frac{1}{2})5 = \frac{5}{2}$

$(\frac{5}{2})^2 = \frac{25}{4}$

y-intercept $f(0) = 0^2 + (5)(0) + 2 = 2$





2. Solve analytically. $|4x - 8| = 12$

$4x - 8 = 12$

$4x = 20$

$x = 5$

$4x - 8 = -12$

$4x = -4$

$x = -1$

check: $|4(5) - 8| \stackrel{?}{=} 12$ | $|4(-1) - 8| \stackrel{?}{=} 12$

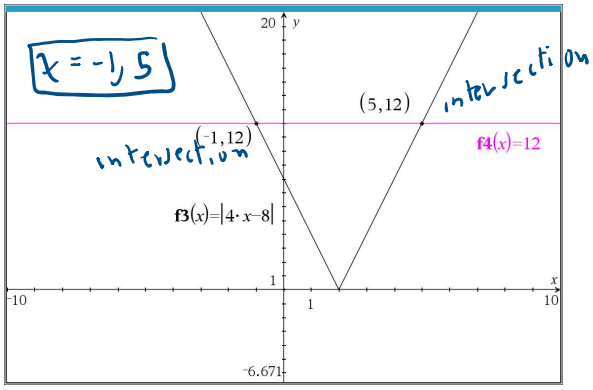
$|20 - 8| \stackrel{?}{=} 12$ | $|-12| \stackrel{?}{=} 12$

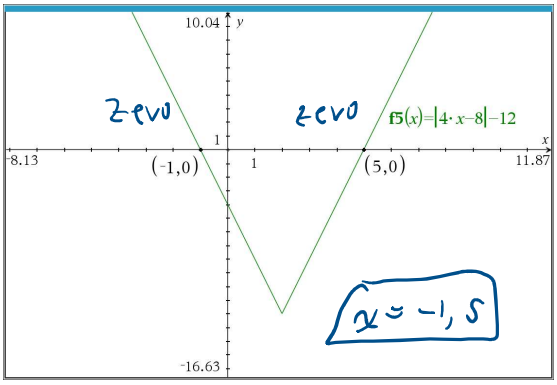
check: $|4(5)-8| \stackrel{?}{=} 12$ | $|4(-1)-8| \stackrel{?}{=} 12$

$|20-8| \stackrel{?}{=} 12$ | $|-12| \stackrel{?}{=} 12$

$12 = 12$ ✓ | $12 = 12$ ✓

3. Solve #2 graphically, sketch a labeled graph.

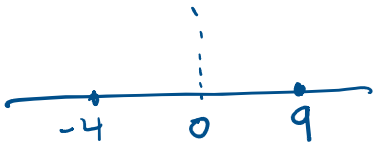




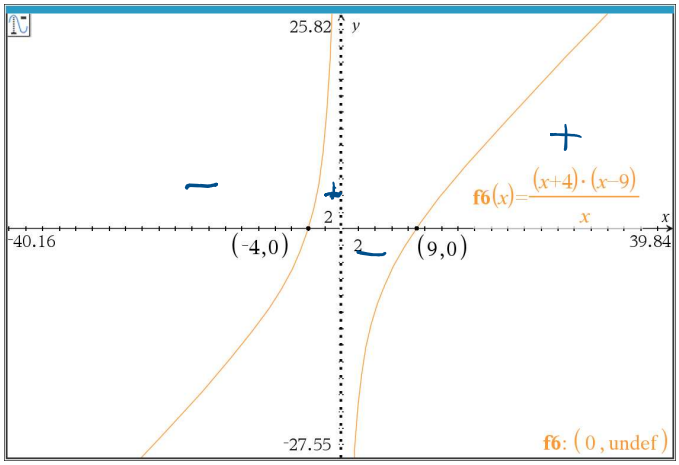
4. Let $f(x) = \frac{(x+4)(x-9)}{x}$. Solve $f(x) \geq 0$ by any convenient method. Show work clearly.

f(0) not defined

f(-4) = f(9) = 0



test values or calculator graph



solution set $[-4, 0) \cup [9, \infty)$