09-25-25 MTH 167-004N

2.3 Quadratic Functions
   2.3.1 Exercises
   page 200 (212): 1, 4, 11, 21, 26

2.4 Inequalities with Absolute Value and Quadratic Functions
   2.4.1 Exercises
   page 220 (232): 1, 8, 17, 34, 36

---

3 Polynomial Functions

3.1 Graphs of Polynomials
   3.1.1 Exercises
   page 246 (258): 3, 7, 13, 21, 27

2.3: 4

In Exercises 1 - 9, graph the quadratic function. Find the x- and y-intercepts of each graph, if any exist. If it is given in general form, convert it into standard form; if it is given in standard form, convert it into general form. Find the domain and range of the function and list the intervals on which the function is increasing or decreasing. Identify the vertex and the axis of symmetry and determine whether the vertex yields a relative and absolute maximum or minimum.

4. $f(x) = -2(x+1)^2 + 4$

y-intercept

$f(0) = -2(0+1)^2 + 4 = -2 + 4 = 2$

x-intercept

$-2(x+1)^2 + 4 = 0$

$\left(2(x+1)^2 - 2(x+1)^2\right) + 4 = 2(x+1)^2$

$0 + 4 = 2(x+1)^2$

$4 = 2(x+1)^2$

$2(x+1)^2 = 4$

$(x+1)^2 = 2$

$x+1 = \pm\sqrt{2}$

$a = b$

$\Rightarrow a + c = b + c$

$x + 1 = \pm\sqrt{2}$

$$\boxed{x = -1 \pm \sqrt{2}}$$

Definition: two equations are equivalent if they have the same solutions.

convert $f(x) = -2(x+1)^2 + 4$ to general form

$f(x) = ax^2 + bx + c$

$f(x) = -2(x^2 + 2x + 1) + 4$

$f(x) = -2x^2 - 4x - 2 + 4$

$$\boxed{f(x) = -2x^2 - 4x + 2}$$

complete the square

$f(x) = (-2x^2 - 4x) + 2$

$f(x) = -2(x^2 + 2x) + 2$

$f(x) = -2(x^2 + 2x + 1 - 1) + 2$

$f(x) = -2(x^2 + 2x + 1) + 2 + 2$

$f(x) = -2(x+1)^2 + 4$

$\left(\frac{1}{2}\right)(2) = 1$

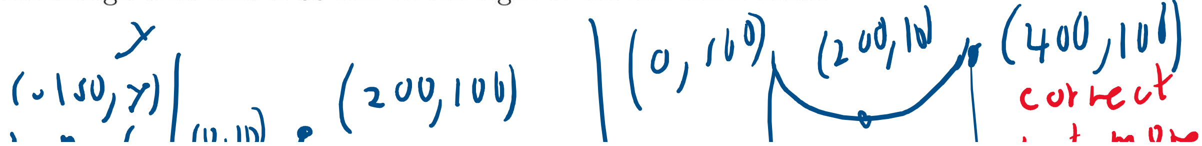
$1^2 = 1$

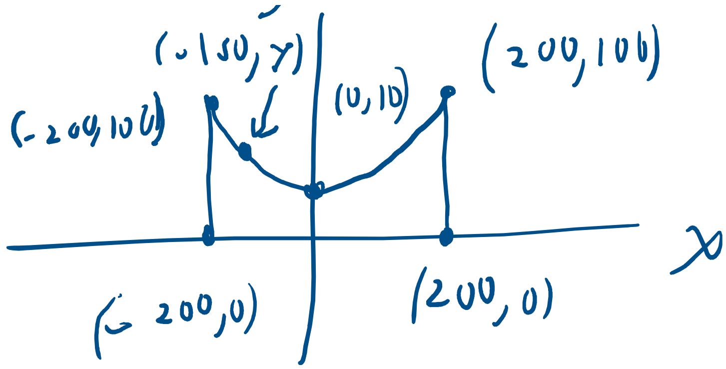
$(p+q)^2$

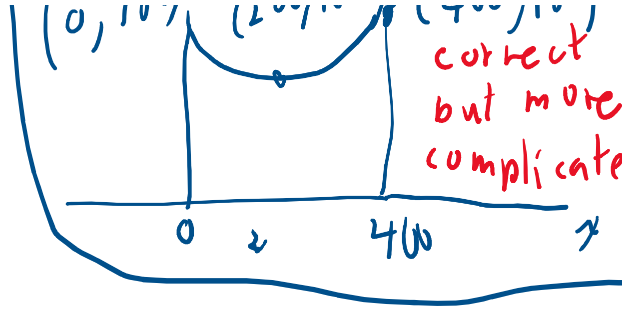
$= p^2 + 2pq + q^2$

2.3: 26

26. The two towers of a suspension bridge are 400 feet apart. The parabolic cable[11] attached to the tops of the towers is 10 feet above the point on the bridge deck that is midway between the towers. If the towers are 100 feet tall, find the height of the cable directly above a point of the bridge deck that is 50 feet to the right of the left-hand tower.







correct but more complicated

$y = ax^2 + bx + c$

$y = a(x-h)^2 + k$

$(h,k) = (0,10)$

$\Rightarrow y = a(x-0)^2 + 10$

$\Rightarrow y = ax^2 + 10$

Find a

Find $y(150)$

Find the equation of the parabola.
Then calculate $y(-150)$

$y = ax^2 + 10$

$100 = a(200)^2 + 10$

solve for a

$a(200)^2 = 90$

$a = \frac{90}{(200)(200)}$

$a = \frac{9}{(20)(200)}$

$\boxed{a = \frac{9}{4000}}$

$y(x) = \left(\frac{9}{4000}\right)x^2 + 10$

$y(-150) = \left(\frac{9}{\ldots}\right)(-150)^2 + 10$

$$y(-150) = \left(\frac{9}{4000}\right)(-150)^2 + 10$$

$$y(-150) = \left(\frac{9}{4000}\right)(225 \times 100) + 10$$

$$y(-150) = \left(\frac{9}{40}\right)(225) + 10$$

$$y(-150) = \left(\frac{9}{8}\right)(45) + 10$$

(9/8)*45+10=60.625

$$\boxed{y(-150) = 60.625}$$

The height of the cable is 60.625 feet

2.3: 11

In Exercises 10 - 14, the cost and price-demand functions are given for different scenarios. For each scenario,

- Find the profit function $P(x)$.
- Find the number of items which need to be sold in order to maximize profit.
- Find the maximum profit.
- Find the price to charge per item in order to maximize profit.
- Find and interpret break-even points.

11. The cost, in dollars, to produce x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is $C(x) = 10x + 100$, $x \geq 0$ and the price-demand function, in dollars per bottle, is $p(x) = 35 - x$, $0 \leq x \leq 35$.

$$P(x) = R(x) - C(x)$$
$$= x\,p(x) - C(x)$$
$$= x(35 - x) - (10x + 100)$$
$$= 35x - x^2 - 10x - 100$$

$$\boxed{P(x) = -x^2 + 25x - 100}$$

$$\boxed{p(x) = -x^2 + 25x - 100}$$

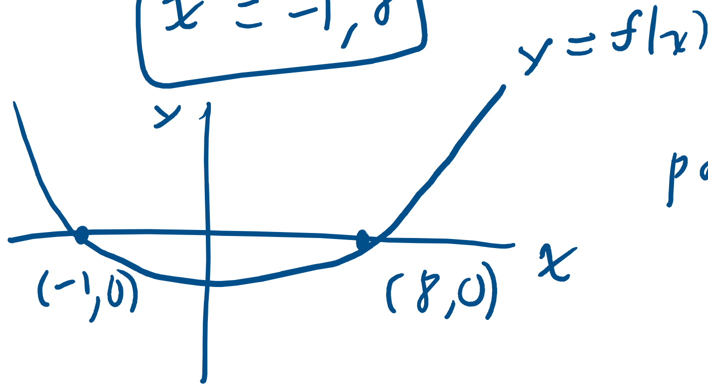
Find the vertex of the parabola

Let $f(x) = (x+1)(x-8) = x^2 - 7x - 8$

Solve $f(x) > 0$

Find the zero of $f(x)$

$$\boxed{x = -1, 8}$$



parabola opens up

$(-\infty, -1) \cup (8, \infty)$

$ab > 0$

$\Leftrightarrow$ $a > 0$ and $b > 0$

or $a < 0$ and $b < 0$

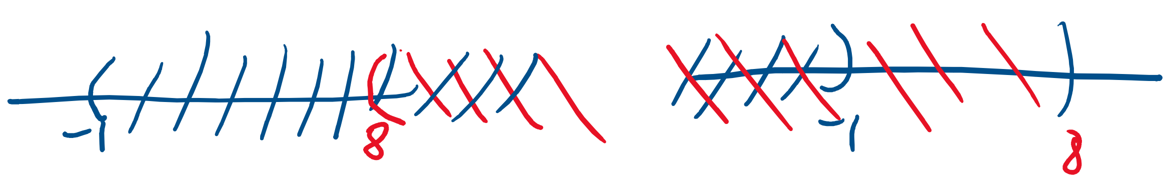
---

algebraic method

$f(x) = (x+1)(x-8)$ solve $f(x) > 0$

$[x+1 > 0$ and $x - 8 > 0]$ or $[x+1 < 0$ and $x - 8 < 0]$

$[x > -1$ and $x > 8]$ or $[x < -1$ and $x < 8]$



$x > 8$ or $x < -1$

$x > 8$ or $x < -1$

$\{x \mid x > 8\}$ $\{x \mid x < -1\}$

$(8, \infty) \cup (-\infty, -1)$

---

numeric value technique

$f(x) = (x+1)(x-8) = x^2 - 7x - 8$

Again find and plot zeros

$\boxed{(-\infty, -1) \cup (8, \infty)}$

Choose a convenient test value in each interval and show that f(test value) is either positive or negative

$f(-2) = (-2+1)(-2-8)$

$= (-)(-) > 0$

$f(0) = (0+1)(0-8)$

$= (+)(-) < 0$

$f(9) = (9+1)(9-8)$

$= (+)(+) > 0$

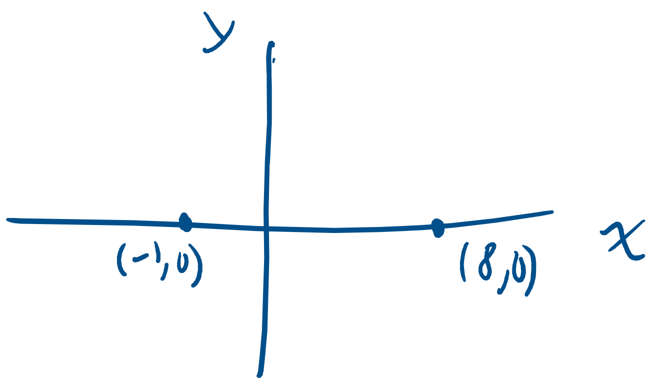
---

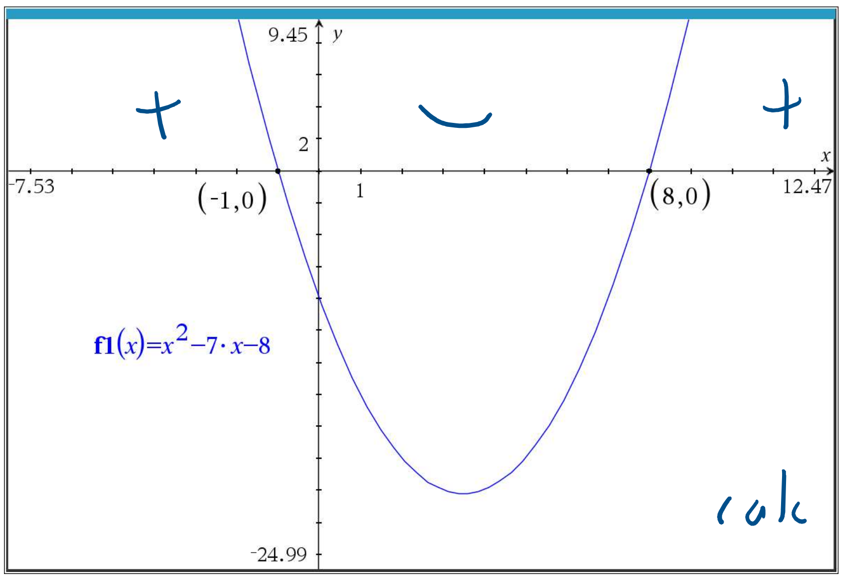
zeros and calculator

Again $f(x) = (x+1)(x-8) = x^2 - 7x - 8$

Find and plot zeros

Find and plot zeros





3.1
memorize

Definition 3.1. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0,$$

where a_0, a_1, $\ldots$, a_n are real numbers and $n \geq 1$ is a natural number. The domain of a polynomial function is $(-\infty, \infty)$.

Memorize

Definition 3.2. Suppose f is a polynomial function.

- Given $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$, we say
  - The natural number n is called the **degree** of the polynomial f.
  - The term $a_n x^n$ is called the **leading term** of the polynomial f.
  - The real number a_n is called the **leading coefficient** of the polynomial f.
  - The real number a_0 is called the **constant term** of the polynomial f.

Definition 3.2. Suppose f is a polynomial function.

- Given $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$, we say
  - The natural number n is called the **degree** of the polynomial f.
  - The term $a_n x^n$ is called the **leading term** of the polynomial f.
  - The real number a_n is called the **leading coefficient** of the polynomial f.
  - The real number a_0 is called the **constant term** of the polynomial f.
- If $f(x) = a_0$, and $a_0 \neq 0$, we say f has degree 0.
- If $f(x) = 0$, we say f has no degree.[a]

[a] Some authors say $f(x) = 0$ has degree $-\infty$ for reasons not even we will go into.

Memorize

End Behavior of functions $f(x) = ax^n$, n even.

Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and n is an even natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:

$a > 0$

$a < 0$

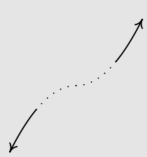
Memorize

End Behavior of functions $f(x) = ax^n$, n odd.

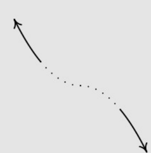
Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and $n \geq 3$ is an odd natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:



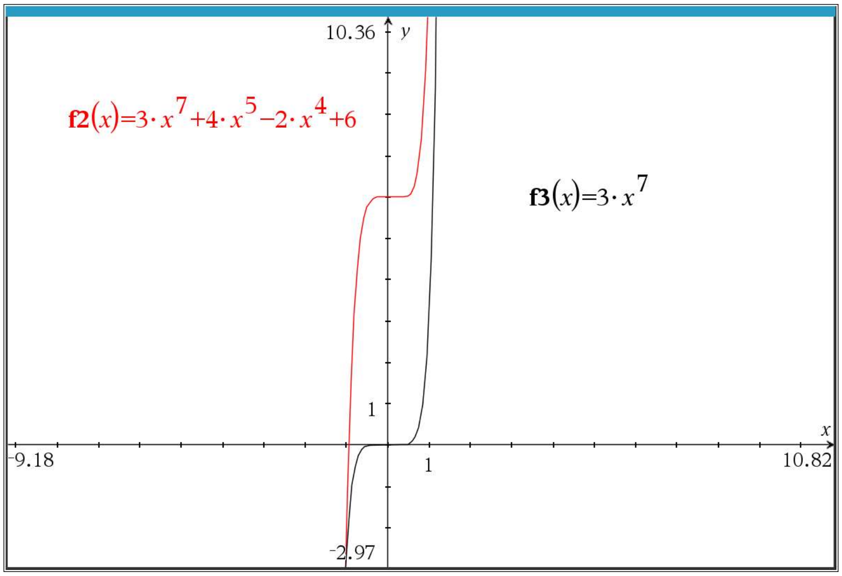
$a > 0$

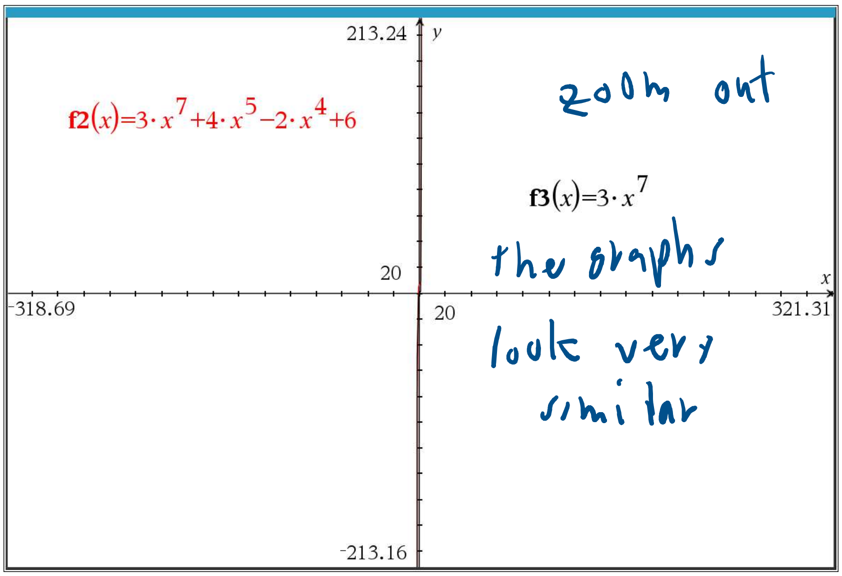


$a < 0$

Memorize

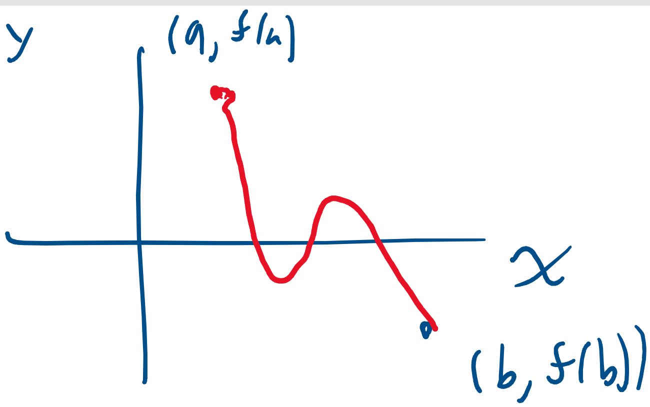
Theorem 3.2. End Behavior for Polynomial Functions: The end behavior of a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$ matches the end behavior of $y = a_n x^n$.





Supplied

Theorem 3.1. The Intermediate Value Theorem (Zero Version): Suppose f is a continuous function on an interval containing $x = a$ and $x = b$ with $a < b$. If $f(a)$ and $f(b)$ have different signs, then f has at least one zero between $x = a$ and $x = b$; that is, for at least one real number c such that $a < c < b$, we have $f(c) = 0$.



Memorize

Definition 3.3. Suppose f is a polynomial function and m is a natural number. If $(x-c)^m$ is a factor of $f(x)$ but $(x-c)^{m+1}$ is not, then we say $x = c$ is a zero of **multiplicity** m.

Memorize

Theorem 3.3. The Role of Multiplicity: Suppose f is a polynomial function and $x = c$ is a zero of multiplicity m.

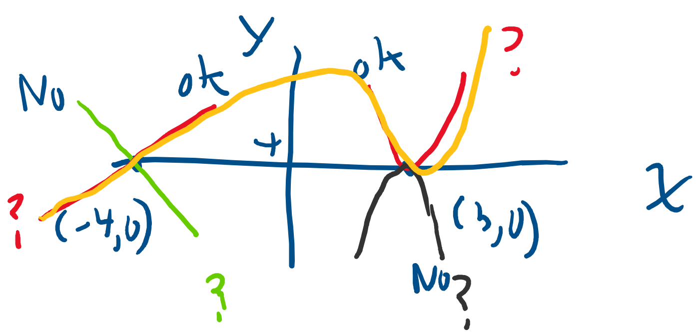
- If m is even, the graph of $y = f(x)$ touches and rebounds from the x-axis at $(c, 0)$.
- If m is odd, the graph of $y = f(x)$ crosses through the x-axis at $(c, 0)$.

Use multiplicities and a sign chart to make a rough sketch of the graph of a polynomial function.

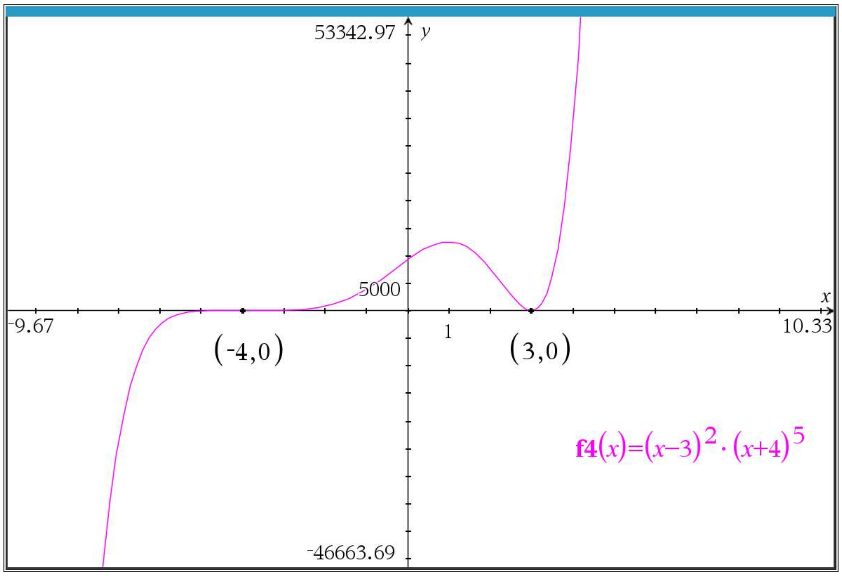
$$f(x) = (x-3)^2(x+4)^5$$

zeros $3, -4$

multiplicity $2, 5$



$$f(0) = (0-3)^2(0+4)^5$$
$$= (+)(+)$$

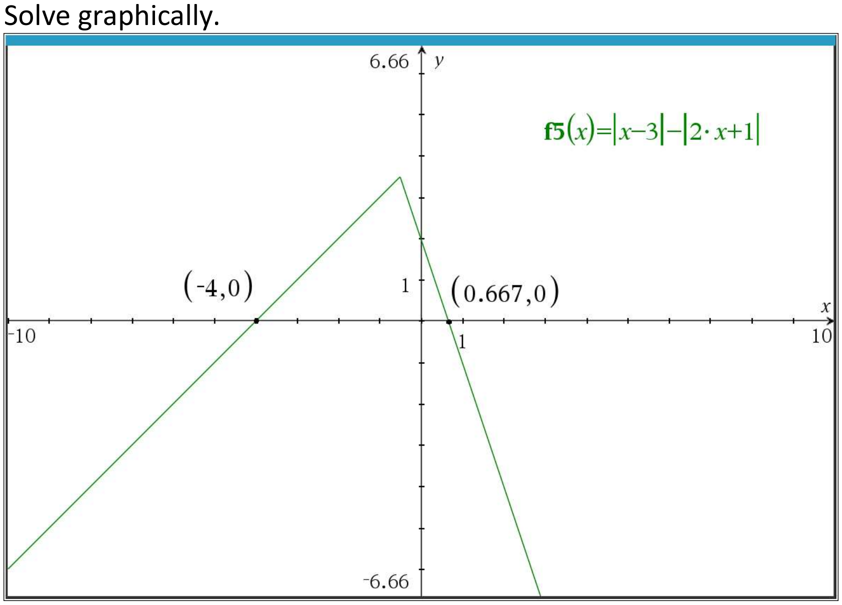


2.410

In Exercises 1 - 32, solve the inequality. Write your answer using interval notation.

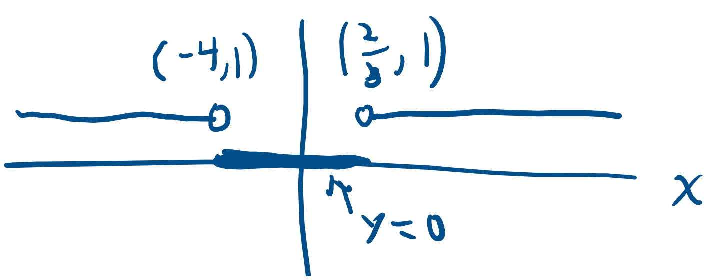
10. $|x-3|-|2x+1|<0$

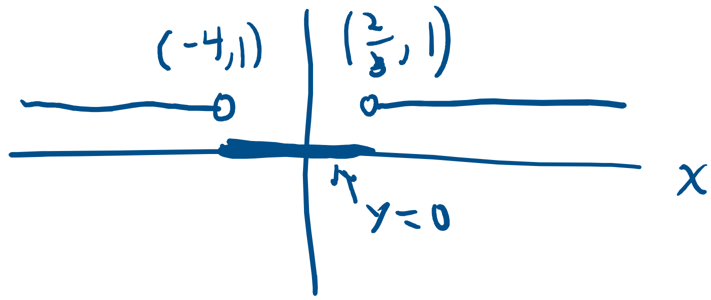
Solve graphically.



Graphical solution: $(-\infty,-4)\cup(0.667,\infty)$

TI-84 : $Y_1 = |x-3| - |2x+1| < 0$




(-4,1)
(2/3, 1)
x
y=0