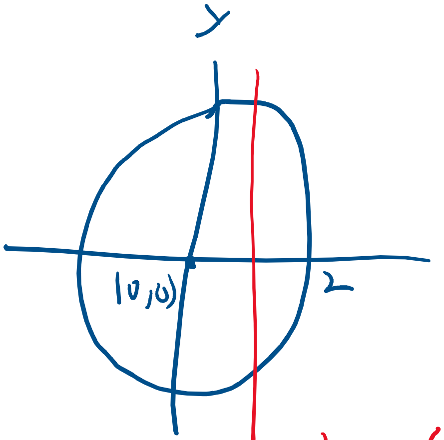
09-17-25 MTH 167-004N

Office hour notes



circle with center $(0,0)$
radius $=2$

$$\boxed{x^2+y^2=4}$$

y-axis symmetry

same

$$(-x)^2+y^2=4$$

$$x^2+y^2=4$$

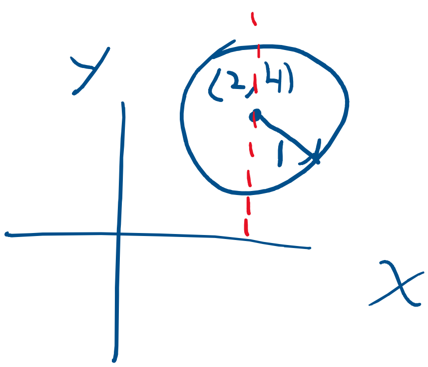
$\therefore$ y-axis symmetry

Likewise, we have
x-axis symmetry
and origin symmetry

graph fails
the vertical line
test

$\therefore$ The relation does not give y as a function of x.

However, we can still check for symmetry.

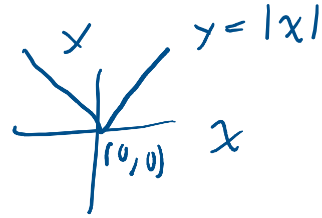


$$(x-2)^2+(y-4)^2=1$$

From the graph, it is clear that we do not have x-axis symmetry, y-axis symmetry, or origin symmetry.

However, this relation is symmetric about the line x=2 and about y=4, and about the center (2,4).

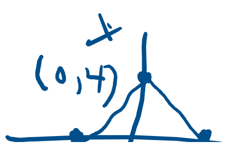
$$f(x)=-2|x|+4$$



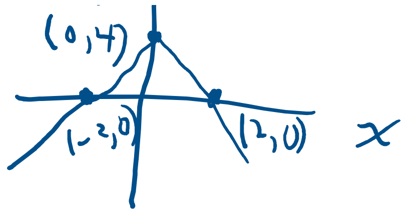
Find x-intercept, y-intercept

y-intercept, find $f(0)$

$$f(0)=-2|0|+4=0+4=4$$



x-intercept
set $y=0$, solve for x



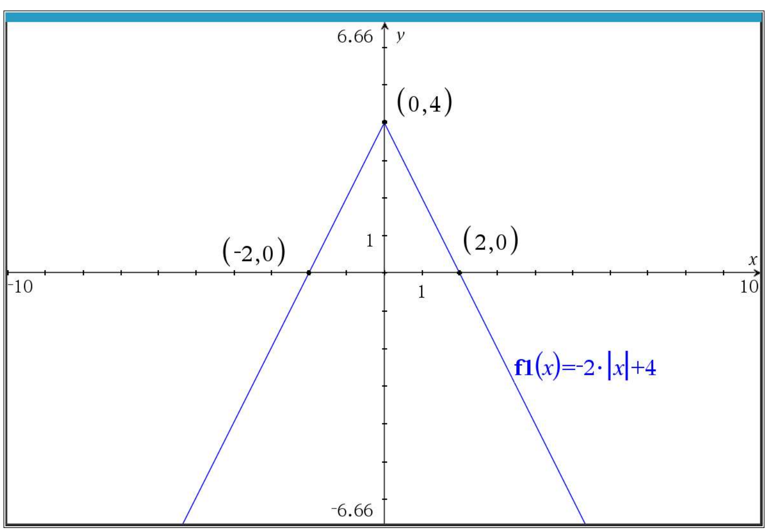
x-intercept
set $y = 0$, solve for x

$$-2|x| + 4 = 0$$
$$-2|x| + 2|x| + 4 = 0 + 2|x|$$
$$(-2|x| + 2|x|) + 4 = 2|x|$$
$$0 + 4 = 2|x|$$
$$4 = 2|x|$$
$$2|x| = 4$$
$$|x| = 2$$
$$\boxed{x = \pm 2}$$

$$f(-2) = -2|-2| + 4$$
$$= -2(2) + 4$$
$$= -4 + 4$$
$$= 0$$

$$f(2) = -2|2| + 4$$
$$= -2(2) + 4$$
$$= -4 + 4$$
$$= 0$$

Here is a graph of the function transformed from $y = |x|$



y-axis symmetry

$$y = -2|x| + 4$$
$$y = -2|-x| + 4$$
$$y = -2|x| + 4$$

same

not same

$\therefore$ y-axis symmetry

x-axis symmetry

$\therefore$ y-axis symmetry

x-axis symmetry

$-y = -2|x| + 4$

$y = 2|x| - 4$

$\therefore$ No x-axis symmetry

2.1: 45

In Exercises 44 - 49, compute the average rate of change of the function over the specified interval.

Definition 2.3. Let f be a function defined on the interval $[a, b]$. The **average rate of change** of f over $[a, b]$ is defined as:

$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

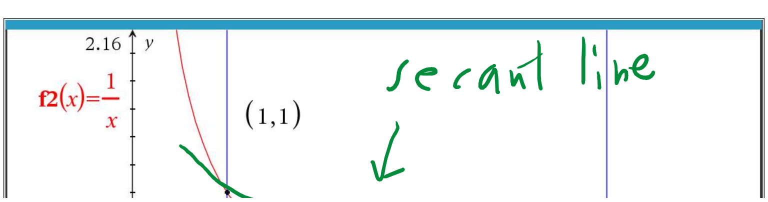
45. $f(x) = \frac{1}{x}$, $[1, 5]$

$$\frac{\Delta f}{\Delta x} = \frac{f(5) - f(1)}{5 - 1}$$

$$= \frac{\frac{1}{5} - \frac{1}{1}}{4}$$

$$= \frac{\frac{1}{5} - \frac{5}{5}}{4}$$

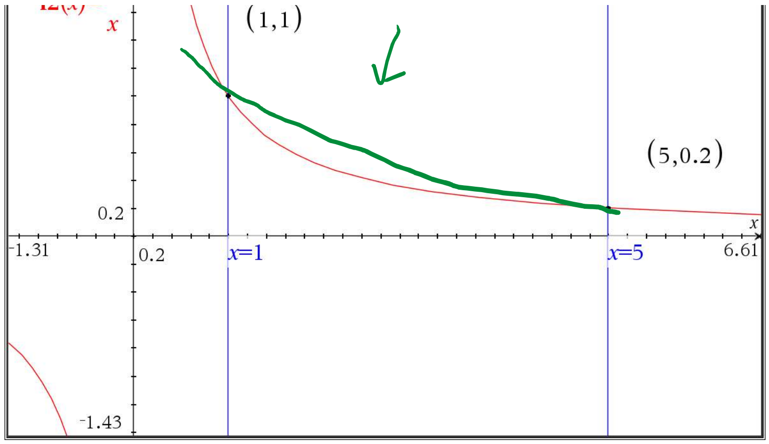
$$= \frac{-\frac{4}{5}}{4} = \frac{-\frac{4}{5}}{\frac{4}{1}} = \left(-\frac{\cancel{4}}{5}\right)\left(\frac{1}{\cancel{4}}\right) = \boxed{-\frac{1}{5}}$$



secant line

slope of secant line

= -1



secant line $= -\frac{1}{5}$

$$f(x) = 3x^2 - 2x$$

Find and simplify $\dfrac{\Delta f}{\Delta x}$

$$\boxed{\begin{aligned} &f(\text{input}) \\ &= 3(\text{input})^2 \\ &\quad - 2(\text{input}) \end{aligned}}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[3(x+h)^2 - 2(x+h)\right] - \left[3x^2 - 2x\right]}{h}$$

$$= \frac{\left[3(x^2 + 2hx + h^2) - 2x - 2h - 3x^2 + 2x\right.}{h}$$

$$= \frac{\cancel{3x^2} + 6hx + 3h^2 \cancel{- 2x} - 2h \cancel{- 3x^2} \cancel{+ 2x}}{h}$$

$$= \frac{6hx + 3h^2 - 2h}{h}$$

$$= \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}}$$

$$= \frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 6x+3h-2}$$

set-builder notation

$A = \{x \mid x \text{ has property } p\}$

A = the set of all x, which is a singular variable, such that x has the property p

interval $(-3, 7) = \{x \mid -3 < x < 7\}$

$= \{x \in \mathbb{R} \mid -3 < x < 7\}$

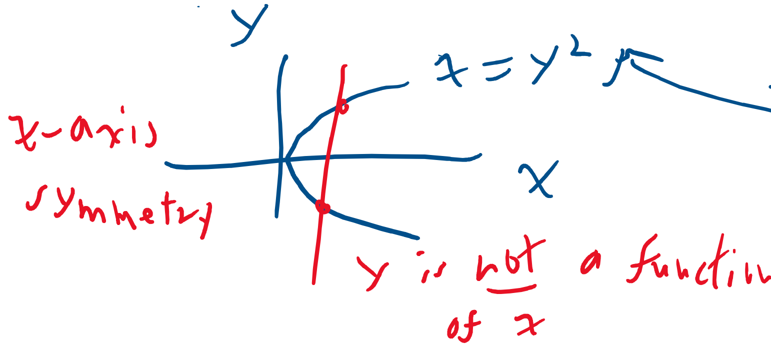
$\mathbb{R}$ = the set of all real numbers

$= \{x \mid x \in \mathbb{R}\}$

$= (-\infty, \infty)$

Def: f is even if $f(-x) = f(x)$, y-axis symmetry

f is odd if $f(-x) = -f(x)$, origin symmetry



same

$x = (-y)^2$

$x = y^2$