09-04-25 MTH 167-004N

1.3 Introduction to Functions
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1.3:39

In Exercises 33 - 47, determine whether or not the equation represents y as a function of x.

39. $x = y^2 + 4$

solve for y

$y^2 = x - 4$

$y = \pm\sqrt{x-4}$

for each x, there are 2 y-values

$\therefore$ y is not a function of x

1.5

Memorize

Function Arithmetic

Suppose f and g are functions and x is in both the domain of f and the domain of g.[a]

- The **sum** of f and g, denoted $f+g$, is the function defined by the formula
$$(f+g)(x) = f(x) + g(x)$$
- The **difference** of f and g, denoted $f-g$, is the function defined by the formula
$$(f-g)(x) = f(x) - g(x)$$
- The **product** of f and g, denoted fg, is the function defined by the formula
$$(fg)(x) = f(x)g(x)$$
- The **quotient** of f and g, denoted $\frac{f}{g}$, is the function defined by the formula
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},$$
provided $g(x) \neq 0$.

[a]Thus x is an element of the intersection of the two domains.

Given function f and g

define $h(x) = (f+g)(x)$

$= f(x) + g(x)$

Let $f(x) = 2x$

Let $g(x) = x + 5$

Let $h = f + g$

$h(x) = (f+g)(x)$

$= f(x) + g(x)$

$= 2x + (x+5)$

$h(x) = 3x + 5$

Let $f(x) = x - 2$

Let $f(x) = x - 2$

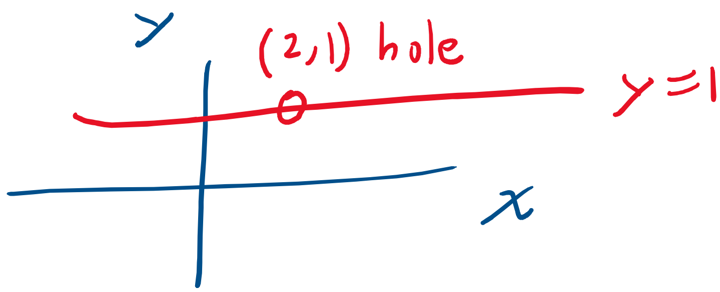
$g(x) = x - 2$

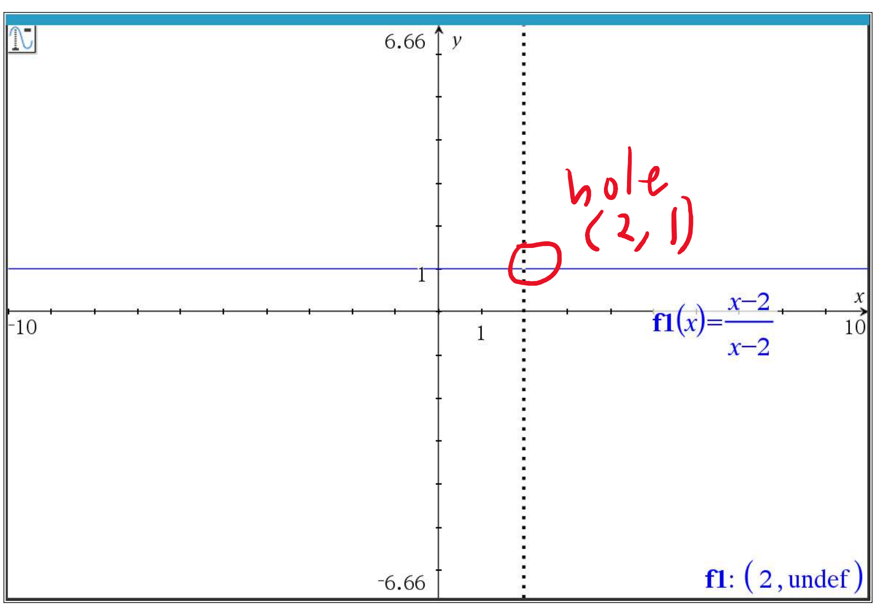
Let $h(x) = \dfrac{f(x)}{g(x)}$

simplify $h(x)$
and find the domain of $h(x)$

Hint graph $h(x)$

$h(x) = \dfrac{x-2}{x-2} = 1$ for $x \neq 2$





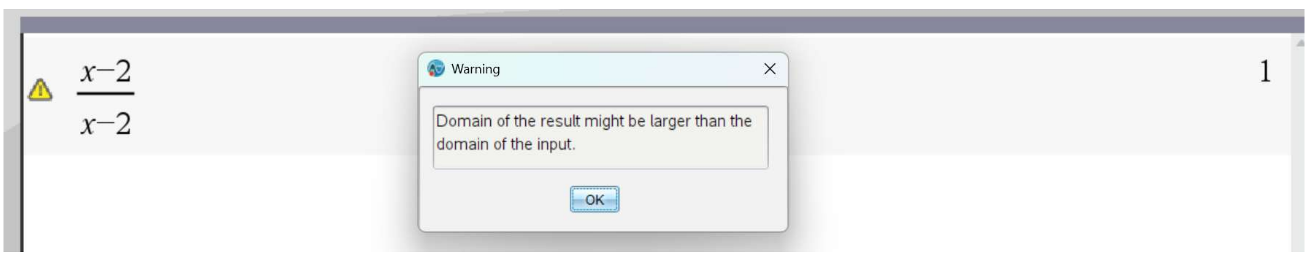
domain of $f = (-\infty, \infty)$

" " " $= (-\infty, \infty)$

$$\text{〃} \quad \text{〃} \quad \text{〃} = (-\infty, \infty)$$

domain of $h \neq (-\infty, \infty) \cap (-\infty, \infty)$

In general domain of $\left\{ \begin{matrix} f+g \\ f\cdot g \\ f\circ g \\ \frac{f}{g} \end{matrix} \right\} \subseteq$ domain f $\cap$ domain g

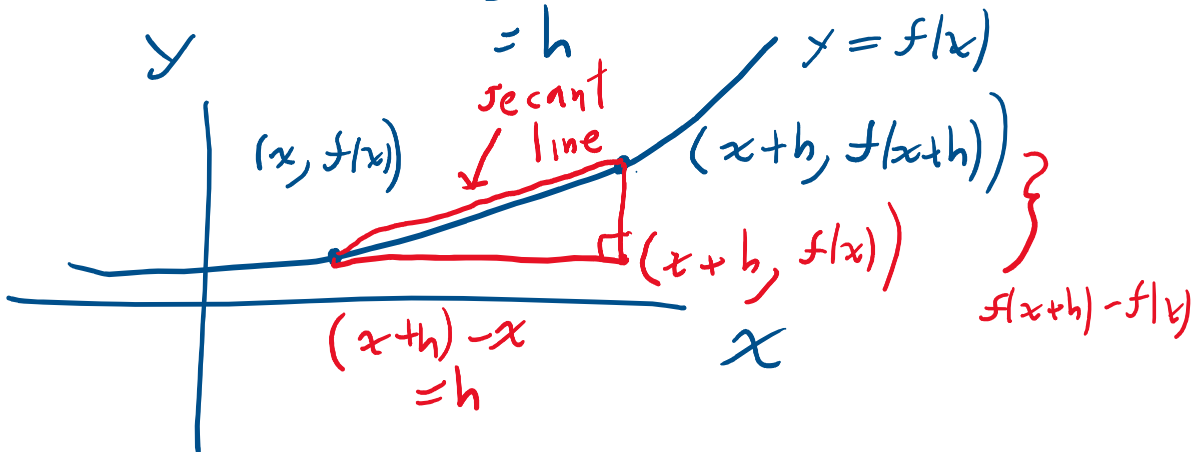


Memorize

Definition 1.8. Given a function f, the **difference quotient** of f is the expression

$$\frac{f(x+h)-f(x)}{h} \qquad h \neq 0$$

$$\begin{aligned} (x+h)-x &= x+h-x \\ &= (x-x)+h \\ &= 0+h \\ &= h \end{aligned}$$



$= h$

slope of secant line joining the points $(x, f(x))$ and $(x+h, f(x+h)) = \frac{\Delta f}{\Delta x} =$ difference quotient

$\Delta =$ "change"

Let $f(x) = 7x - 5$ $\quad f(\text{input}) = (7)(\text{input}) - 5$

Find and simplify the difference quotient $\frac{\Delta f}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

$$= \frac{[7(x+h) - 5] - [7x - 5]}{h}$$

$$= \frac{\cancel{7x} + 7h \cancel{- 5} \cancel{- 7x} \cancel{+ 5}}{h}$$

$$= \frac{7\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 7}$$

Let $f(x) = x^2$

Find and simplify $\frac{\Delta f}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2x + h}$$

Let $f(x) = \frac{1}{x}$, $x \neq 0$

find and simplify $\frac{\Delta f}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \left(\frac{1}{x+h}\right)\left(\frac{x}{x}\right) - \left(\frac{1}{x}\right)\left(\frac{x+h}{x+h}\right)$$

$$= \frac{\left(\overline{x+h}\right)\left(\overline{x}\right) - \left(\overline{x}\right)\left(\overline{x+h}\right)}{h}$$

$$= \frac{\frac{x - (x+h)}{(x+h)x}}{h} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$$

$$= \frac{\frac{-h}{(x+h)x}}{h} = \frac{\frac{-h}{(x+h)x}}{\frac{h}{1}}$$

$$= \frac{-\cancel{h}}{(x+h)x} \cdot \frac{1}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta a} = \frac{-1}{(x+h)x}}$$

Let $h = 0$

$\Rightarrow \frac{\Delta f}{\Delta x} \rightarrow -\frac{1}{x^2}$

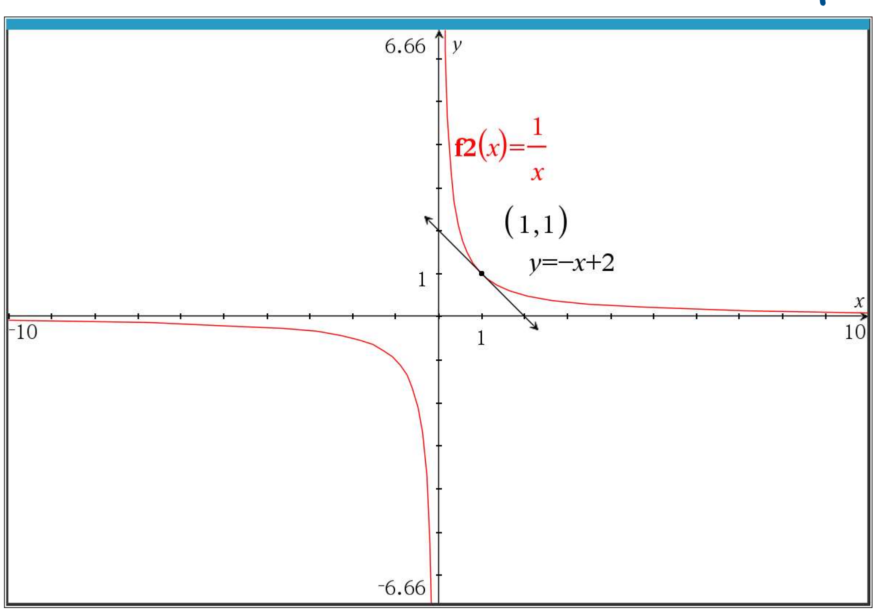
$x = 1 \Rightarrow -\frac{1}{x^2}$

$= \frac{-1}{1^2}$

$= \boxed{-1}$

slope of tangent line

Calculus preview



Supplied

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

1.6

Memorize

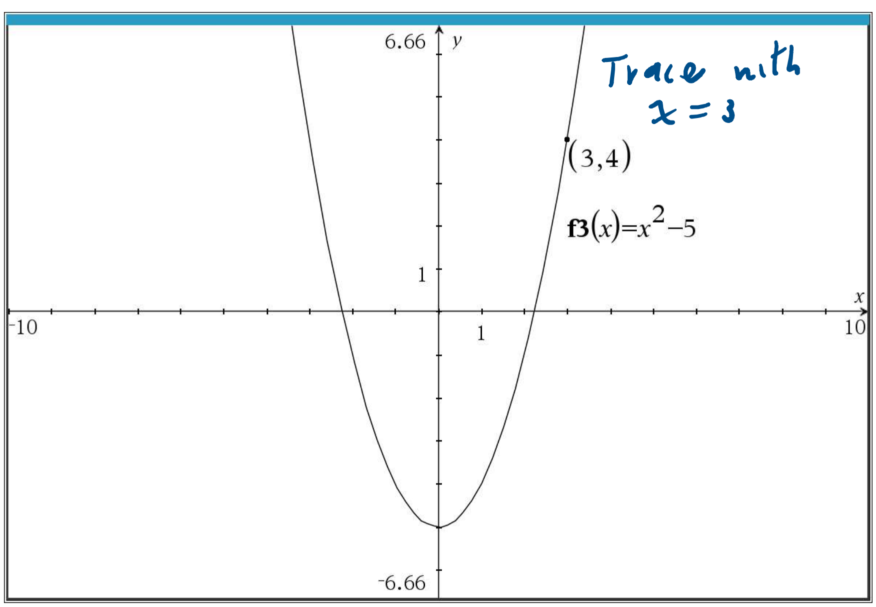
The Fundamental Graphing Principle for Functions

The graph of a function f is the set of points which satisfy the equation $y = f(x)$. That is, the point (x, y) is on the graph of f if and only if $y = f(x)$.

Is the point $(3, 4)$ on the graph of $y = f(x) = x^2 - 5$?

$$4 \stackrel{?}{=} 3^2 - 5$$
$$4 \stackrel{?}{=} 9 - 5$$
$$4 = 4 \checkmark$$

$\therefore$ $(3,4)$ is on the graph $y = x^2 - 5$



Piecewise-defined function

Let $f(x) = \begin{cases} -x^2 & \text{for } x < 1 \\ 4 & \text{for } x \geq 1 \end{cases}$

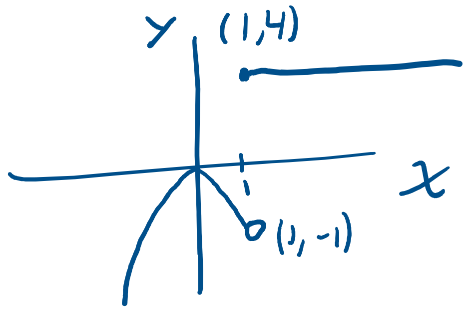
$f(0) = -(0)^2 = -0 = 0$

$0 < 1 \Rightarrow$ use top formula

$f(6) = 4$

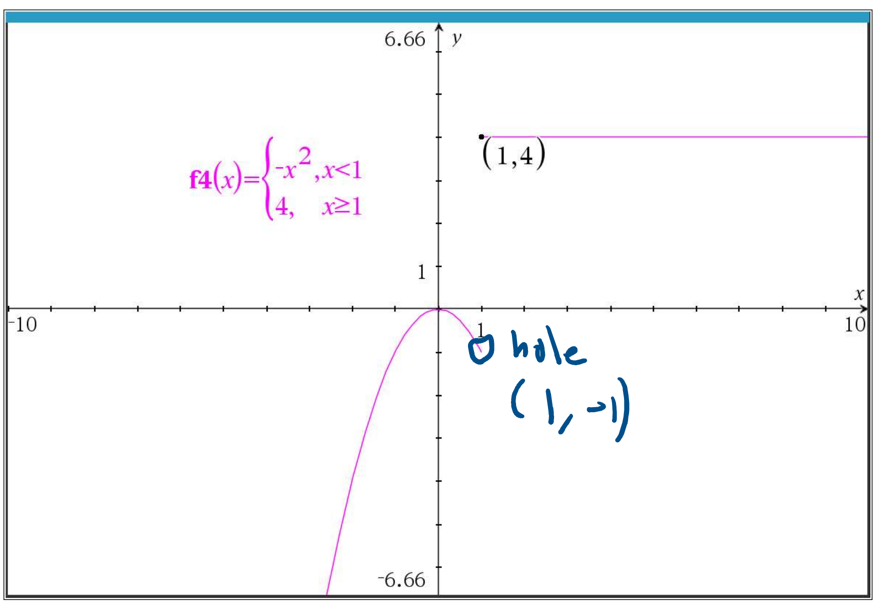
$4 \geq 1 \Rightarrow$ use bottom formula

domain of $f = (-\infty \ \infty)$



$Y_1 = -x^2 * (x < 1) + 4 * (x \geq 1)$

Graph



Memorize

> **Definition 1.9.** The **zeros** of a function f are the solutions to the equation $f(x) = 0$. In other words, x is a zero of f if and only if $(x, 0)$ is an x-intercept of the graph of $y = f(x)$.

iff

$\Leftrightarrow$

Testing the Graph of a Function for Symmetry

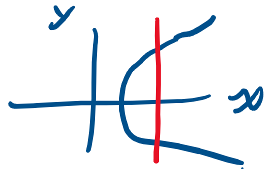
The graph of a function f is symmetric

even

- about the y-axis if and only if $f(-x) = f(x)$ for all x in the domain of f.

odd

- about the origin if and only if $f(-x) = -f(x)$ for all x in the domain of f.



x-axis symmetry

fails the vertical line test
$\Rightarrow$ y is not a function of x