09-92-25 MTH 167-004N

1 Relations and Functions
1.1 Sets of Real Numbers and the Cartesian Coordinate Plane
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1.3 Introduction to Functions
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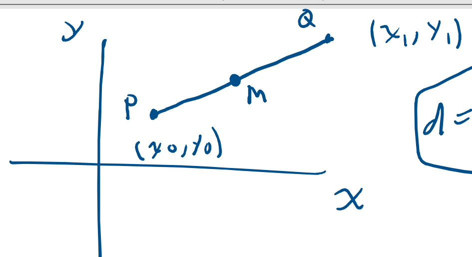
1.1: 36

36. Verify the Midpoint Formula by showing the distance between $P(x_1, y_1)$ and M and the distance between M and $Q(x_2, y_2)$ are both half of the distance between P and Q.

> **Equation 1.2. The Midpoint Formula:** The midpoint M of the line segment connecting $P(x_0, y_0)$ and $Q(x_1, y_1)$ is:
>
> $$M = \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right)$$



$$\boxed{d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}}$$

$$\text{dist}(P, M) = \sqrt{\left(\frac{x_0 + x_1}{2} - x_0\right)^2 + \left(\frac{y_0 + y_1}{2} - y_0\right)^2}$$

$$\text{dist}(M, Q) = \sqrt{\left(x_1 - \frac{x_0 + x_1}{2}\right)^2 + \left(y_1 - \frac{y_0 + y_1}{2}\right)^2}$$

$$\text{dist}(P, Q) = \sqrt{(y_1 - y_0)^2 + (x_1 - x_0)^2}$$

$$\text{dist}(P, M) = \sqrt{\left(\frac{x_0 + x_1}{2}\right)^2 - 2x_0\left(\frac{x_0 + x_1}{2}\right) + (x_0)^2 + \left(\frac{y_0 + y_1}{2}\right)^2 - 2y_0\left(\frac{y_0 + y_1}{2}\right) + (y_0)^2}$$

$$= \sqrt{\frac{x_0^2 + 2x_0x_1 + (x_1)^2}{4} - \cancel{x_0^2} - x_0x_1 + \cancel{(x_0)^2} + \frac{y_0^2 + 2y_0y_1 + (y_1)^2}{4} - \cancel{y_0^2} - y_0y_1 + \cancel{(y_0)^2}}$$

$$\sqrt{\cdots + \frac{y_0^2 + 2y_0y_1 + (y_1)^2}{4} - \cancel{x_0^2} - y_0y_1 + \cancel{(y_0)^2}}$$

$$= \sqrt{\frac{x_0^2 + 2x_0x_1 + (x_1)^2 - 4x_0x_1}{4} + \frac{y_0^2 + 2y_0y_1 + (y_1)^2 - 4y_0y_1}{4}}$$

$$= \sqrt{\frac{(x_0)^2 - 2x_0x_1 + (x_1)^2}{4} + \frac{(y_0)^2 - 2y_0y_1 + (y_1)^2}{4}}$$

$$= \frac{1}{2}\sqrt{\left((x_0)^2 - 2x_0x_1 + (x_1)^2\right) + \left(y_0^2 - 2y_0y_1 + (y_1)^2\right)}$$

$$= \frac{1}{2}\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

$$= \frac{1}{2}\,\mathrm{dist}(P, Q)$$

Likewise, $\mathrm{dist}(M, Q) = \frac{1}{2}\,\mathrm{dist}(P, Q)$

1.2: 50

For each equation given in Exercises 41 - 52:

- Find the x- and y-intercept(s) of the graph, if any exist.
- Follow the procedure in Example 1.2.3 to create a table of sample points on the graph of the equation.
- Plot the sample points and create a rough sketch of the graph of the equation.
- Test for symmetry. If the equation appears to fail any of the symmetry tests, find a point on the graph of the equation whose reflection fails to be on the graph as was done at the end of Example 1.2.4

50. $x^2 - y^2 = 1$

x-intercept set $y=0$
solve for x

$x^2-(0)^2=1$

$x^2=1$

$\boxed{x=\pm 1}$

or $(-1,0),(1,0)$

y-intercept
set $x=0$
solve for y

$0^2-y^2=1$

$-y^2=1$

$y^2=-1$

$y=\pm\sqrt{-1}=\pm i \notin \mathbb{R}$

$\therefore$ No y-intercept

x	y
-1	0
1	0
$-\sqrt{2}$	-1
$\sqrt{2}$	-1
$-\sqrt{2}$	1
$\sqrt{2}$	1

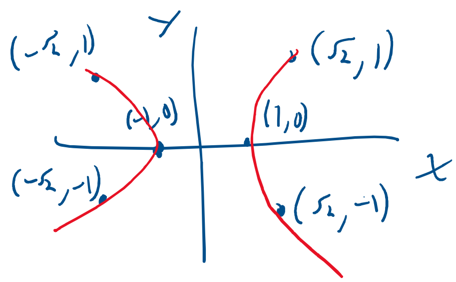
$x^2-y^2=1$

$x^2-(\pm 1)^2=1$

$x^2-1=1$

$x^2=2$

$x=\pm\sqrt{2}$



y-axis symmetry

$\boxed{x^2-y^2=1}$

same

$(-x)^2-y^2=1$

$\boxed{x^2-y^2=1}$

$\therefore$ y-axis symmetry

To graph on calculator
solve for y

$-y^2=1-x^2$

$y^2=-1+x^2$

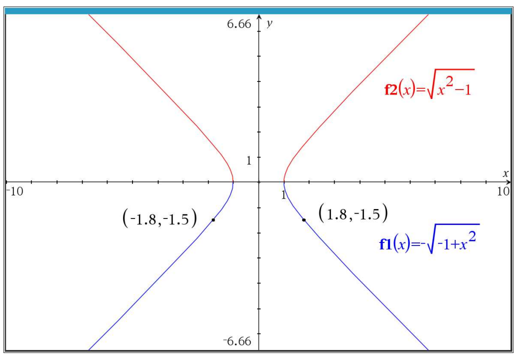
$$-y^2 = 1 - x^2$$

$$y^2 = -1 + x^2$$

$$\boxed{y = \pm\sqrt{-1+x^2}}$$

$$Y_1 = -\sqrt{-1+x^2}$$

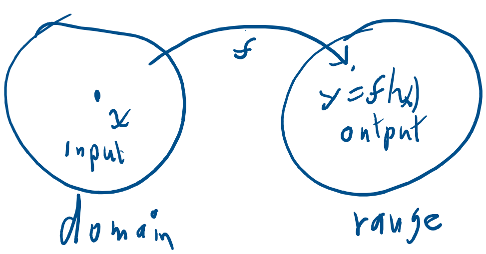
$$Y_2 = \sqrt{-1+x^2}$$



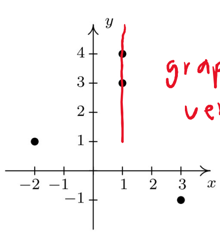
1.3 memorize

Definition 1.6. A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a **function** of x.

General definition: a function is a rule that associates to each element of a set, called the domain, a unique element in another set, called the range.



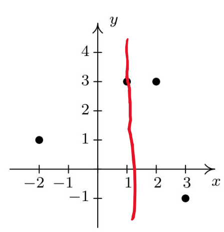
Theorem 1.1. The Vertical Line Test: A set of points in the plane represents y as a function of x if and only if no two points lie on the same vertical line.



The graph of R_1

graph fails vertical line test

y is not a function of x



The graph of R_2

graph passes the vertical line test

y is a function of x

Definition 1.7. Suppose F is a relation which describes y as a function of x.

- The set of the x-coordinates of the points in F is called the **domain** of F.

Definition 1.7. Suppose F is a relation which describes y as a function of x.

- The set of the x-coordinates of the points in F is called the **domain** of F.
- The set of the y-coordinates of the points in F is called the **range** of F.

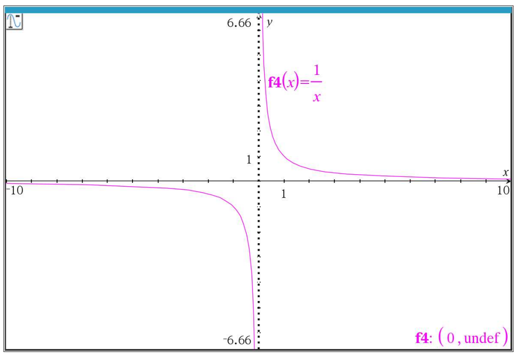
We can generalize this to relations that are not functions.

Definition : (implied) domain of a function is the largest subset of real numbers for which the function is well-defined, that is, for which the output can be calculated.

Find the domain of $f(x) = \frac{1}{x}$

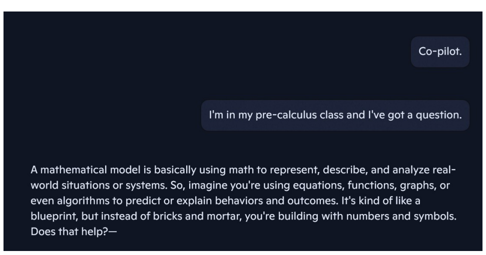
domain $= \{x \mid x \neq 0\}$ set-builder notation

$= (-\infty, 0) \cup (0, \infty)$



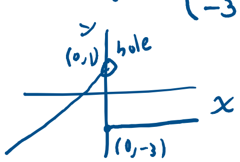
When finding a domain, exclude values of x that result in division by zero or taking the square root of a negative number.

Definition: a mathematical model is a set of equations and formulas that describe a real-world system.



Piece-wise defined function

$$f(x) = \begin{cases} x+1 & \text{for } x<0 \\ -3 & \text{for } x \geq 0 \end{cases}$$



domain $= (-\infty, \infty)$
$= \mathbb{R}$

TI $\quad Y_1 = (x+1)*(x<0) - 3*(x \geq 0)$

Your name MTH 167-004N quiz 1

1. Let $R = \{(1,2), (2,3), (1,2)\}$.
   Does R represent y as a function of x? Why or why not?

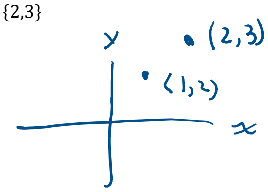
Yes. Each input x has exactly one output y. The fact that (1,2) is repeated

does not change this.
In terms of sets, $\{(1,2),(2,3),(1,2)\}=\{(1,2),(2,3)\}$.

2. What is the domain of R?
   domain of R = $\{1,2\}$

3. What is the range of R?
   range of R = $\{2,3\}$

4. Graph R .



Note that I did not plot the point (1,2) twice.

5. What is the domain of $g(x)=\frac{x}{\sqrt{x-1}}$?
   To avoid division by zero, we cannot have x = 1.
   $g(1)=\frac{1}{\sqrt{1-1}}=\frac{1}{\sqrt{0}}=\frac{1}{0}$, which is not defined.
   To avoid taking the square root of a negative number,
   we must have $x-1\geq 0 \Leftrightarrow x\geq 1$.
   Combining these conditions, the domain of g = $\{x|x>1\}=(1,\infty)$

   We can check this with a graph.