08-28-25 MTH 167-004N

1 Relations and Functions
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1.1 memorize

Definition 1.1. A **set** is a well-defined collection of objects which are called the 'elements' of the set. Here, 'well-defined' means that it is possible to determine if something belongs to the collection or not, without prejudice.

Memorize

Ways to Describe Sets

1. **The Verbal Method:** Use a sentence to define a set.
2. **The Roster Method:** Begin with a left brace '{', list each element of the set *only once* and then end with a right brace '}'.
3. **The Set-Builder Method:** A combination of the verbal and roster methods using a "dummy variable" such as x.

Roster method $A = \{2,4,6\} = \{6,4,2\} = \{2,4,6,2\}$

Set-builder notation
$A = \{x | x \text{ is an even number and } 2 \leq x \leq 6\}$

Verbal method: A is the set of all even numbers greater than or equal to 2 and less than or equal to 6

Roster method: the set of all stars in our galaxy

Set-builder: {California, 3.14159, ice cream cone, NOVA}

Copilot

{California, 3.14159, ice cream cone, NOVA} What property defines the elements in this set?

This set is delightfully eclectic, but there *is* a unifying thread: each element can be associated with **"things that can be featured in a PBS documentary or educational program."** Here's how:

- 🌉 **California** — Geography, environment, history, or social issues. Plenty of PBS specials explore California's ecosystems, culture, and politics.
- 🔢 **3.14159** — That's π (pi), a mathematical constant. Perfect for a NOVA episode on math, physics, or the nature of the universe.
- 🍦 **Ice cream cone** — Food science, cultural history, or even economics. PBS has covered quirky topics like the history of desserts or innovations in food.
- 🌌 **NOVA** — This one's meta: NOVA is *itself* a PBS science documentary series.

So the defining property could be:

> **Each item is a subject or symbol that could appear in an educational or science-themed documentary.**

Want to explore other possible interpretations, like symbolic, linguistic, or even humorous ones?

Memorize

Sets of Numbers

1. The **Empty Set**: $\emptyset = \{\} = \{x \,|\, x \neq x\}$. This is the set with no elements. Like the number '0,' it plays a vital role in mathematics.[a]
2. The **Natural Numbers**: $\mathbb{N} = \{1, 2, 3, \ldots\}$ The periods of ellipsis here indicate that the natural numbers contain 1, 2, 3, 'and so forth'.
3. The **Whole Numbers**: $\mathbb{W} = \{0, 1, 2, \ldots\}$
4. The **Integers**: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
5. The **Rational Numbers**: $\mathbb{Q} = \left\{\frac{a}{b} \,|\, a \in \mathbb{Z} \text{ and } b \in \mathbb{Z}\right\}$. Rational numbers are the ratios of integers (provided the denominator is not zero!) It turns out that another way to describe the rational numbers[b] is:

$$\mathbb{Q} = \{x \,|\, x \text{ possesses a repeating or terminating decimal representation.}\}$$

6. The **Real Numbers**: $\mathbb{R} = \{x \,|\, x \text{ possesses a decimal representation.}\}$
7. The **Irrational Numbers**: $\mathbb{P} = \{x \,|\, x \text{ is a non-rational real number.}\}$ Said another way, an irrational number is a decimal which neither repeats nor terminates.[c]
8. The **Complex Numbers**: $\mathbb{C} = \{a + bi \,|\, a, b \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$ Despite their importance, the complex numbers play only a minor role in the text.[d]

[a] ...which, sadly, we will not explore in this text.
[b] See Section 9.2.
[c] The classic example is the number π (See Section 10.1), but numbers like $\sqrt{2}$ and $0.101001000100001\ldots$ are other fine representatives.
[d] They first appear in Section 3.4 and return in Section 11.7.

$\mathbb{N} \subset \mathbb{W} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$\mathbb{P} \subset \mathbb{R}$

Memorize

Interval Notation

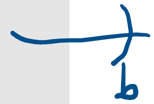
Let a and b be real numbers with $a < b$.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \,\|\, a < x < b\}$	(a, b)	a b
$\{x \,\|\, a \leq x < b\}$	$[a, b)$	a b
$\{x \,\|\, a < x \leq b\}$	$(a, b]$	a b
$\{x \,\|\, a \leq x \leq b\}$	$[a, b]$	a b
$\{x \,\|\, x < b\}$	$(-\infty, b)$	b

a b

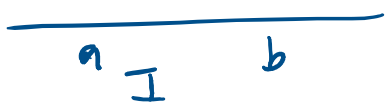
a b

$\{x \mid a \le x \le b\}$	$[a,b]$	a b
$\{x \mid x < b\}$	$(-\infty, b)$	b
$\{x \mid x \le b\}$	$(-\infty, b]$	b
$\{x \mid x > a\}$	(a, ∞)	a
$\{x \mid x \ge a\}$	$[a, \infty)$	a
$\mathbb{R}$	$(-\infty, \infty)$	

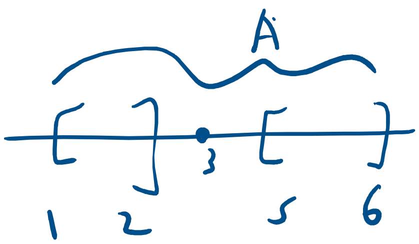


extra $\Big\{$ Def: An interval is a set of numbers A such that if $a, b \in A$ and $a < c < b$, then $c \in A$

$a < c < b$



$a, b \in I \Rightarrow c \in I$



Is A an interval?

$2 < 3 < 5$

$2 \in A$, $5 \in A$

but $3 \notin A$

$\therefore$ No

Notation $x \in A$ means x is an element (member) of set A

$x \notin A$ x is not an element of A

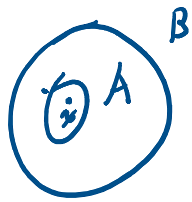
$x \notin A$ 'x is <u>not</u> an element of A

Def $A \subseteq B$ A is a subset of B

ie $x \in A \Rightarrow x \in B$

implies

Venn diagram



Def $A \subset B$ if $A \subseteq B$ but $A \neq B$

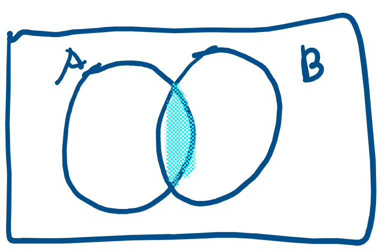
Proper subset

Memorize

Definition 1.2. Suppose A and B are two sets.

- The **intersection** of A and B: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- The **union** of A and B: $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ (or both)}\}$

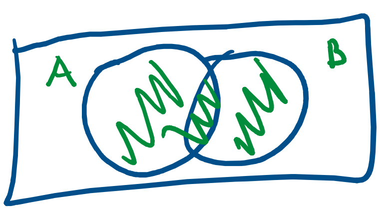
$\hookrightarrow$ inclusive or



U = universal set

or set of discourse

$A \cap B$



$A \cup B$

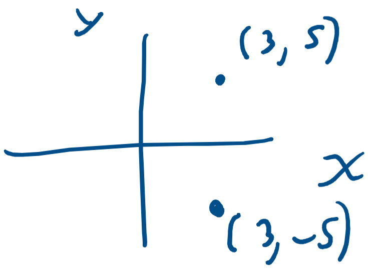
Memorize

Definition 1.3. Two points (a, b) and (c, d) in the plane are said to be

- **symmetric about the x-axis** if $a = c$ and $b = -d$
- **symmetric about the y-axis** if $a = -c$ and $b = d$
- **symmetric about the origin** if $a = -c$ and $b = -d$

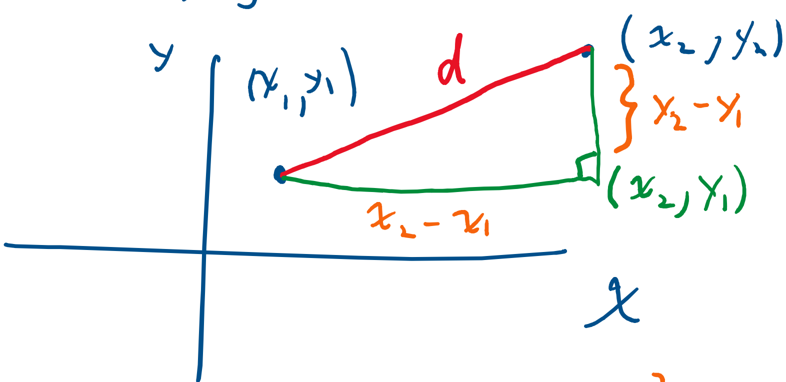
y . (2, 5)

- **symmetric about the origin** if $a = -c$ and $b = -d$



x-axis symmetry

Find the distance d between the points (x_1, y_1) and (x_2, y_2).



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$ Pytha. thm.

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

distance formula

memorize

$$\sqrt{9+36} = \sqrt{45} = \sqrt{(9)(5)}$$

$$= \sqrt{9}\,\sqrt{5}$$

$$= \boxed{3\sqrt{5}}$$

45 → 9, 5; 9 → 3, 3

Memorize

Equation 1.2. The Midpoint Formula: The midpoint M of the line segment connecting $P(x_0, y_0)$ and $Q(x_1, y_1)$ is:

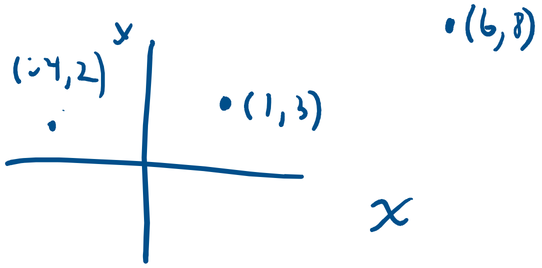
$$M = \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right)$$

1.2

Memorize

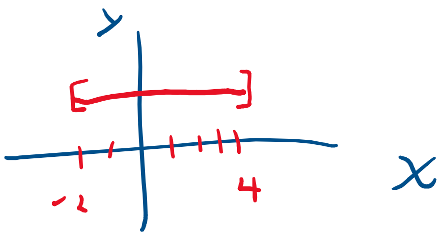
Definition 1.4. A **relation** is a set of points in the plane.

$R = \{(1,3), (-4,2), (6,8)\}$



Example 1.2.1. Graph the following relations.

2. $HLS_1 = \{(x,3) \mid -2 \leq x \leq 4\}$



Memorize

The Fundamental Graphing Principle

The graph of an equation is the set of points which satisfy the equation. That is, a point (x, y) is on the graph of an equation if and only if x and y satisfy the equation.

Memorize

Definition 1.5. Suppose the graph of an equation is given.

- A point on a graph which is also on the x-axis is called an ***x*-intercept** of the graph.
- A point on a graph which is also on the y-axis is called an ***y*-intercept** of the graph.

Finding the Intercepts of the Graph of an Equation

Given an equation involving x and y, we find the intercepts of the graph as follows:

- x-intercepts have the form $(x, 0)$; set $y = 0$ in the equation and solve for x.
- y-intercepts have the form $(0, y)$; set $x = 0$ in the equation and solve for y.

Find the x-intercept and y-intercept of the line $2x + 3y = 6$.
Then, plot the intercepts and graph the line.

x-intercept
set $y=0$
solve for x

$2x + (3)(0) = 6$

$2x = 6$

$\boxed{x=3}$

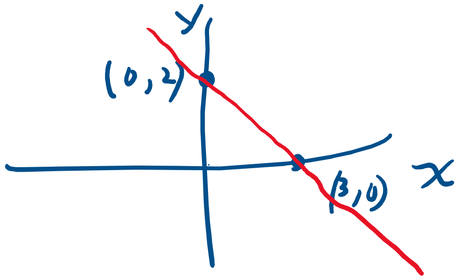
y-intercept
set $x=0$
solve for y

$(2)(0) + 3y = 6$

$3y = 6$

$\boxed{y=2}$

$2x = 6$ | $3y = 6$
$\boxed{x = 3}$ | $\boxed{y = 2}$
or the point $(3,0)$ | or the point $(0,2)$



$2x + 3y = 6$

Solve for y

$3y = -2x + 6$

$$\boxed{y = -\frac{2x}{3} + 2}$$

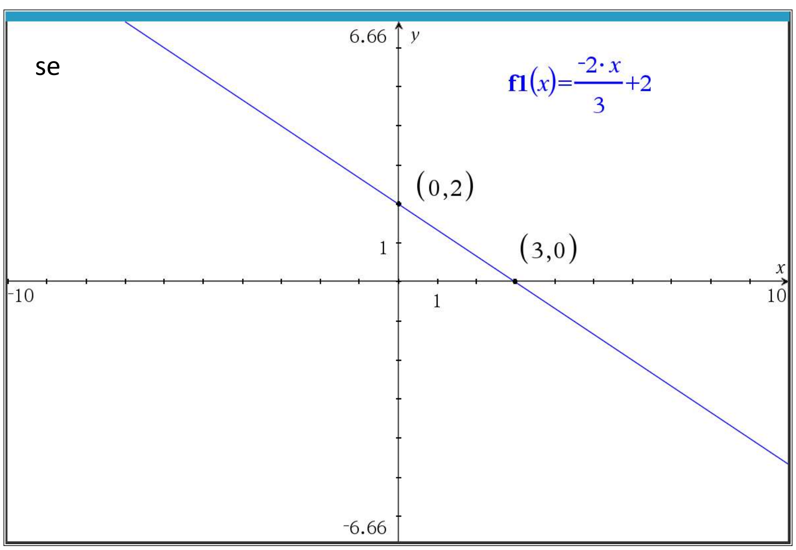
Notation

$-\frac{2x}{3} = \left(-\frac{2}{3}\right)x$

Good

$-2/3\,x$ bad

I changed the equation to slope-intercept form, and used Trace with x = 0 and x = 3 to verify the intercepts.



Memorize

Testing the Graph of an Equation for Symmetry

To test the graph of an equation for symmetry

- about the y-axis – substitute $(-x, y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the y-axis.
- about the x-axis – substitute $(x, -y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the x-axis.
- about the origin - substitute $(-x, -y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the origin.

such that

1.1
$A = \{x | x = 2n \text{ for } n \in \mathbb{Z}\}$ = the set of all even numbers

$\boxed{\text{True}}$ or false $6 \in A$

$6 = (2)(3)$

$3 \in \mathbb{Z}$

$A = \{0, \pm 2, \pm 4, \ldots\}$

$A = \{\ldots -4, -2, 0, 2, 4, \ldots\}$

$\boxed{\text{True}}$ or false $5 \notin A$

$5 = 2n$

$n = \frac{5}{2} \notin \mathbb{Z}$

$\therefore 5 \notin A$

$A = \{x | x \text{ has property } p\}$

"A is the set of all x such that x has property p"

$A = \{a, b, c, d\}$

$B = \{b, c, m, n\}$

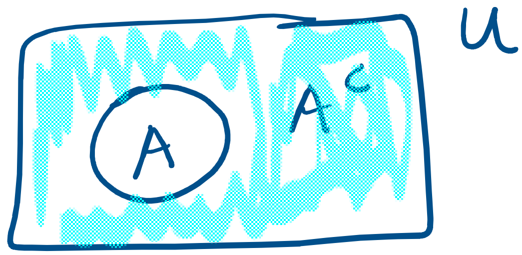
$A \cap B = \{b, c\}$

$A \cap B = \{ b, c \}$

$A \cup B = \{a, b, c, d, m, n\}$

Def. A^c = complement of A

$= \{x \in U \mid x \notin A\}$



$A \cup A^c = U$

$A \cap A^c = \emptyset$