11-25-25 MTH 167-002N

10.7 Trigonometric Equations and Inequalities
10.7.1 Exercises
page 874 (886): 1, 8, 26, 39, 62, 69, 73
11 Applications of Trigonometry
11.1 Applications of Sinusoids
11.1.2 Exercises
page 891 (903):1, 2, 3
11.2 The Law of Sines
11.2.1 Exercises
page 904 (916): 1, 3, 25, 26

11.3 The Law of Cosines
11.3.1 Exercises
page 916 (928): 1, 7, 11, 19
11.4 Polar Coordinates
11.4.1 Exercises
page 930 (942): 2, 11, 17, 19, 22, 37, 57, 64, 72, 85
11.5 Graphs of Polar Equations
11.5.1 Exercises
page 958 (972): 1, 3, 9, 21, 32

Before class notes
10.6

In Exercises 41 - 48, assume that the range of arcsecant is $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ and that the range of arccosecant is $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ when finding the exact value.

41. $\operatorname{arcsec}(-2)$

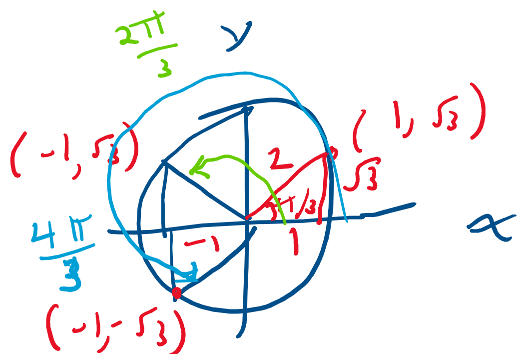
Let $\theta = \operatorname{arcsec}(-2)$

$\sec(\theta) = -2$

$\dfrac{1}{\cos(\theta)} = -2$

$\cos(\theta) = -\dfrac{1}{2}$

which angle, $\dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$, is in $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$?



$$\boxed{\therefore \operatorname{arcsec}(-2) = \frac{4\pi}{3}}$$

41. $\operatorname{arcsec}(-2) = \dfrac{4\pi}{3}$

Textbook answer

$\sec^{-1}(-2)$	$\dfrac{2\cdot\pi}{3}$

TI uses "trig friendly" range of arcsec.
This exercise uses the "calculus friendly" range of arcsec.

10.6

In Exercises 57 - 86, find the exact value or state that it is undefined.

66. $\cos(\arccos(\pi))$

Let $\theta = \arccos(\pi)$

$\cos\theta = \pi \approx 3.14 \notin [-1,1]$

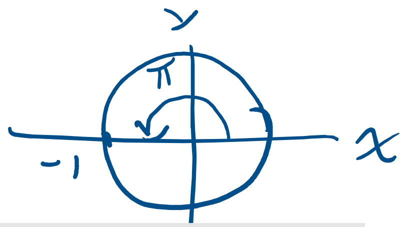
$\therefore$ No solution

$\arccos(\cos(\pi))$

Let $\theta = \arccos(\cos \pi)$

$\cos\theta = \cos\pi$

$\Rightarrow \cos\theta = -1$



- Properties of $F(x) = \arccos(x)$
  - Domain: $[-1, 1]$
  - Range: $[0, \pi]$
  - $\arccos(x) = t$ if and only if $0 \leq t \leq \pi$ and $\cos(t) = x$
  - $\cos(\arccos(x)) = x$ provided $-1 \leq x \leq 1$
  - $\arccos(\cos(x)) = x$ provided $0 \leq x \leq \pi$

$\therefore \theta = \pi$

$\cos^{-1}(\cos(\pi))$	π

In Exercises 216 - 221, rewrite the given function as a sinusoid of the form $S(x) = A\sin(\omega x + \phi)$ using Exercises 35 and 36 in Section 10.5 for reference. Approximate the value of ϕ (which is in radians, of course) to four decimal places.

216. $f(x) = 5\sin(3x) + 12\cos(3x)$

Properties of the Sinusoid $S(t) = A\sin(\omega t + \phi) + B$

- The **amplitude** is $|A|$
- The **angular frequency** is ω and the **ordinary frequency** is $f = \dfrac{\omega}{2\pi}$
- The **period** is $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- The **phase** is ϕ and the **phase shift** is $-\dfrac{\phi}{\omega}$
- The **vertical shift** or **baseline** is B

$\sin(\omega x + \phi)$

- The **phase** is φ and the **phase shift** is ω
- The **vertical shift** or **baseline** is B

$$f(x) = A\sin(wx + \varphi)$$
$$= A\left(\sin(wx)\cos(\varphi) + \cos(wx)\sin(\varphi)\right)$$
$$= A\sin(wx)\cos(\varphi) + A\cos(wx)\sin(\varphi)$$

$\boxed{w = 3}$ $\boxed{B = 0}$

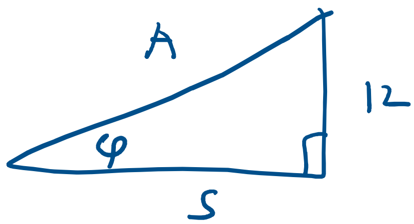
$$f(x) = A\cos(\varphi)\sin(3x) + A\sin(\varphi)\cos(3x)$$

$A\cos(\varphi) = 5$ | $A\sin(\varphi) = 12$

$\cos(\varphi) = \frac{5}{A}$

$\sin(\varphi) = \frac{12}{A}$

$$A = \frac{5}{\cos(\varphi)} = \frac{12}{\sin(\varphi)}$$
$$\frac{\sin(\varphi)}{\cos(\varphi)} = \frac{12}{5}$$
$$\tan(\varphi) = \frac{12}{5}$$



$$A = \sqrt{144 + 25} = \sqrt{169} = 13$$

$\boxed{A = 13}$

$\cos(\varphi) = \frac{5}{13}$ $\sin(\varphi) = \frac{12}{13}$

$$\tan(\varphi) = \frac{opp}{adj} = \frac{12}{5}$$

$$\boxed{\varphi = \arctan\left(\frac{12}{5}\right) \approx 1.1760}$$

$\tan^{-1}\left(\frac{12}{5}\right)$	$\frac{\pi}{2} - \tan^{-1}\left(\frac{5}{12}\right)$
$\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{5}{12}\right)\right)$ ▸Decimal	1.17601

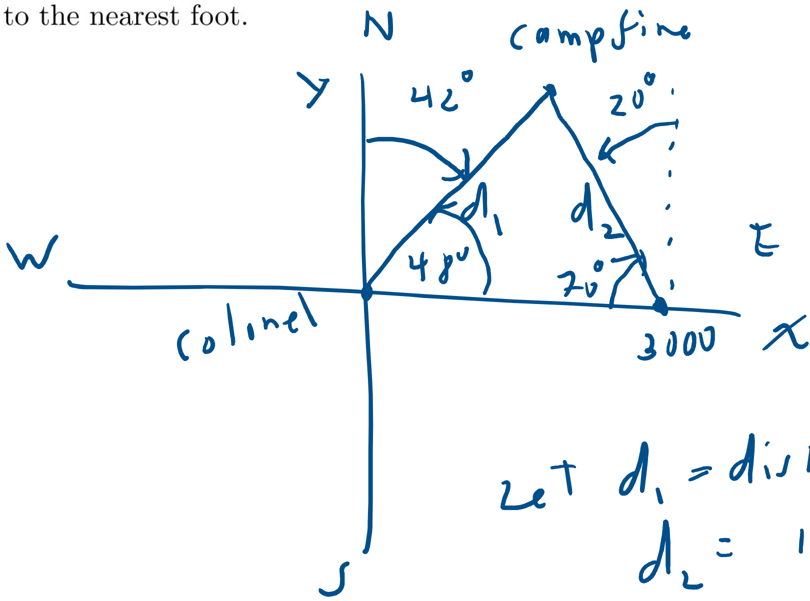
$\left|\left(2 \quad \tan \left(12\right)\right)\right|$ Decimal

$$\boxed{f(x) \approx 13(\sin(3x) + 1.1760)}$$

216. $f(x) = 5\sin(3x) + 12\cos(3x) = 13\sin\left(3x + \arcsin\left(\frac{12}{13}\right)\right) \approx 13\sin(3x + 1.1760)$

11.2: 26

26. The Colonel spots a campfire at a of bearing N42°E from his current position. Sarge, who is positioned 3000 feet due east of the Colonel, reckons the bearing to the fire to be N20°W from his current position. Determine the distance from the campfire to each man, rounded to the nearest foot.



Let d_1 = distance (colonel, campfire)

d_2 = " (sarge, campfire)

Find d_1, d_2

$$\frac{d_1}{\sin(70^\circ)} = \frac{d_2}{\sin(48^\circ)} = \frac{3000}{\sin(62^\circ)}$$

$180^\circ - 48^\circ - 70^\circ$

$= 180^\circ - 118^\circ$

$= 62^\circ$

$$d_1 = \frac{3000\sin(70^\circ)}{\sin(62^\circ)} \approx 3193 \text{ ft}$$

$$d_2 = \frac{3000\sin(48^\circ)}{\sin(62^\circ)} \approx 2525 \text{ ft}$$

$$\frac{3000\cdot\sin(70°)}{\sin(62°)} \qquad \frac{3000\cdot\cos\left(\frac{\pi}{9}\right)}{\cos\left(\frac{7\cdot\pi}{45}\right)}$$

$$\frac{3000\cdot\cos\left(\frac{\pi}{9}\right)}{\cos\left(\frac{7\cdot\pi}{45}\right)}\blacktriangleright\text{Decimal} \qquad 3192.8$$

$$\frac{3000\cdot\sin(48°)}{\sin(62°)}\blacktriangleright\text{Decimal} \qquad 2524.99$$

26. The Colonel is about 3193 feet from the campfire.
    Sarge is about 2525 feet to the campfire.

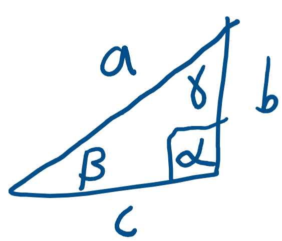
11.3
Supplied

Theorem 11.5. Law of Cosines: Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following equations hold

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha) \qquad b^2 = a^2 + c^2 - 2ac\cos(\beta) \qquad c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

or, solving for the cosine in each equation, we have

$$\cos(\alpha) = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}$$



$\alpha = \frac{\pi}{2}$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos\left(\frac{\pi}{2}\right)$

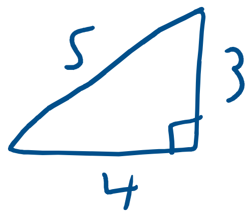
$a^2 = b^2 + c^2 - (2bc)(0)$

$a^2 = b^2 + c^2$

Supplied

Theorem 11.6. Heron's Formula: Suppose a, b and c denote the lengths of the three sides of a triangle. Let s be the semiperimeter of the triangle, that is, let $s = \frac{1}{2}(a+b+c)$. Then the area A enclosed by the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$



Area of $\Delta = \left(\frac{1}{2}\right)(4)(3) = \boxed{6}$

$s = \frac{1}{2}(3+4+5) = \frac{1}{2}(12) = 6$

$A = \sqrt{6(6-3)(6-4)(6-5)}$

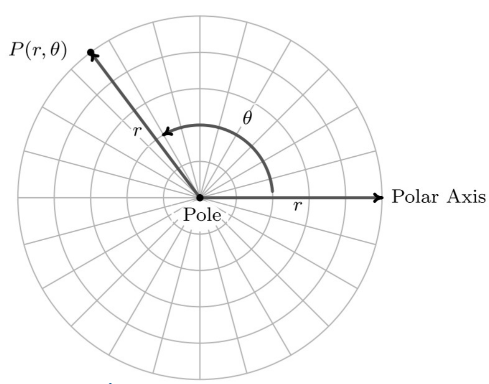
$A = \sqrt{(6)(3)(2)(1)}$

$A = \sqrt{6^2}$

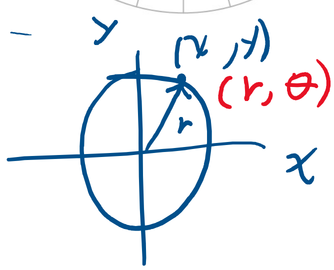
$\boxed{A = 6}$

11.4

Memorize

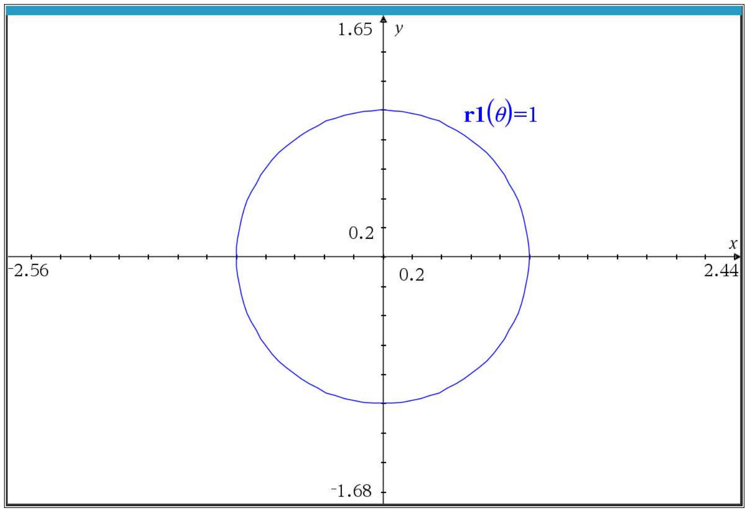


r and θ are the polar coordinated of a point

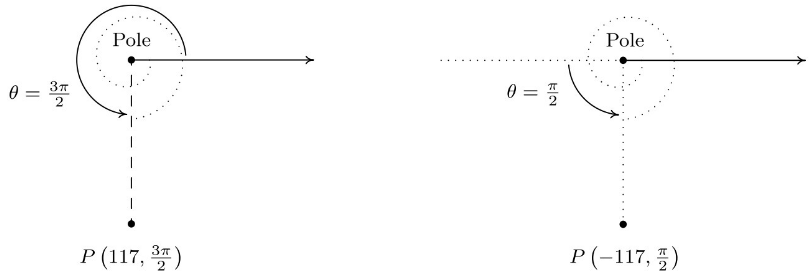


$x^2 + y^2 = r^2$

$r = r_0$



We can have more than one polar representation for a given point.



Memorize

Equivalent Representations of Points in Polar Coordinates

Suppose (r, θ) and (r', θ') are polar coordinates where $r \neq 0$, $r' \neq 0$ and the angles are measured in radians. Then (r, θ) and (r', θ') determine the same point P if and only if one of the following is true:

- $r' = r$ and $\theta' = \theta + 2\pi k$ for some integer k
- $r' = -r$ and $\theta' = \theta + (2k+1)\pi$ for some integer k

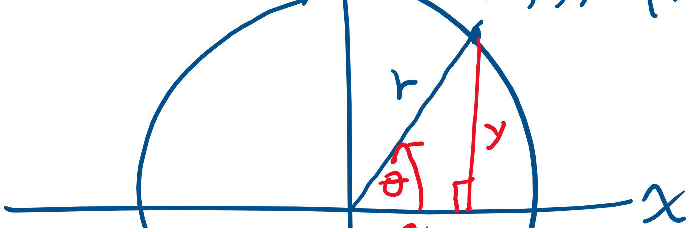
All polar coordinates of the form $(0, \theta)$ represent the pole regardless of the value of θ.

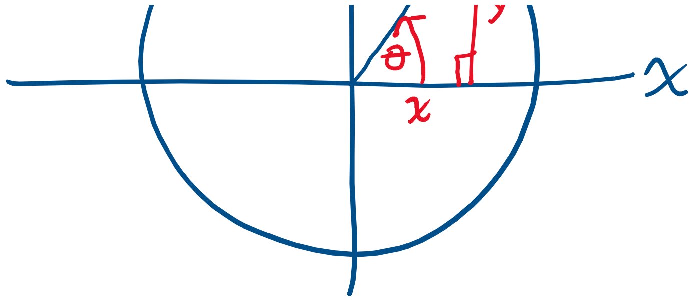
Memorize

Theorem 11.7. Conversion Between Rectangular and Polar Coordinates: Suppose P is represented in rectangular coordinates as (x, y) and in polar coordinates as (r, θ). Then

- $x = r\cos(\theta)$ and $y = r\sin(\theta)$
- $x^2 + y^2 = r^2$ and $\tan(\theta) = \dfrac{y}{x}$ (provided $x \neq 0$)

$(x, y) = (r\cos\theta, \sin\theta)$





11.5

memorize

The Fundamental Graphing Principle for Polar Equations

The graph of an equation in polar coordinates is the set of points which satisfy the equation. That is, a point $P(r, \theta)$ is on the graph of an equation if and only if there is a representation of P, say (r', θ'), such that r' and θ' satisfy the equation.

Memorize

Theorem 11.8. Graphs of Constant r and θ: Suppose a and α are constants, $a \neq 0$.

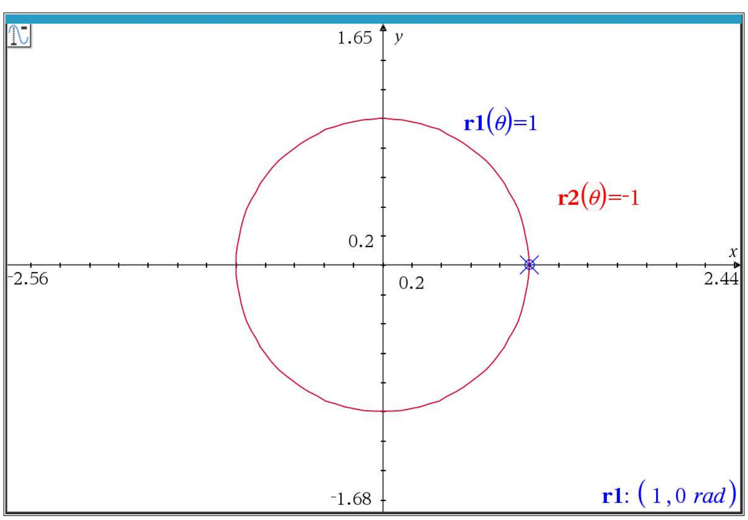
- The graph of the polar equation $r = a$ on the Cartesian plane is a circle centered at the origin of radius $|a|$.
- The graph of the polar equation $\theta = \alpha$ on the Cartesian plane is the line containing the terminal side of α when plotted in standard position.

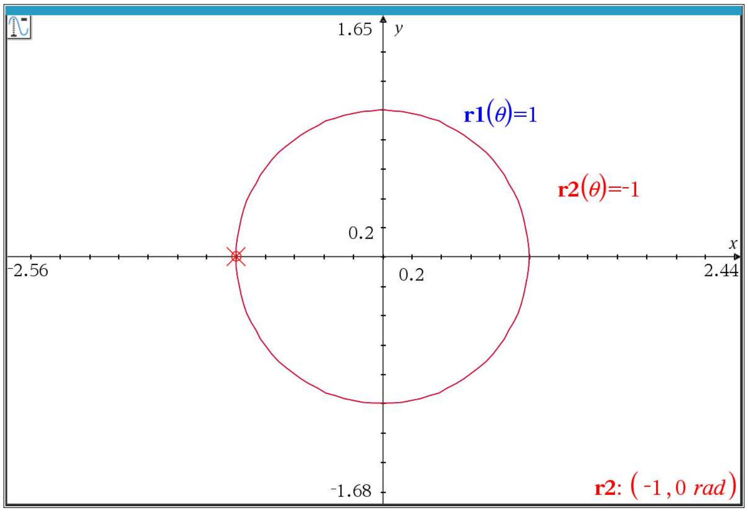
supplied

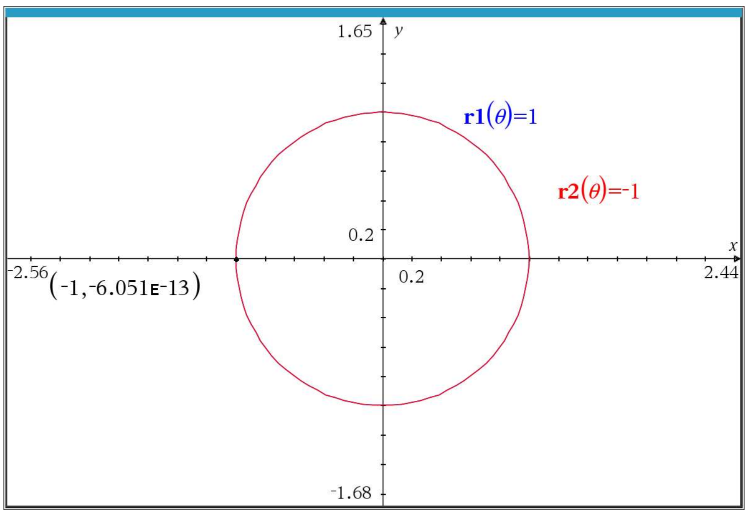
Guidelines for Finding Points of Intersection of Graphs of Polar Equations

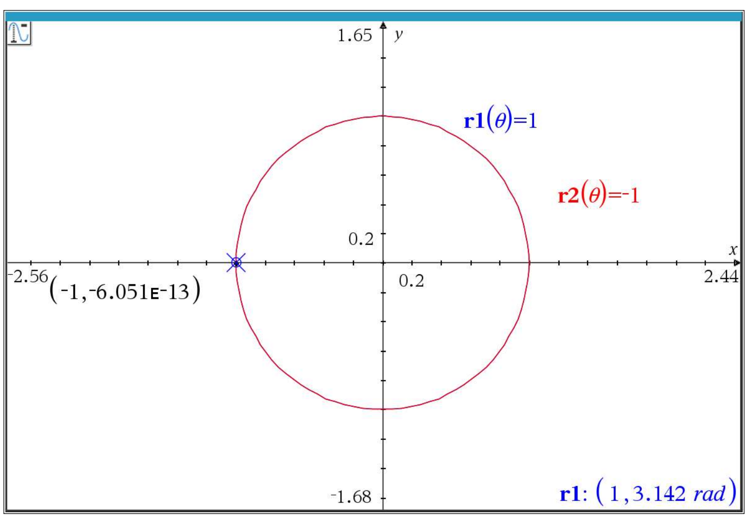
To find the points of intersection of the graphs of two polar equations E_1 and E_2:

- Sketch the graphs of E_1 and E_2. Check to see if the curves intersect at the origin (pole).
- Solve for pairs (r, θ) which satisfy both E_1 and E_2.
- Substitute $(\theta + 2\pi k)$ for θ in either one of E_1 or E_2 (but not both) and solve for pairs (r, θ) which satisfy both equations. Keep in mind that k is an integer.
- Substitute $(-r)$ for r and $(\theta + (2k+1)\pi)$ for θ in either one of E_1 or E_2 (but not both) and solve for pairs (r, θ) which satisfy both equations. Keep in mind that k is an integer.









11.5.1 Exercises

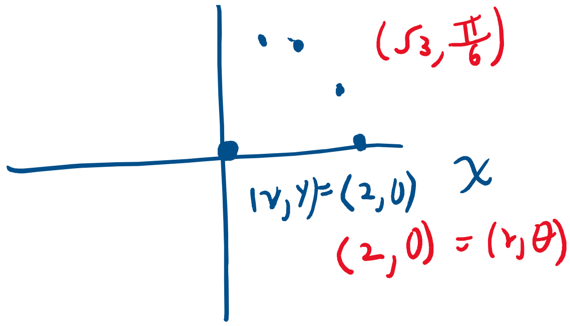
In Exercises 1 - 20, plot the graph of the polar equation by hand. Carefully label your graphs.

2. Circle: $r = 2\cos(\theta)$

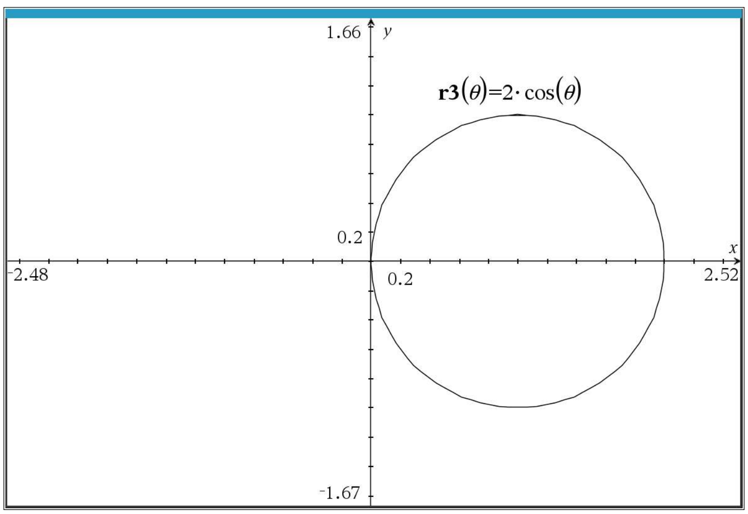
y

$(\sqrt{3}, \frac{\pi}{6})$

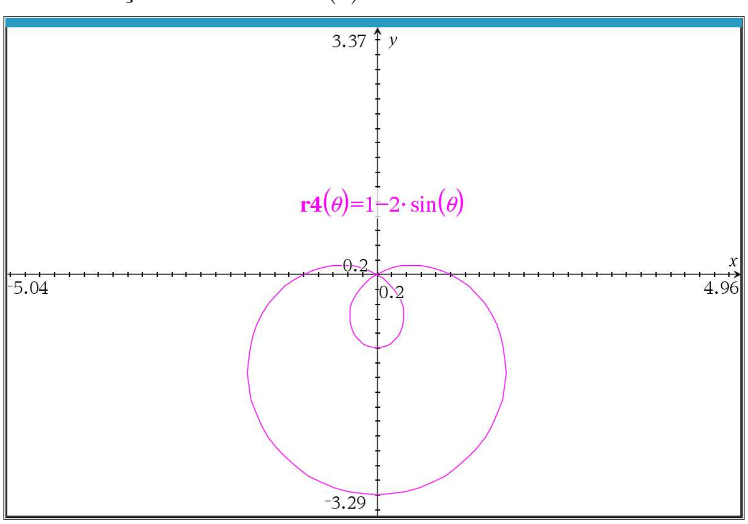
θ	$2\cos\theta$
0	2
π	–

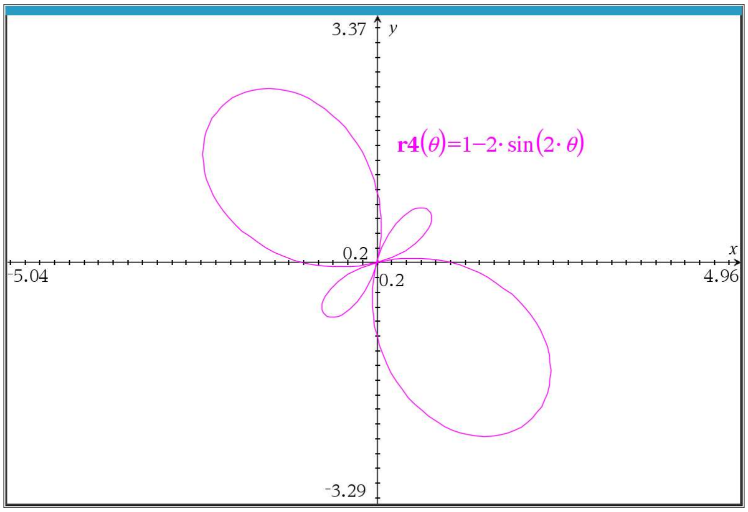


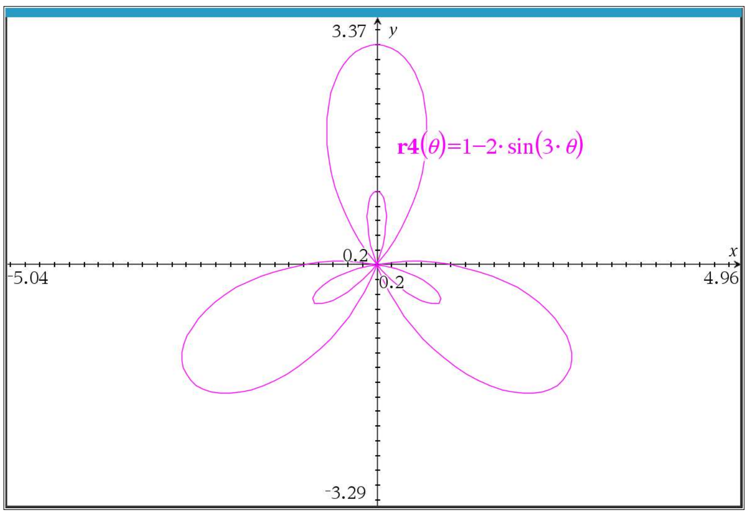
0	2
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$-\frac{\pi}{2}$	0

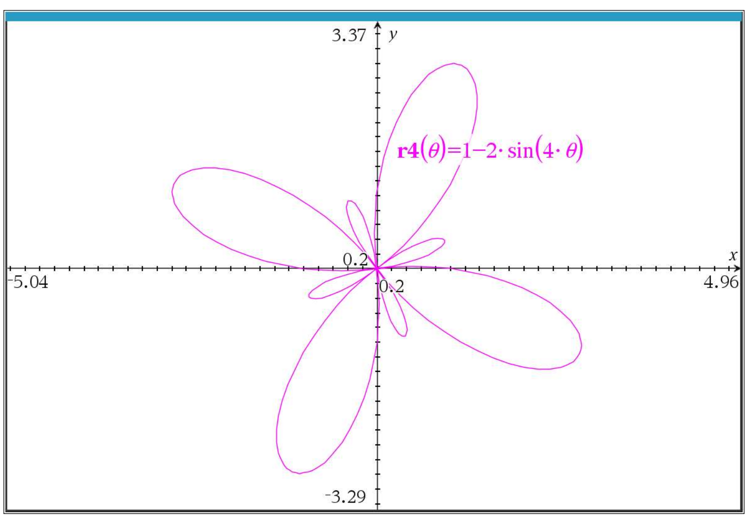


14. Limaçon: $r = 1 - 2\sin(\theta)$









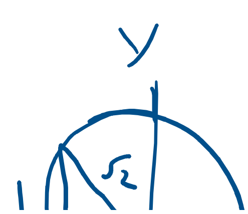
In Exercises 21 - 30, find the exact polar coordinates of the points of intersection of graphs of the polar equations. Remember to check for intersection at the pole (origin).

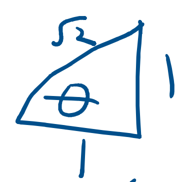
22. $r = 1 + \sin(\theta)$ and $r = 1 - \cos(\theta)$

$1 + \sin(\theta) = 1 - \cos(\theta)$

$\sin(\theta) = -\cos(\theta)$

$\sin(\theta)$





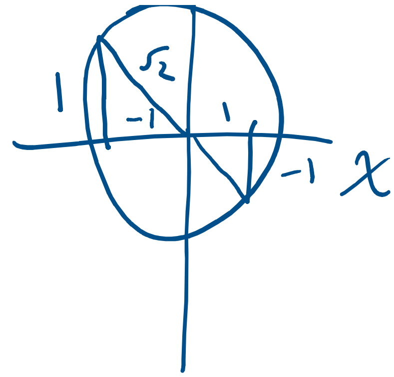
$\theta = \frac{\pi}{4}$

$\sin(\theta) = -\cos(\theta)$

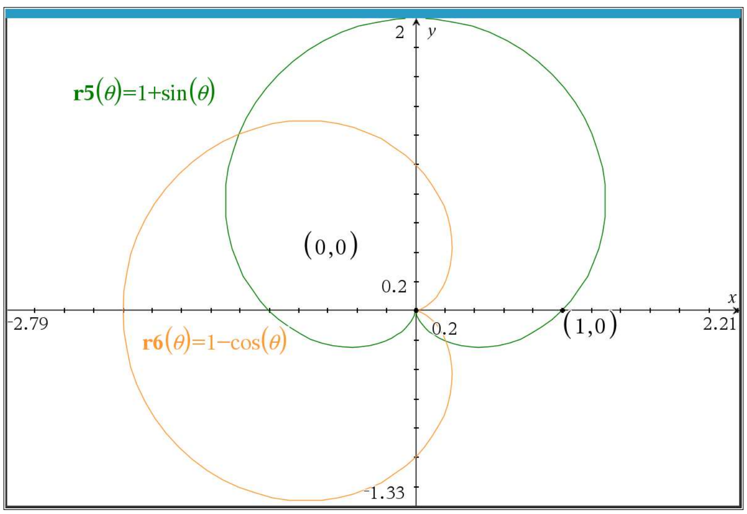
$$\frac{\sin(\theta)}{\cos(\theta)} = -1$$

$\tan(\theta) = -1$

$$\boxed{(r,\theta) = \left(\frac{2+\sqrt{2}}{2}, \frac{3\pi}{4}\right) \quad \left(\frac{2-\sqrt{2}}{2}, -\frac{\pi}{4}\right)}$$



$\theta = \frac{\pi}{4}$

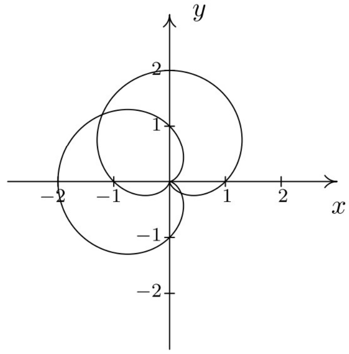


check at the origin

set $r=0$

$0 = 1+\sin(\theta)$	$0 = 1-\cos(\theta)$
$\sin(\theta) = -1$	$\cos\theta = 1$
$\boxed{\theta = \frac{3\pi}{2}}$	$\boxed{\theta = 0}$
$(r,\theta) = \left(0, \frac{3\pi}{2}\right)$	$(r,\theta) = (0,0)$

22. $r = 1 + \sin(\theta)$ and $r = 1 - \cos(\theta)$ $\qquad \left(\dfrac{2+\sqrt{2}}{2}, \dfrac{3\pi}{4}\right), \left(\dfrac{2-\sqrt{2}}{2}, \dfrac{7\pi}{4}\right)$, pole



$r = 1 + \sin\left(\frac{3\pi}{4}\right)$

$= 1 + \frac{\sqrt{2}}{2}$

$= \frac{2+\sqrt{2}}{2}$

$r = 1 - \cos\left(-\frac{\pi}{4}\right)$

$= 1 - \frac{\sqrt{2}}{2}$

$= \frac{2-\sqrt{2}}{2}$

Quiz 7 open homework

10.7.1 Exercises

In Exercises 1 - 18, find all of the exact solutions of the equation and then list those solutions which are in the interval $[0, 2\pi)$.

1. $\sin(5x) = 0$

Make a labeled sketch from your calculator.

Let $u = 5x$

$\sin(u) = 0$

$\Rightarrow u = k\pi$

$\Rightarrow 5x = k\pi$

$\Rightarrow \boxed{x = \frac{k\pi}{5}, \ k \in \mathbb{Z}}$ all solutions

$\boxed{x = \frac{k\pi}{5}, \ k = 0, 1, 2, \ldots 9}$

solutions in $[0, 2\pi)$

k	x
0	0
1	$\frac{\pi}{5}$
2	$\frac{2\pi}{5}$
3	$\frac{3\pi}{5}$
4	$\frac{4\pi}{5}$

Pi/5=0.628318530717959
2*Pi/5=1.256637061435917
⋮
9*Pi/5=5.654866776461628

3	$\frac{3\pi}{5}$
4	$4\pi/5$
5	π
6	$6\pi/5$
7	$7\pi/5$
8	$8\pi/5$
9	$9\pi/5$
10	$2\pi \notin [0, 2\pi)$

2*Pi/5=1.256637061435917
⋮
9*Pi/5=5.654866776461628