

**10.5 Graphs of the Trigonometric Functions**

10.5.4 Exercises

page 809 (821): 2, 8, 13, 25

**10.6 The Inverse Trigonometric Functions**

10.6.5 Exercises

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**10.7 Trigonometric Equations and Inequalities**

10.7.1 Exercises

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**11 Applications of Trigonometry****11.1 Applications of Sinusoids**

11.1.2 Exercises

page 891 (903): 1, 2, 3

**11.2 The Law of Sines**

11.2.1 Exercises

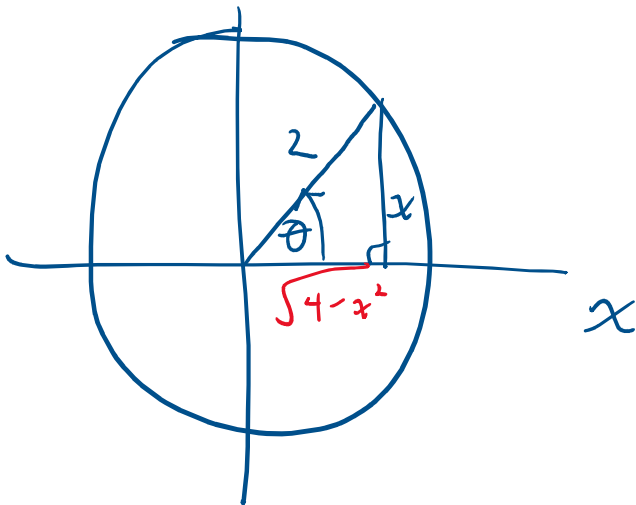
page 904 (916): 1, 3, 25, 26

10 textbook sections

5 class meetings before final exam

2-3 sections/class + 1 day of review

10.6: 185

185. If  $\sin(\theta) = \frac{x}{2}$  for  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ , find an expression for  $\theta + \sin(2\theta)$  in terms of  $x$ .

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= \left(2\right) \left(\frac{x}{2}\right) \frac{\sqrt{4-x^2}}{2}\end{aligned}$$

$$\boxed{\sin(2\theta) = \frac{x \sqrt{4-x^2}}{2}}$$

$$\sin \theta = \frac{x}{2}$$

$$\arcsin(\sin \theta) = \arcsin\left(\frac{x}{2}\right)$$

$$\boxed{\theta = \arcsin\left(\frac{x}{2}\right)}$$

$$\boxed{\theta + \sin(2\theta) = \arcsin\left(\frac{x}{2}\right) + \frac{x \sqrt{4-x^2}}{2}}$$

Copilot

To find an expression for  $\theta + \sin(2\theta)$  in terms of  $x$ , given that  $\sin(\theta) = \frac{x}{2}$  and  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ , we can proceed as follows:

### Step-by-step Derivation

1. Use the identity for  $\sin(2\theta)$ :

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

We already know:

$$\sin(\theta) = \frac{x}{2}$$

2. Find  $\cos(\theta)$  using the Pythagorean identity:

$$\cos(\theta) = \sqrt{1 - \sin^2(\theta)} = \sqrt{1 - \left(\frac{x}{2}\right)^2} = \sqrt{1 - \frac{x^2}{4}} = \frac{\sqrt{4 - x^2}}{2}$$

Since  $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ,  $\cos(\theta)$  is non-negative.

3. Substitute into  $\sin(2\theta)$ :

$$\sin(2\theta) = 2 \cdot \frac{x}{2} \cdot \frac{\sqrt{4 - x^2}}{2} = \frac{x\sqrt{4 - x^2}}{2}$$

4. Now express  $\theta$  in terms of  $x$ :

$$\theta = \arcsin\left(\frac{x}{2}\right)$$

5. Final expression:

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$$\theta + \sin(2\theta) = \arcsin\left(\frac{x}{2}\right) + \frac{x\sqrt{4 - x^2}}{2}$$

 **Final Answer**

10.7

supplied

### Strategies for Solving Basic Equations Involving Trigonometric Functions

- To solve  $\cos(u) = c$  or  $\sin(u) = c$  for  $-1 \leq c \leq 1$ , first solve for  $u$  in the interval  $[0, 2\pi)$  and add integer multiples of the period  $2\pi$ . If  $c < -1$  or of  $c > 1$ , there are no real solutions.
- To solve  $\sec(u) = c$  or  $\csc(u) = c$  for  $c \leq -1$  or  $c \geq 1$ , convert to cosine or sine, respectively, and solve as above. If  $-1 < c < 1$ , there are no real solutions.
- To solve  $\tan(u) = c$  for any real number  $c$ , first solve for  $u$  in the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  and add integer multiples of the period  $\pi$ .
- To solve  $\cot(u) = c$  for  $c \neq 0$ , convert to tangent and solve as above. If  $c = 0$ , the solution to  $\cot(u) = 0$  is  $u = \frac{\pi}{2} + \pi k$  for integers  $k$ .

# 10.7.1 EXERCISES

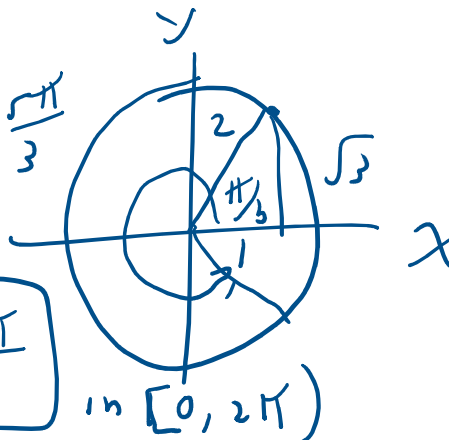
In Exercises 1 - 18, find all of the exact solutions of the equation and then list those solutions which are in the interval  $[0, 2\pi)$ .

$$2. \cos(3x) = \frac{1}{2}$$

$$\text{Let } u = 3x$$

$$\cos(u) = \frac{1}{2}$$

$$u = \frac{\pi}{3}, \frac{5\pi}{3} \text{ in } [0, 2\pi)$$



$$\text{all solutions } u = \frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k$$

$$3x = \frac{\pi}{3} + 2\pi k, \quad 3x = \frac{5\pi}{3} + 2\pi k$$

$$\text{all solutions } x = \frac{\pi}{9} + \frac{2\pi k}{3}, \quad x = \frac{5\pi}{9} + \frac{2\pi k}{3}$$

$$k=0 \quad x = \frac{\pi}{9}, \frac{5\pi}{9}$$

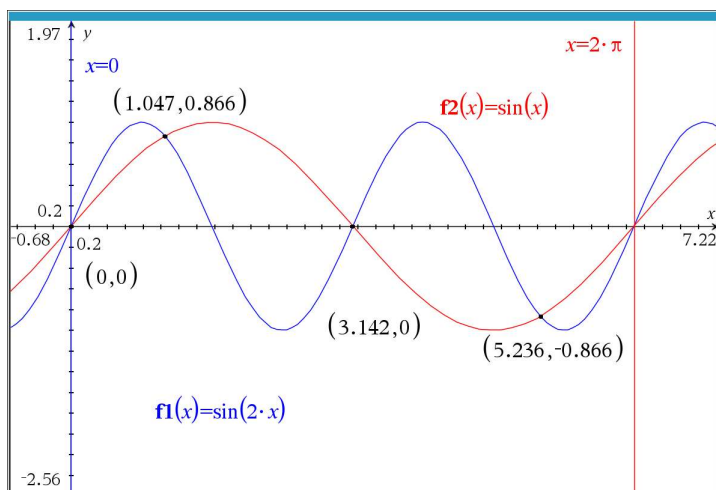
$$k=1 \quad \begin{array}{l} \frac{\pi}{9} + \frac{2\pi}{3} \\ = \frac{\pi}{9} + \frac{6\pi}{9} \\ = \frac{7\pi}{9} \end{array} \quad \begin{array}{l} \frac{5\pi}{9} + \frac{2\pi}{3} \\ = \frac{5\pi}{9} + \frac{6\pi}{9} \\ = \frac{11\pi}{9} \end{array}$$

$$k=-1: \frac{\pi}{9} - \frac{6\pi}{9} = -\frac{5\pi}{9} \text{ No} \quad \begin{array}{l} k=2 \\ \frac{\pi}{9} + \frac{4\pi}{3} = \frac{13\pi}{9} \\ \frac{5\pi}{9} + \frac{4\pi}{3} = \frac{17\pi}{9} \end{array}$$

$$2. \quad x = \frac{\pi}{9} + \frac{2\pi k}{3} \text{ or } x = \frac{5\pi}{9} + \frac{2\pi k}{3}; \quad x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$$

In Exercises 19 - 42, solve the equation, giving the exact solutions which lie in  $[0, 2\pi)$

$$20. \sin(2x) = \sin(x)$$



$$2 \sin x \cos x = \sin x$$

$$2 \sin x \cos x - \sin x = 0$$

$$(\sin x)(2 \cos x - 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad 2 \cos x - 1 = 0$$

$$\boxed{x = 0, \pi}$$

$$\cos x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

$$\pi/3 = 1.047197551196598$$

$$5\pi/3 = 5.235987755982988$$

11.1

supplied (already given previously)

### Properties of the Sinusoid $S(t) = A \sin(\omega t + \phi) + B$

- The **amplitude** is  $|A|$
- The **angular frequency** is  $\omega$  and the **ordinary frequency** is  $f = \frac{\omega}{2\pi}$
- The **period** is  $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- The **phase** is  $\phi$  and the **phase shift** is  $-\frac{\phi}{\omega}$
- The **vertical shift** or **baseline** is  $B$

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Supplied

**Theorem 11.1. Equation for Free Undamped Harmonic Motion:** Suppose an object of mass  $m$  is suspended from a spring with spring constant  $k$ . If the initial displacement from the equilibrium position is  $x_0$  and the initial velocity of the object is  $v_0$ , then the displacement  $x$  from the equilibrium position at time  $t$  is given by  $x(t) = A \sin(\omega t + \phi)$  where

- $\omega = \sqrt{\frac{k}{m}}$  and  $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$
- $A \sin(\phi) = x_0$  and  $A\omega \cos(\phi) = v_0$ .

6. Suppose an object weighing 10 pounds is suspended from the ceiling by a spring which stretches 2 feet to its equilibrium position when the object is attached.
- (a) Find the spring constant  $k$  in  $\frac{\text{lbs.}}{\text{ft.}}$  and the mass of the object in slugs.
  - (b) Find the equation of motion of the object if it is released from 1 foot *below* the equilibrium position from rest. When is the first time the object passes through the equilibrium position? In which direction is it heading?
  - (c) Find the equation of motion of the object if it is released from 6 inches *above* the equilibrium position with a *downward* velocity of 2 feet per second. Find when the object passes through the equilibrium position heading downwards for the third time.

(a)

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{k = m \omega^2}$$

$$\text{weight} = \text{force} = (\text{mass})(g)$$

Hooke's Law  $F = kx$   
force =  $k$  (displacement)

$$10 \text{ lb} = k(2 \text{ ft})$$

$$k = \frac{10 \text{ lb}}{2 \text{ ft}}$$

$$\boxed{k = \frac{5 \text{ lb}}{\text{ft}}}$$

$$\text{weight} = \text{force} = (mass)(g) \quad \boxed{F_t}$$

$$10 \text{ lb} = (m) 32 \frac{\text{ft}}{\text{sec}^2}$$

$$m = \frac{10 \text{ lb} \cdot \text{sec}^2}{32 \text{ ft}}$$

$$\frac{10}{32} = \frac{5}{16}$$

$$m = \left( \frac{5}{16} \right) \text{ slug}$$

Follow textbook example to finish this.

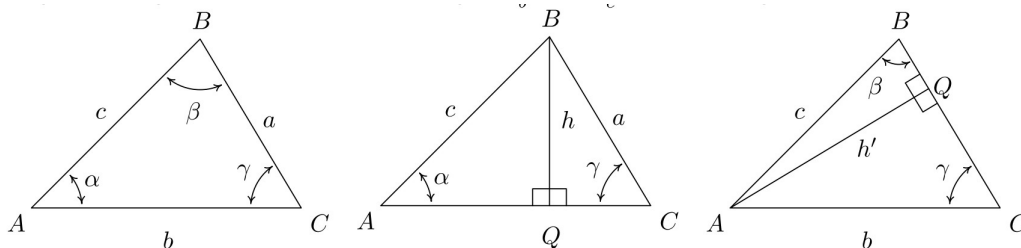
## 11.2 memorize

**Theorem 11.2. The Law of Sines:** Given a triangle with angle-side opposite pairs  $(\alpha, a)$ ,  $(\beta, b)$  and  $(\gamma, c)$ , the following ratios hold

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

or, equivalently,

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$



$$\sin(\alpha) = \frac{\text{opp}}{\text{hyp}} = \frac{h}{c} \quad \bigg| \quad \sin(\gamma) = \frac{\text{opp}}{\text{hyp}} = \frac{h}{a}$$

$\triangle ABQ \qquad \qquad \triangle BCQ$

$$h = c \sin(\alpha) = a \sin(\gamma)$$

$$\Rightarrow \frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c}$$

Do the same for the third triangle to complete the

theorem.

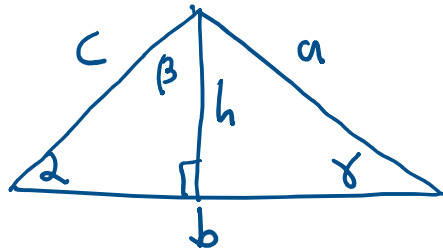
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**Theorem 11.3.** Suppose  $(\alpha, a)$  and  $(\gamma, c)$  are intended to be angle-side pairs in a triangle where  $\alpha$ ,  $a$  and  $c$  are given. Let  $h = c \sin(\alpha)$

- If  $a < h$ , then no triangle exists which satisfies the given criteria.
- If  $a = h$ , then  $\gamma = 90^\circ$  so exactly one (right) triangle exists which satisfies the criteria.
- If  $h < a < c$ , then two distinct triangles exist which satisfy the given criteria.
- If  $a \geq c$ , then  $\gamma$  is acute and exactly one triangle exists which satisfies the given criteria

**Theorem 11.4.** Suppose  $(\alpha, a)$ ,  $(\beta, b)$  and  $(\gamma, c)$  are the angle-side opposite pairs of a triangle. Then the area  $A$  enclosed by the triangle is given by

$$A = \frac{1}{2}bc \sin(\alpha) = \frac{1}{2}ac \sin(\beta) = \frac{1}{2}ab \sin(\gamma)$$



$$A = \left(\frac{1}{2}\right)(\text{base})(\text{height})$$

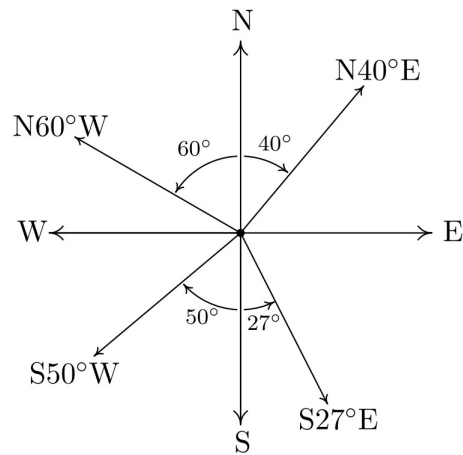
$$A = \left(\frac{1}{2}\right)(bh)$$

$$\sin(\alpha) = \frac{h}{c}$$

$$h = c \sin(\alpha)$$

$$A = \left(\frac{1}{2}\right)(bc \sin(\alpha))$$

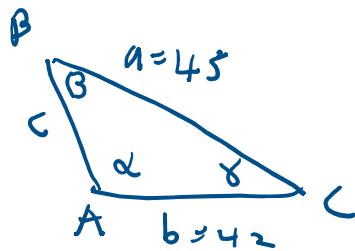
Compass bearings (supplied)



### 11.2.1 EXERCISES

In Exercises 1 - 20, solve for the remaining side(s) and angle(s) if possible. As in the text,  $(\alpha, a)$ ,  $(\beta, b)$  and  $(\gamma, c)$  are angle-side opposite pairs.

6.  $\alpha = 117^\circ$ ,  $a = 45$ ,  $b = 42$



$$\frac{\sin(\alpha)}{a} = \frac{\sin(117^\circ)}{45} = \frac{\sin(\beta)}{42} = \frac{\sin(\gamma)}{c}$$

$$\sin(\beta) = \frac{42 \sin(117^\circ)}{45} = 0.832$$

$$\beta = \arcsin(0.832)$$

Finish at home. Check Theorem 11.3 to see how many triangles we have here.