11-18-25 MTH 167-002N

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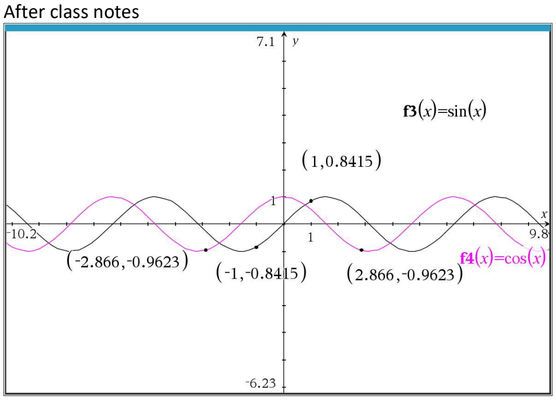
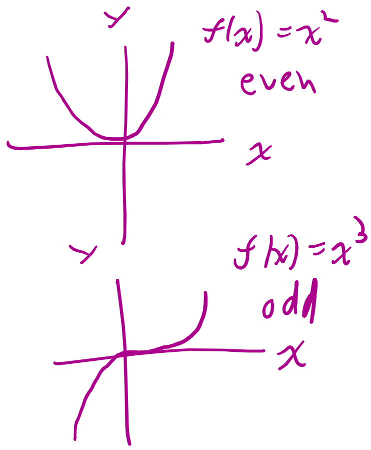
After class notes





Definition f is even if $f(-x) = f(x)$
f is odd if $f(-x) = -f(x)$

Cos (x) is even
Sin(x) is odd

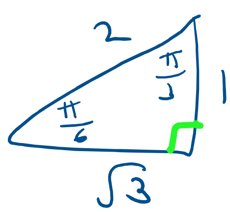
10.2: 38

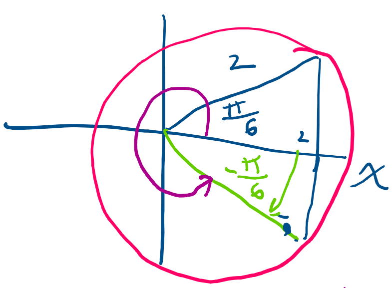
10.2 The Unit Circle: Cosine and Sine

In Exercises 31 - 39, find all of the angles which satisfy the given equation.

38. $\cos(\theta) = \frac{\sqrt{3}}{2}$

y 2π





$\theta = \frac{\pi}{6} + 2\pi k,$ $k \in \mathbb{Z}$
k is an integer

$\theta = -\frac{\pi}{6} + 2\pi k$

$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{11\pi}{6} + 2\pi k$

10.4: 14

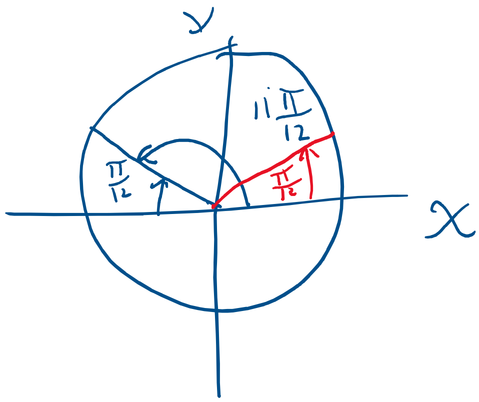
In Exercises 7 - 21, use the Sum and Difference Identities to find the exact value. You may have need of the Quotient, Reciprocal or Even / Odd Identities as well.

$\left(11\pi \right)$

In Exercises 7 - 21, use the Sum and Difference Identities to find the exact value. You may have need of the Quotient, Reciprocal or Even / Odd Identities as well.

14. $\sin\left(\frac{11\pi}{12}\right)$

$$\sin\left(\frac{11\pi}{12}\right) = \sin\frac{\pi}{12}$$



$$\frac{\pi}{3} \qquad \frac{\pi}{4} \qquad \frac{\pi}{6}$$

$$\underbrace{\frac{4\pi}{12} \qquad \frac{3\pi}{12}} \qquad \frac{2\pi}{12}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$\sin\left(\frac{11\cdot\pi}{12}\right) \qquad \frac{(\sqrt{3}-1)\cdot\sqrt{2}}{4}$$

10.3: 55

In Exercises 43 - 57, find all of the angles which satisfy the equation.

55. $\tan(\theta) = -\sqrt{3}$

In Exercises 43 - 57

55. tan(θ

Alright Donald, let's tackle this trig problem step by step.

We're solving:

$$\tan(\theta) = -\sqrt{3}$$

🔎 Step 1: Recall tangent values

- $\tan(\theta) = \sqrt{3}$ at **60° (π/3 radians)** in the first quadrant.
- Since tangent is **negative**, we're looking at quadrants **II and IV** (because tangent = sine/cosine, and signs differ there).

🔎 Step 2: Reference angle

The reference angle is $60°$ (or $\pi/3$).

🔎 Step 3: Apply quadrant rules

- In **Quadrant II**, tangent is negative → angle = $180° - 60° = 120°$ (or $2\pi/3$).
- In **Quadrant IV**, tangent is negative → angle = $360° - 60° = 300°$ (or $5\pi/3$).

🔎 Step 4: General solution

Tangent has a period of π. So the general solution is:

$$\theta = \frac{2\pi}{3} + n\pi \quad \text{or} \quad \theta = \frac{5\pi}{3} + n\pi, \quad n \in \mathbb{Z}$$

✅ Final Answer

- In degrees: $\theta = 120° + 180°n$ or $\theta = 300° + 180°n$.
- In radians: $\theta = \frac{2\pi}{3} + n\pi$ or $\theta = \frac{5\pi}{3} + n\pi$.

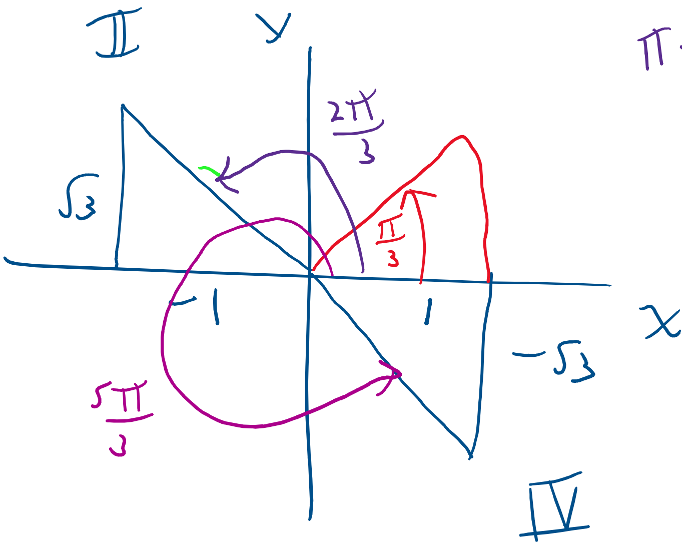
Since you like weaving humor into math, here's a quick mnemonic-style limerick for tangent signs:

Copy

```
In I it's plus, in II it's minus,
In III it's plus, in IV decline us.
So when tan's negative,
Quadrants II and IV give—
Angles where slopes don't align us.
```

π. y.

π – π – 3π – π – 2π



$$\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$$

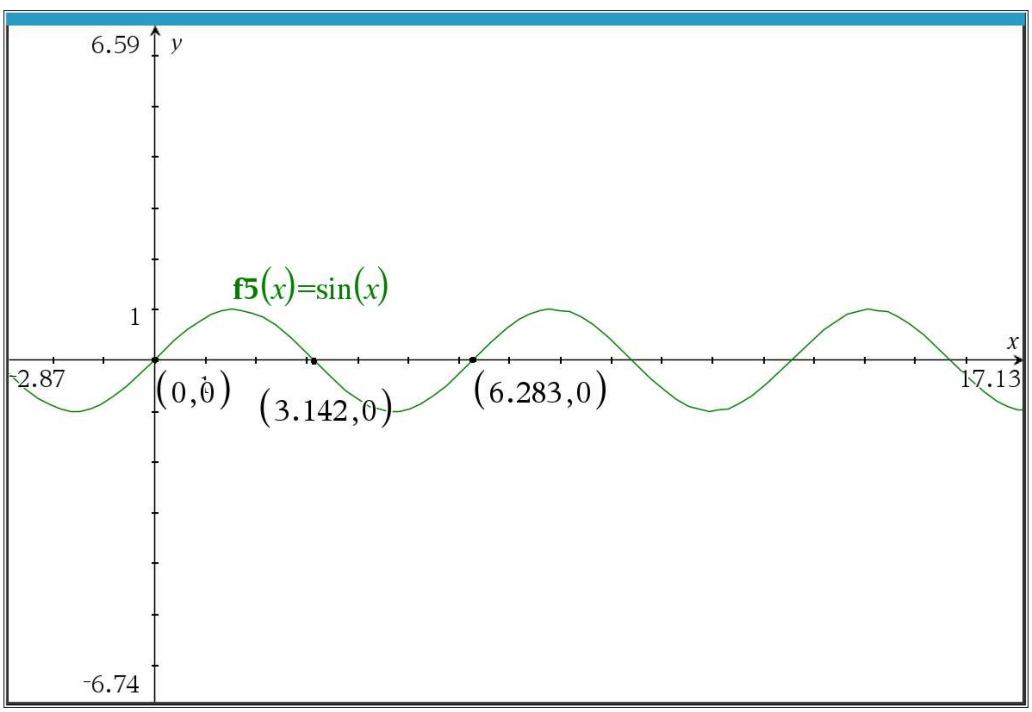
$$\boxed{\begin{array}{c} \theta = \frac{2\pi}{3} + \pi k \\ \text{or} \\ \theta = \frac{5\pi}{3} + \pi k \end{array}}$$

$$2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

10.5
Memorize

Definition 10.3. Periodic Functions: A function f is said to be **periodic** if there is a real number c so that $f(t+c) = f(t)$ for all real numbers t in the domain of f. The smallest positive number p for which $f(t+p) = f(t)$ for all real numbers t in the domain of f, if it exists, is called the **period** of f.

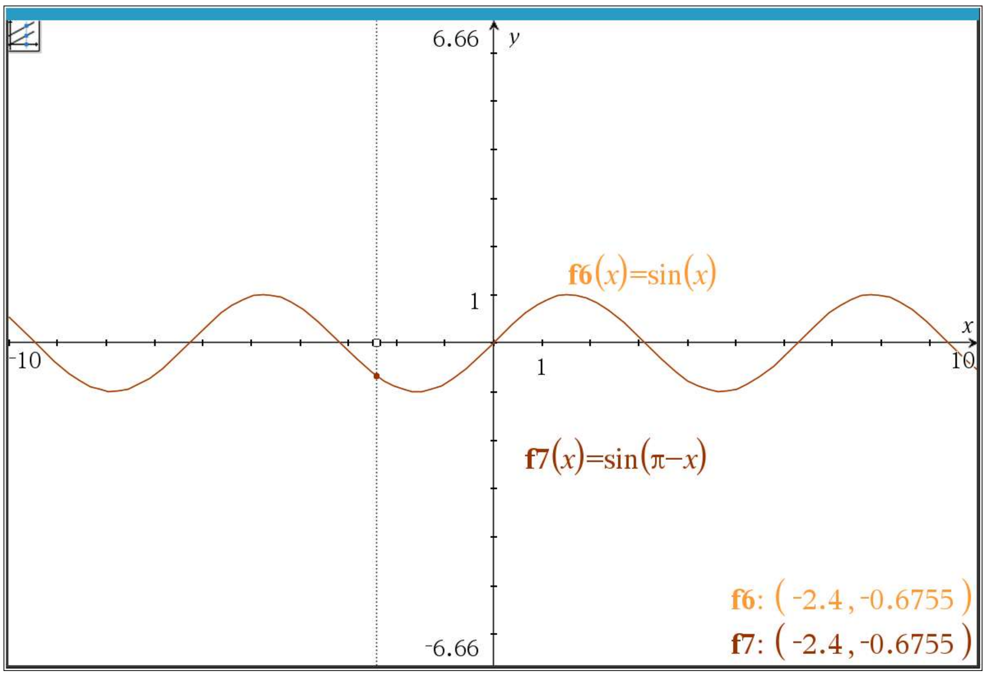
Sin(x) is periodic with period = 2π

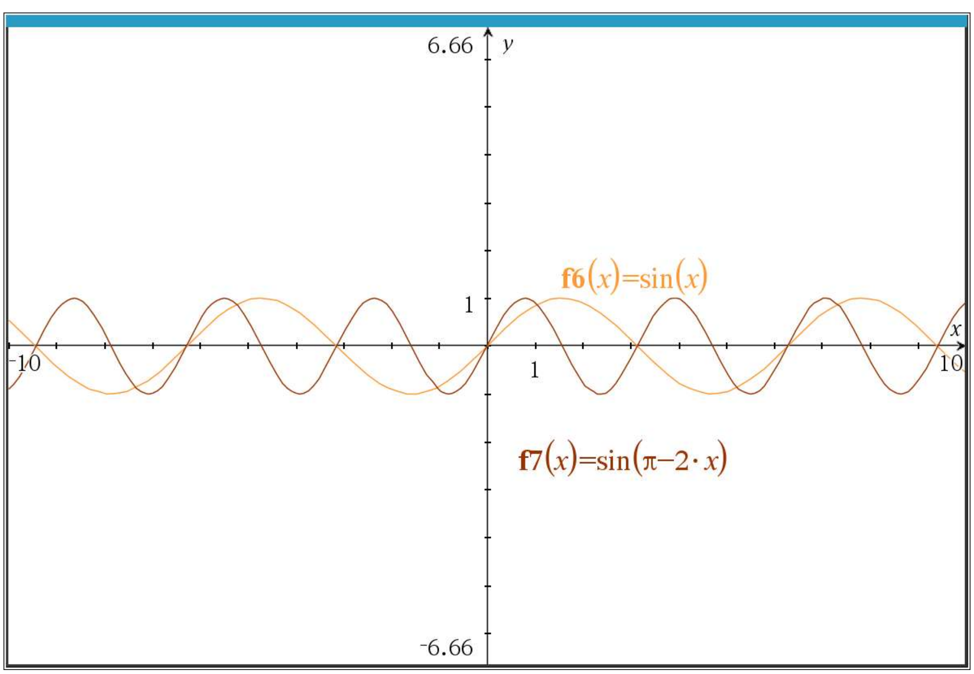


Memorize

Theorem 10.22. Properties of the Cosine and Sine Functions

- The function $f(x) = \cos(x)$
  - has domain $(-\infty, \infty)$
  - has range $[-1, 1]$
  - is continuous and smooth
  - is even
  - has period 2π
- The function $g(x) = \sin(x)$
  - has domain $(-\infty, \infty)$
  - has range $[-1, 1]$
  - is continuous and smooth
  - is odd
  - has period 2π





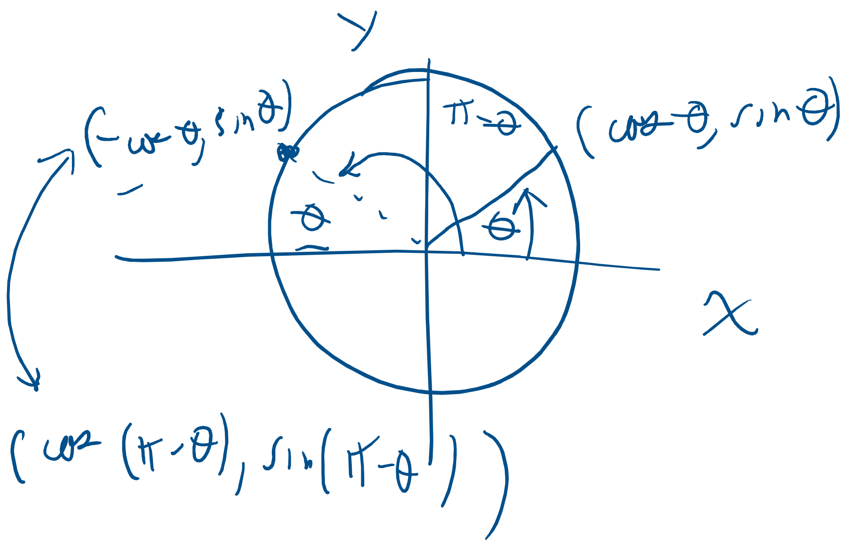
show $\sin(\pi - x) = \sin x$

$$= \sin\pi \cos x - \cos\pi \sin x$$

$$= (0)\cos x - (-1)(\sin x)$$

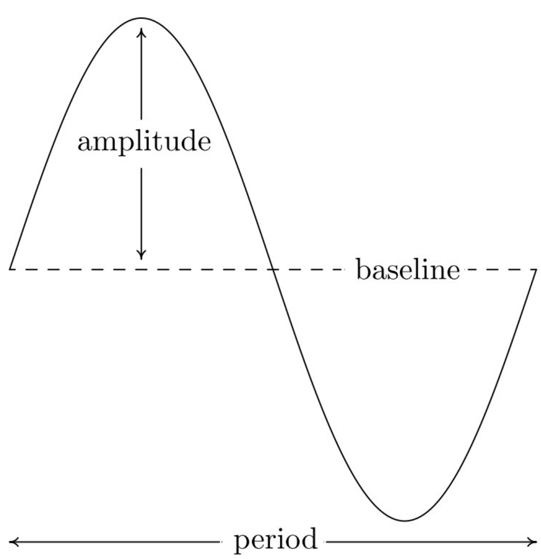
$$= 0 + \sin x$$

$$= \sin x$$



$$\therefore \cos(\pi - \theta) = -\cos\theta$$

Memorize



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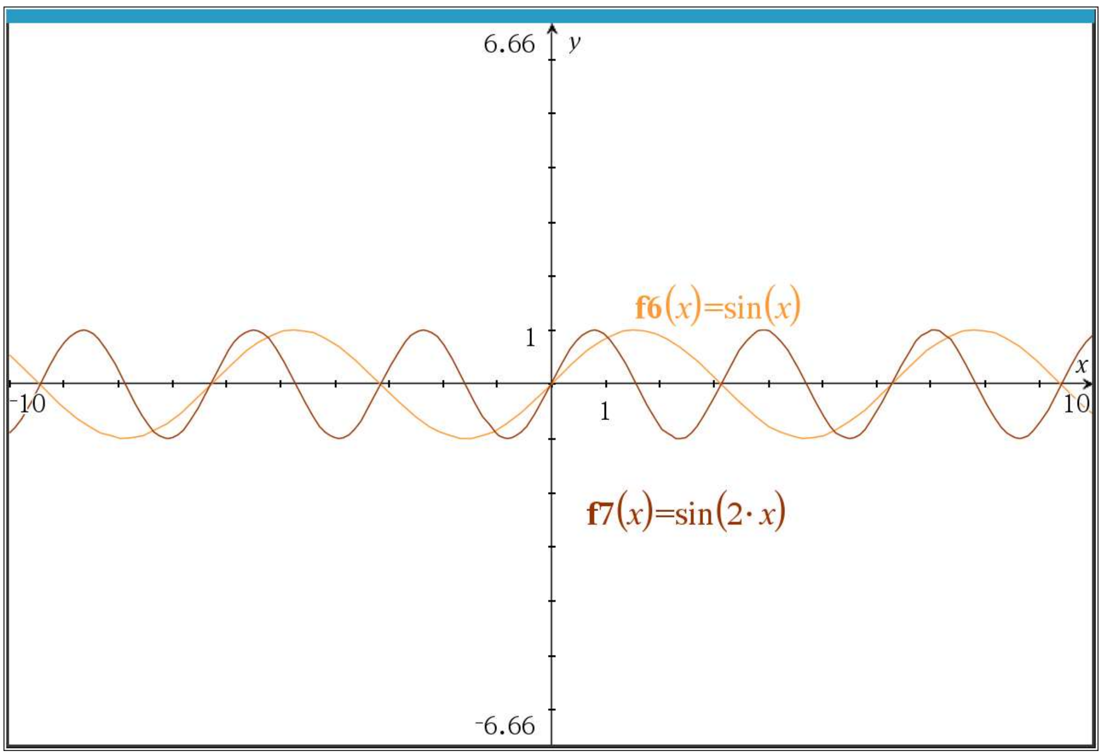
Theorem 10.23. For $\omega > 0$, the functions

$$C(x) = A\cos(\omega x + \phi) + B \quad \text{and} \quad S(x) = A\sin(\omega x + \phi) + B$$

- have period $\dfrac{2\pi}{\omega}$
- have phase shift $-\dfrac{\phi}{\omega}$
- have amplitude $|A|$
- have vertical shift B

$\omega =$ angular frequency $\quad \phi = phase$



Your Name MTH 167-002N bonus quiz 2 Write the instructions. No calculator.

1. Convert 45° into radians. $\boxed{\frac{\pi}{4}}$

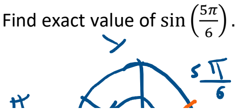
$\pi = 180°$

$\frac{\pi}{180} = 1°$

$\frac{45\pi}{180} = 45°$

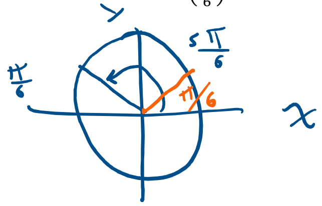
$\boxed{\frac{\pi}{4} = 45°}$

2. Find exact value of $\sin\left(\frac{5\pi}{6}\right)$. Make a relevant sketch of a unit circle.



$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

2. Find exact value of $\sin\left(\frac{5\pi}{6}\right)$. Make a relevant sketch of a unit circle.

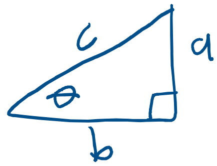


$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin\left(\frac{5\pi}{6}\right) = \boxed{\frac{1}{2}}$

3.



Find the 6 basic trig functions of θ.

$\sin\theta = \frac{opp}{hyp} = \frac{a}{c}$

$\cos\theta = \frac{adj}{hyp} = \frac{b}{c}$

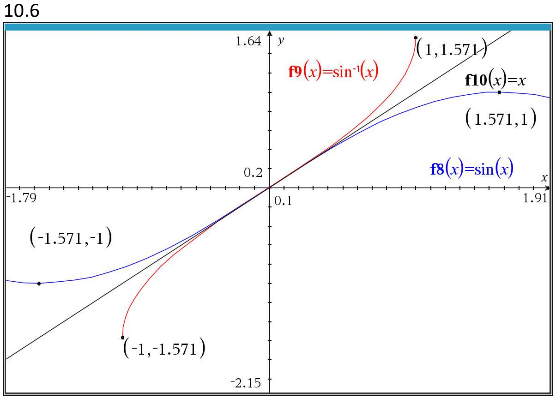
$\tan\theta = \frac{opp}{adj} = \frac{a}{b}$

$\cot\theta = \frac{adj}{opp} = \frac{b}{a}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{c}{b}$

$\csc = \frac{1}{\sin\theta} = \frac{c}{a}$

10.6



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Theorem 10.26. Properties of the Arccosine and Arcsine Functions

- Properties of $F(x) = \arccos(x)$
  - Domain: $[-1, 1]$
  - ✓ Range: $[0, \pi]$ supplied
  - $\arccos(x) = t$ if and only if $0 \leq t \leq \pi$ and $\cos(t) = x$
  - $\cos(\arccos(x)) = x$ provided $-1 \leq x \leq 1$
  - $\arccos(\cos(x)) = x$ provided $0 \leq x \leq \pi$
- Properties of $G(x) = \arcsin(x)$
  - Domain: $[-1, 1]$
  - ✓ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ supplied
  - $\arcsin(x) = t$ if and only if $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ and $\sin(t) = x$
  - $\sin(\arcsin(x)) = x$ provided $-1 \leq x \leq 1$
  - $\arcsin(\sin(x)) = x$ provided $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
  - additionally, arcsine is odd

10.6

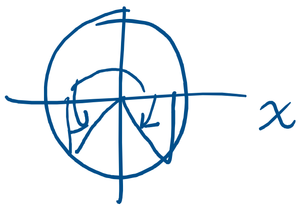
In Exercises 57 - 86, find the exact value or state that it is undefined.

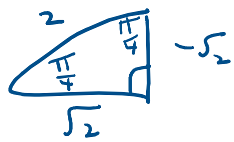
59. $\sin\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{3}{5}$

58. $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

Let $\theta = \arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$\Rightarrow \sin\theta = -\frac{\sqrt{2}}{2}$





$\theta = -\frac{\pi}{4}$ or $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

61. $\sin\left(\arcsin\left(\frac{5}{4}\right)\right)$ is undefined.

Let $\theta = \arcsin\left(\frac{5}{4}\right)$

$\sin\theta = \frac{5}{4} \notin [-1, 1] = \text{range of } \sin\theta$

66. $\cos(\arccos(\pi))$ is undefined.

Let $\theta = \arccos \pi$

$\Rightarrow \cos\theta = \pi \approx 3.14 \notin \text{range of } \cos\theta$