11-11-25 MTH 167-002N

7.3 Parabolas

7.3.1 Exercises

page 512 (524): 1, 4, 11, 19

7.4 Ellipses

7.4.1 Exercises

page 525 (537): 2, 6, 9, 16

7.5 Hyperbolas

7.5.1 Exercises

page 541(553): 3, 10, 14

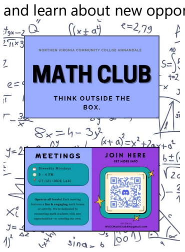
10 Foundations of Trigonometry

10.1 Angles and their Measure

10.1.2 Exercises

page 709 (721): 9, 15, 17, 30, 35, 39, 41, 50

Math Club: Our club is open to all students, regardless of their current math level. We provide a fun and engaging space to explore math topics, connect with peers, and learn about new opportunities. We meet biweekly on Mondays from 3:00 PM to 4:00 PM in CT-121 (MDE Lab).

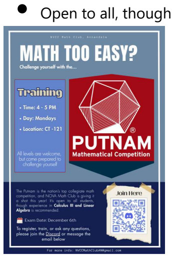


2nd Annual Integration Bee: This is a fun and challenging competition with over **$300** in prizes. It's a great event for students to test their calculus skills and even challenge their professors! We'll have snacks, and all are welcome to attend, even just to watch.

- Date: Friday, November 21st
- Time: 3:00 PM
- Location: CA - 302 (Annandale Campus)
- Calculus II experience is recommended for competitors.

Putnam Mathematical Competition: We are actively recruiting and training students for the Putnam, the nation's top collegiate math competition. This is a fantastic opportunity for students seeking a serious challenge.

- Exam Date: Saturday, December 6th
- Training Sessions: Mondays from 4:00 PM to 5:00 PM in CT-121
- Open to all, though experience in Calculus III and Linear Algebra is recommended.



We would be very grateful if you could forward this email, along with the attached flyers, to the Annandale math faculty. Their support is invaluable for getting students involved.

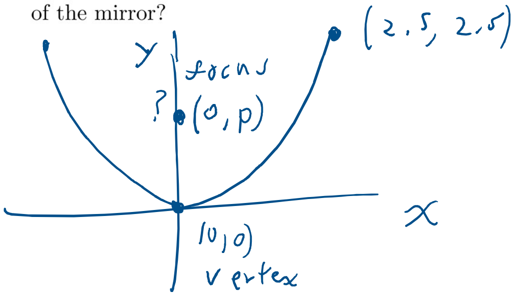
Thank you for your time and for supporting student engagement in mathematics.

Best regards,

Emiliano Mercado
NVCCMathClubAN@gmail.com

7.3: 19

19. The mirror in Carl's flashlight is a paraboloid of revolution. If the mirror is 5 centimeters in diameter and 2.5 centimeters deep, where should the light bulb be placed so it is at the focus of the mirror?



Equation 7.2. The Standard Equation of a Vertical[a] Parabola: The equation of a (vertical) parabola with vertex (h, k) and focal length $|p|$ is

$$(x - h)^2 = 4p(y - k)$$

If $p > 0$, the parabola opens upwards; if $p < 0$, it opens downwards.

[a] That is, a parabola which opens either upwards or downwards.

Find p

$h = 0,\ k = 0$

$$\boxed{x^2 = 4py}$$

$$p = \frac{x^2}{4y}$$

$$p = \frac{(2.5)^2}{4(2.5)} = \frac{2.5}{4}$$

$$\boxed{p = 0.625}$$

The light bulb should be placed 0.625 cm above the vertex of the parabolic mirror.

7.4: 2

7.4.1 Exercises

In Exercises 1 - 8, graph the ellipse. Find the center, the lines which contain the major and minor axes, the vertices, the endpoints of the minor axis, the foci and the eccentricity.

2. $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

Equation 7.4. The Standard Equation of an Ellipse: For positive unequal numbers a and b, the equation of an ellipse with center (h, k) is

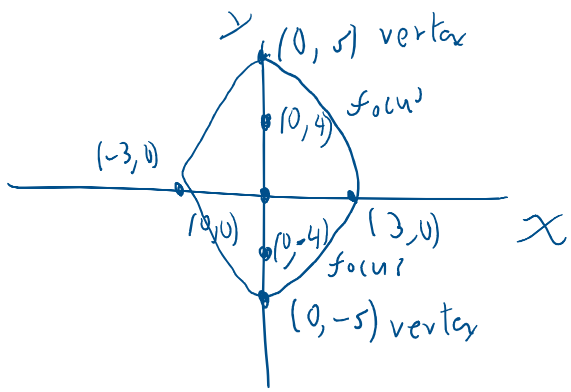
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

the focus, c, can be found by $c = \sqrt{a^2 - b^2}$. If a > b

$c = \sqrt{b^2 - a^2}$. If b >a $\Rightarrow c = \sqrt{25-9} = \sqrt{16} = 4$

Definition 7.5. The **eccentricity** of an ellipse, denoted e, is the following ratio:

$$e = \frac{\text{distance from the center to a focus}}{\text{distance from the center to a vertex}} = \frac{c}{b}$$



$(h, k) = (0, 0)$ center

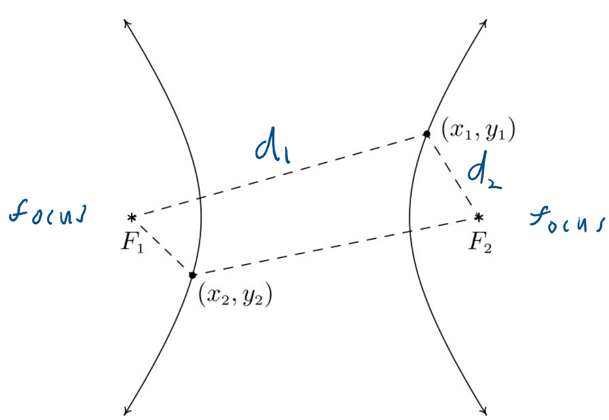
$a^2 = 9 \Rightarrow a = 3$

$b^2 = 25 \Rightarrow b = 5$

$$\boxed{e = \frac{c}{b} = \frac{4}{5}}$$

7.5 supplied locus definition

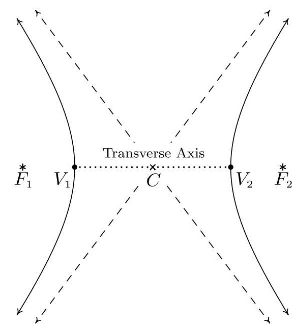
Definition 7.6. Given two distinct points F_1 and F_2 in the plane and a fixed distance d, a **hyperbola** is the set of all points (x, y) in the plane such that the absolute value of the difference of each of the distances from F_1 and F_2 to (x, y) is d. The points F_1 and F_2 are called the **foci** of the hyperbola.



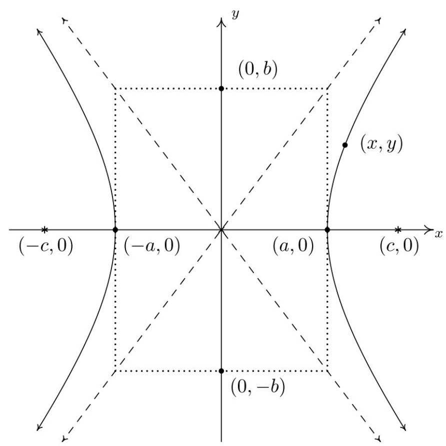
$|d_1 - d_2| = d$

$d =$ constant

Supplied



A hyperbola with center C; foci F_1, F_2; and vertices V_1, V_2 and asymptotes (dashed)



Supplied

Equation 7.6. The Standard Equation of a Horizontal[a] Hyperbola For positive numbers a and b, the equation of a horizontal hyperbola with center (h, k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

[a]That is, a hyperbola whose branches open to the left and right

Supplied

Equation 7.7. The Standard Equation of a Vertical Hyperbola For positive numbers a and b, the equation of a vertical hyperbola with center (h, k) is:

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Supplied

Strategies for Identifying Conic Sections

Suppose the graph of equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is a non-degenerate conic section.[a]

- If just *one* variable is squared, the graph is a parabola. Put the equation in the form of Equation 7.2 (if x is squared) or Equation 7.3 (if y is squared).

If *both* variables are squared, look at the coefficients of x^2 and y^2, A and $\cancel{B}$ C.

- If $A = \cancel{B}$ C, the graph is a circle. Put the equation in the form of Equation 7.1.
- If $A \neq \cancel{B}$ C but A and $\cancel{B}$ C have the *same sign*, the graph is an ellipse. Put the equation in the form of Equation 7.4.
- If A and $\cancel{B}$ C have the *different signs*, the graph is a hyperbola. Put the equation in the form of either Equation 7.6 or Equation 7.7.

[a]That is, a parabola, circle, ellipse, or hyperbola – see Section 7.1.

of either Equation 7.6 or Equation 7.7.

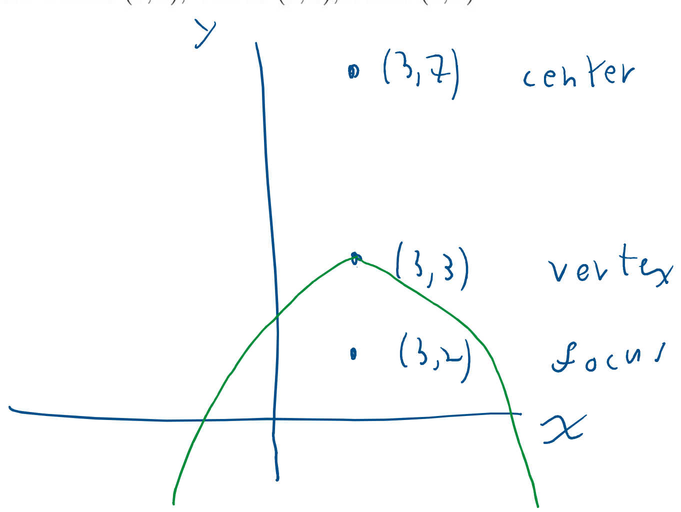
[a]That is, a parabola, circle, ellipse, or hyperbola – see Section 7.1.

A more general equation is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Bxy rotates the conic section

In Exercises 13 - 18, find the standard form of the equation of the hyperbola which has the given properties.

13. Center $(3, 7)$, Vertex $(3, 3)$, Focus $(3, 2)$



Equation 7.7. The Standard Equation of a Vertical Hyperbola For positive numbers a and b, the equation of a vertical hyperbola with center (h, k) is:

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$(h,k) = (3,7)$

$$\frac{(y-7)^2}{b^2} - \frac{(x-3)^2}{a^2} = 1$$

$b^2 = c^2 - a^2$

b = distance from center to vertex

$b = 7 - 3 = \boxed{4 = b}$

c = distance from center to focus $= 7 - 2 = \boxed{5 = c}$

$a = \sqrt{c^2 - b^2} = \sqrt{25 - 16} = \sqrt{9} = \boxed{3 = a}$

$= \sqrt{25-16}$

$$\boxed{\frac{(y-7)^2}{4^2} - \frac{(x-3)^2}{3^2} = 1}$$

13. $\dfrac{(y-7)^2}{16} - \dfrac{(x-3)^2}{9} = 1$

9.5

In Exercises 9 - 12, put the equation in standard form. Find the center, the lines which contain the transverse and conjugate axes, the vertices, the foci and the equations of the asymptotes.

11. $9x^2 - 25y^2 - 54x - 50y - 169 = 0$

$$(9x^2 - 54x) + (-25y^2 - 50y) = 169$$

$$9(x^2 - 6x) - 25(y^2 + 2y) = 169$$

$$9(x^2 - 6x + 9 - 9) - 25(y^2 + 2y + 1 - 1) = 169$$

$$9(x^2 - 6x + 9) - 81 - 25(y^2 + 2y + 1) + 25 = 169$$

$$9(x-3)^2 - 25(y+1)^2 = 169 + 81 - 25$$

$$9(x-3)^2 - 25(y+1)^2 = 225$$

$$\frac{9(x-3)^2}{225} - \frac{25(y+1)^2}{225} = 1$$

$$\frac{(x-3)^2}{\frac{225}{9}} - \frac{(y+1)^2}{\frac{225}{25}} = 1$$

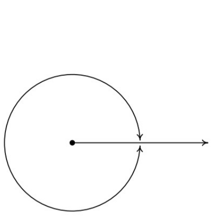
$$\frac{(x-3)^2}{\left(\frac{15}{3}\right)^2} - \frac{(y+1)^2}{\left(\frac{15}{5}\right)^2} = 1$$

$$\boxed{\frac{(x-3)^2}{5^2} - \frac{(y+1)^2}{3^2} = 1}$$

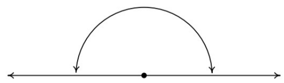
$$\begin{array}{r} 169 \\ +81 \\ \hline 250 \\ -25 \\ \hline 225 \end{array}$$

$$\frac{15^2}{225}$$

10.1

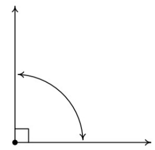


One revolution $\leftrightarrow 360^\circ$



180°

straight angle



90°

right angle

memorize

DD = decimal degree

DMS = degree - minute - second

$$1^\circ = 60'$$

$$1' = 60''$$

$$1' = \frac{1}{60}^\circ$$

$$1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ$$

Memorize

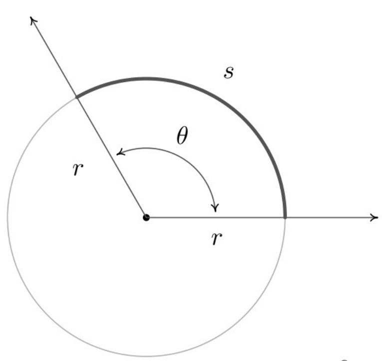
Definition 10.1. The real number π is defined to be the ratio of a circle's circumference to its diameter. In symbols, given a circle of circumference C and diameter d,

$$\pi = \frac{C}{d}$$

d = diameter = (2)(radius) = $2r$

$$\pi = \frac{C}{2r} \Rightarrow \boxed{C = 2\pi r}$$

Memorize



The radian measure of θ is $\frac{s}{r}$.

θ = central angle

s = arc length

r = radius

$$\theta = \frac{s}{r}$$

$$\Leftrightarrow \boxed{s = r\theta}$$

Let $s = C$

$C = r\theta$

$2\pi r = r\theta$

$\theta = 2\pi = 360^\circ$

$\boxed{\pi \text{ radians} = 180^\circ}$ = straight angle

$\boxed{\frac{\pi}{2} = 90^\circ = \text{right angle}}$

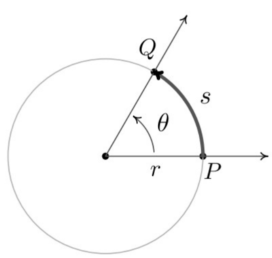
Memorize

Equation 10.1. Degree - Radian Conversion:

- To convert degree measure to radian measure, multiply by $\dfrac{\pi \text{ radians}}{180^\circ}$
- To convert radian measure to degree measure, multiply by $\dfrac{180^\circ}{\pi \text{ radians}}$

Memorize

Equation 10.2. Velocity for Circular Motion: For an object moving on a circular path of radius r with constant angular velocity ω, the (linear) velocity of the object is given by $v = r\omega$.



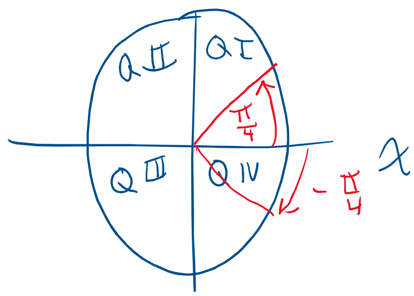
$\omega = \dfrac{\Delta\theta}{\Delta t}$ = angular velocity

$\dfrac{\Delta s}{\Delta t}$ = linear velocity = V

$\Delta s = r\Delta\theta$

$\Rightarrow \dfrac{\Delta s}{\Delta t} = \dfrac{r\Delta\theta}{\Delta t} = \boxed{r\omega = V}$

(90°)▸DD	90°
(90°)▸Rad	$\left(\frac{\pi}{2}\right)^r$
$\frac{\pi}{2}$▸DD	90°


Q II
Q I
Q III
Q IV
π/4
- π/4
x