11-04-25 MTH 167-002N

6.3 Exponential Equations and Inequalities
6.3.1 Exercises
page 456 (468): 1, 11, 19, 36, 40, 44
6.4 Logarithmic Equations and Inequalities
6.4.1 Exercises
page 466 (488): 3, 9, 22, 26, 33

6.5 Applications of Exponential and Logarithmic Functions
6.5.3 Exercises
page 482 (494): 1, 14, 23, 27
7 Hooked on Conics
7.1 Introduction to Conics
7.2 Circles
7.2.1 Exercises
page 502 (514): 1, 8, 13

6.3: 11

6.3.1 EXERCISES

In Exercises 1 - 33, solve the equation analytically.

11. $5^x = -2$

$\ln(5^x) = \underbrace{\ln(-2)}_{\text{not defined}}$

$5^x > 0$ all x

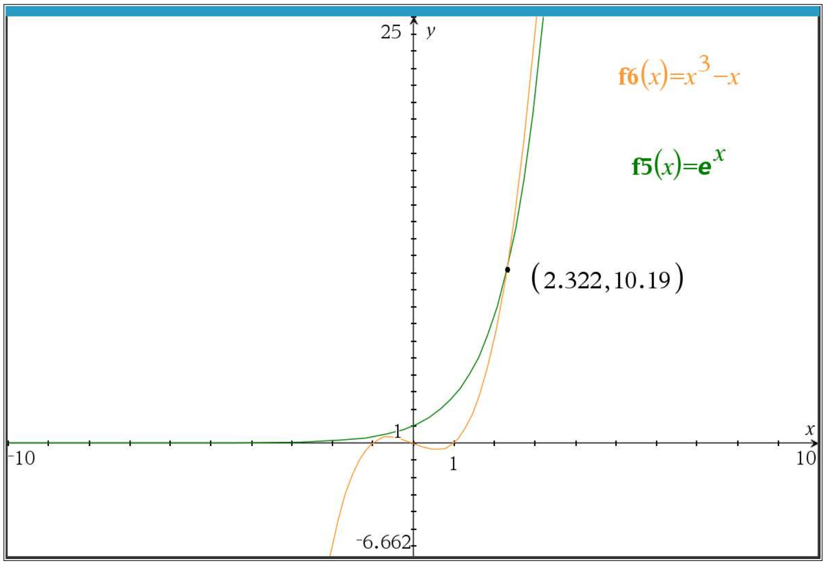
$\therefore$ No solution

6.3: 45

In Exercises 40 - 45, use your calculator to help you solve the equation or inequality.

45. $e^x < x^3 - x$

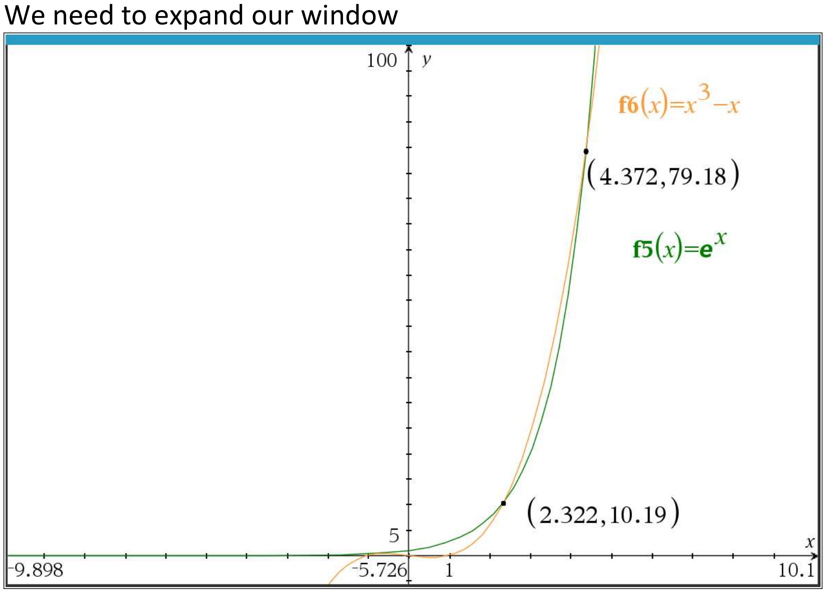
From graph solution set $\approx \{x | x > 2.322\}$

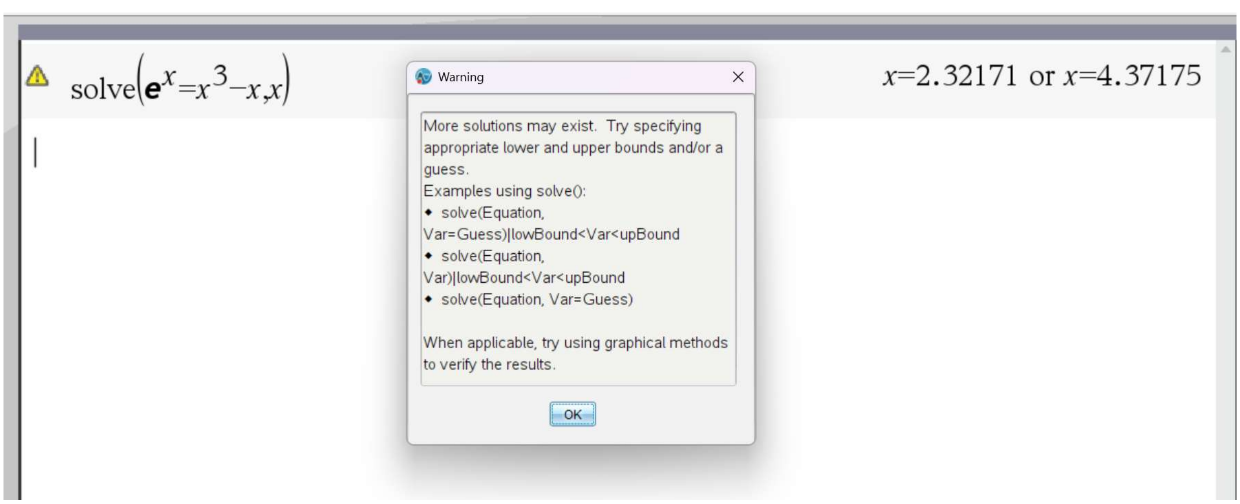


Textbook answer

45. $\approx (2.3217, 4.3717)$ ← interval, not point

We need to expand our window





6.4: 26

In Exercises 25 - 30, solve the inequality analytically.

26. $x\ln(x) - x > 0$

$x(\ln(x) - 1) > 0$

solve $x(\ln(x) - 1) = 0$

$\boxed{x = 0}$ or $\ln(x) - 1 = 0$

$\ln(x) = 1$

$\boxed{x = e}$

not defined $\quad$ $-$ $\quad$ $+$

$-1 \quad 0 \quad e \quad e^2$ ← test value

test value

Let $f(x) = x(\ln x - 1)$

$f(-1) = (-1)(\ln(-1) - 1)$

not defined

$f(1) = (1)(\ln 1 - 1)$

$f(1) = 0 - 1 = -1 < 0$

$f(e^2) = e^2(\ln e^2 - 1)$

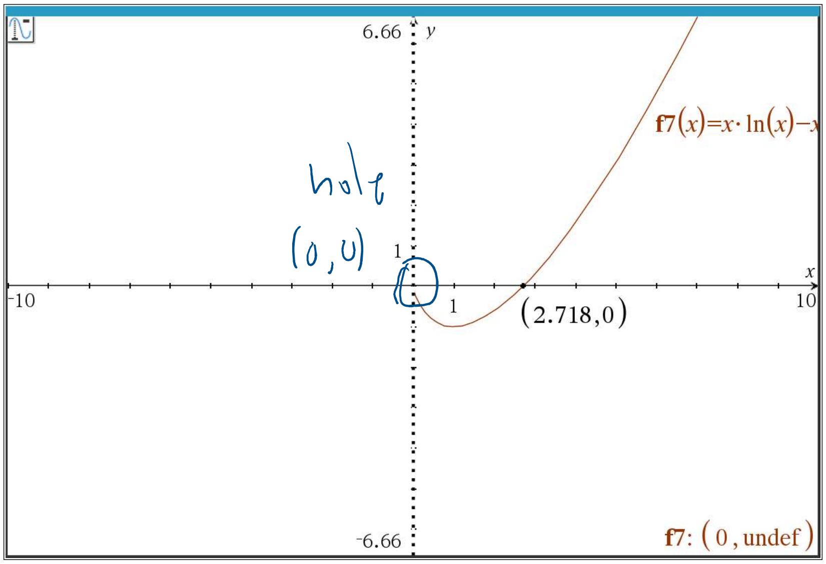
$= e^2(2 - 1)$

$= e^2 > 0$

$f(x) > 0$ on (e, ∞)

$f(x) > 0$ on (e, ∞)

$= \{x \mid x > e\}$



hole $(0,0)$

$x \approx 2.718 \approx e$

6.4: 33

In Exercises 31 - 34, use your calculator to help you solve the equation or inequality.

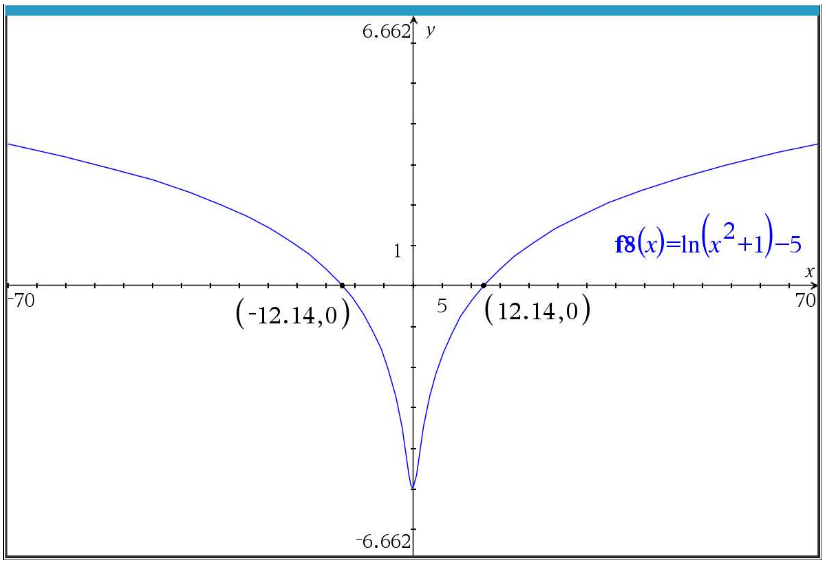
33. $\ln(x^2 + 1) \geq 5$

$x^2 + 1 \geq e^5$

$x^2 \geq e^5 - 1$

$\sqrt{x^2} \geq \sqrt{e^5 - 1}$

$|x| \geq \sqrt{e^5 - 1}$ $\quad (-\infty, -\sqrt{e^5 - 1}) \cup (\sqrt{e^5 - 1}, \infty)$



$\left(\sqrt{e^5-1}\right)$ ▸Decimal	12.1414

6.4: 37

37. In Example 6.4.4 we found the inverse of $f(x) = \dfrac{\log(x)}{1-\log(x)}$ to be $f^{-1}(x) = 10^{\frac{x}{x+1}}$.

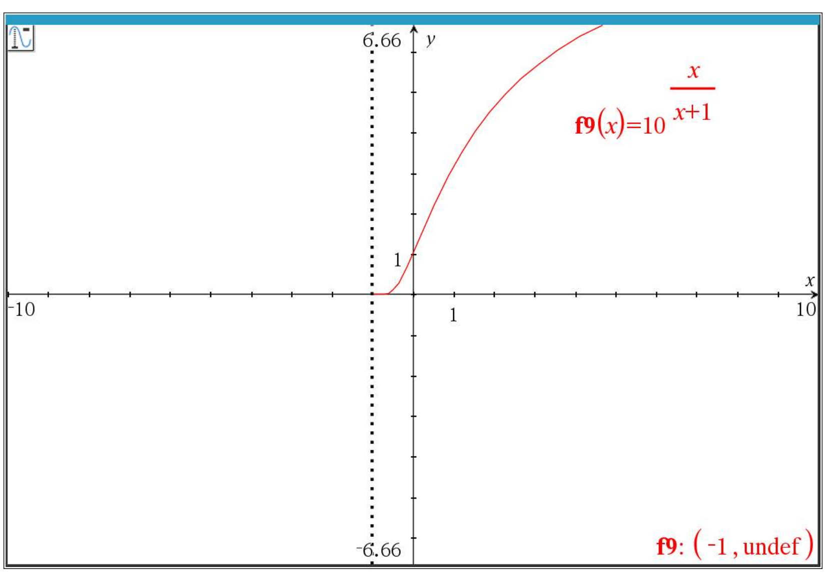
(a) Show that $\left(f^{-1} \circ f\right)(x) = x$ for all x in the domain of f and that $\left(f \circ f^{-1}\right)(x) = x$ for all x in the domain of f^{-1}.

(b) Find the range of f by finding the domain of f^{-1}.

(c) Let $g(x) = \dfrac{x}{1-x}$ and $h(x) = \log(x)$. Show that $f = g \circ h$ and $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$. (We know this is true in general by Exercise 31 in Section 5.2, but it's nice to see a specific example of the property.)

(b) range of f = domain of f^{-1}

$f^{-1}(x) = 10^{\frac{x}{x+1}}$ not defined for $x = -1$



domain of $f^{-1} = \{x \mid x \neq -1\}$

domain of $f^{-1} = \{x \mid x \neq -1\}$

6.5

Supplied

Equation 6.1. Simple Interest The amount of interest I accrued at an annual rate r on an investment[a] P after t years is

$$I = Prt$$

The amount A in the account after t years is given by

$$A = P + I = P + Prt = P(1 + rt)$$

[a]Called the **principal**

Supplied

Equation 6.2. Compounded Interest: If an initial principal P is invested at an annual rate r and the interest is compounded n times per year, the amount A in the account after t years is

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

Supplied

Equation 6.3. Continuously Compounded Interest: If an initial principal P is invested at an annual rate r and the interest is compounded continuously, the amount A in the account after t years is

$$A(t) = Pe^{rt}$$

$$A(0) = Pe^{(r)(0)}$$
$$= Pe^{0}$$
$$= (P)(1)$$
$$= P$$

$\therefore A(t) = A_0 e^{rt}$

Supplied

Equation 6.4. Uninhibited Growth: If a population increases according to The Law of Uninhibited Growth, the number of organisms N at time t is given by the formula

$$N(t) = N_0 e^{kt},$$

where $N(0) = N_0$ (read 'N nought') is the initial number of organisms and $k > 0$ is the constant of proportionality which satisfies the equation

$$(\text{instantaneous rate of change of } N(t) \text{ at time } t) = k\,N(t)$$

Supplied

Equation 6.5. Radioactive Decay The amount of a radioactive element A at time t is given by the formula

$$A(t) = A_0 e^{kt},$$

where $A(0) = A_0$ is the initial amount of the element and $k < 0$ is the constant of proportionality which satisfies the equation

$$(\text{instantaneous rate of change of } A(t) \text{ at time } t) = k\,A(t)$$

Supplied

Equation 6.6. Newton's Law of Cooling (Warming): The temperature T of an object at time t is given by the formula

$$T(t) = T_a + (T_0 - T_a)\, e^{-kt},$$

where $T(0) = T_0$ is the initial temperature of the object, T_a is the ambient temperature[a] and $k > 0$ is the constant of proportionality which satisfies the equation

$$\text{(instantaneous rate of change of } T(t) \text{ at time } t) = k\,(T(t) - T_a)$$

[a]That is, the temperature of the surroundings.

Supplied

Equation 6.7. Logistic Growth: If a population behaves according to the assumptions of logistic growth, the number of organisms N at time t is given by the equation

$$N(t) = \frac{L}{1 + Ce^{-kLt}},$$

where $N(0) = N_0$ is the initial population, L is the limiting population,[a] C is a measure of how much room there is to grow given by

$$C = \frac{L}{N_0} - 1.$$

and $k > 0$ is the constant of proportionality which satisfies the equation

$$\text{(instantaneous rate of change of } N(t) \text{ at time } t) = k\,N(t)\,(L - N(t))$$

[a]That is, as $t \to \infty$, $N(t) \to L$

6.5

In Exercises 14 - 18, we list some radioactive isotopes and their associated half-lives. Assume that each decays according to the formula $A(t) = A_0 e^{kt}$ where A_0 is the initial amount of the material and k is the decay constant. For each isotope:

- Find the decay constant k. Round your answer to four decimal places.
- Find a function which gives the amount of isotope A which remains after time t. (Keep the units of A and t the same as the given data.)
- Determine how long it takes for 90% of the material to decay. Round your answer to two decimal places. (HINT: If 90% of the material decays, how much is left?)

15. Phosphorus 32, used in agriculture, initial amount 2 milligrams, half-life 14 days.

$A(t) = A_0 e^{kt}$

$A_0 = 2$

$A(t) = 2e^{kt}$

$A(14) = \boxed{2e^{14k} = 1}$

$e^{14k} = \frac{1}{2}$

$\ln(e^{14k}) = \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2)$

$= 0 - \ln(2)$

$= -\ln(2)$

$14k = -\ln(2)$

$k = -\frac{\ln(2)}{14} \approx \boxed{-0.04951 \approx k}$

$\frac{-\ln(2)}{14}$ ▸Decimal -0.049511

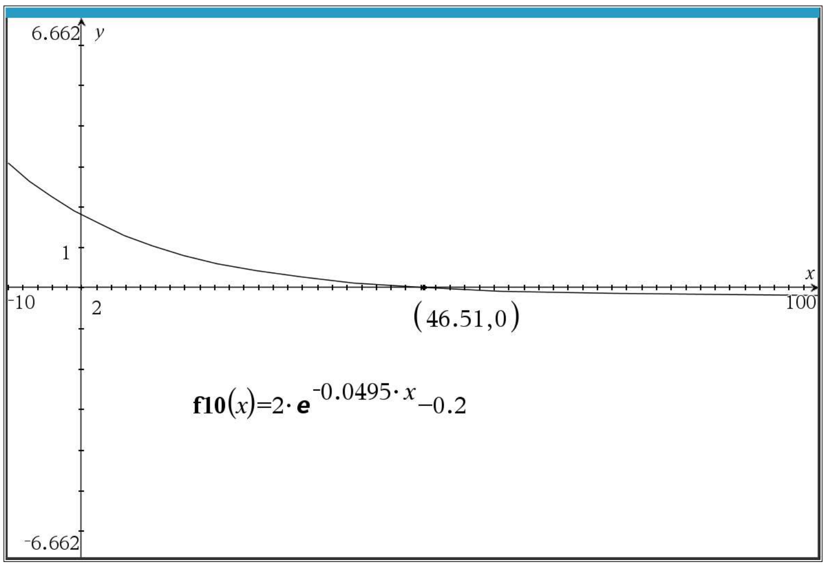
$\boxed{A(t) = 2e^{-0.04951t}}$

$$\boxed{A(t) = 2e^{-0.04951 t}}$$

Find t such that $A(t) = (10\%)(2\text{ mg})$
$= 0.2\text{ mg}$

solve $2e^{-0.04951 t} = 0.2$

$e^{-0.04951 t} = 0.1$

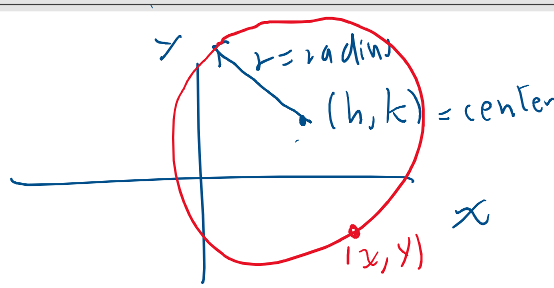


90% of the Phosphorus 32 will decay in about 47 days.

- $k = \frac{\ln(1/2)}{14} \approx -0.0495$
- $A(t) = 2e^{-0.0495t}$
- $t = \frac{\ln(0.1)}{-0.0495} \approx 46.52$ days.

7.1 discussed briefly
7.2
memorize

Definition 7.1. A **circle** with center (h, k) and radius $r > 0$ is the set of all points (x, y) in the plane whose distance to (h, k) is r.



$\text{dist}((h,k), (x,y)) = r$

$\text{dist}((h,k),(x,y)) = r$

$$\sqrt{(x-h)^2+(y-k)^2} = r$$

$$\boxed{(x-h)^2+(y-k)^2 = r^2}$$

equation of a circle
with center (h,k)
and radius r

Memorize

Definition 7.2. The **Unit Circle** is the circle centered at $(0,0)$ with a radius of 1. The standard equation of the Unit Circle is $x^2+y^2=1$.