10-30-25 MTH 167-002N

6 Exponential and Logarithmic Functions
6.1 Introduction to Exponential and Logarithmic Functions
6.1.1 Exercises
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6.2 Properties of Logarithms
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6.3 Exponential Equations and Inequalities
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6.4 Logarithmic Equations and Inequalities
6.4.1 Exercises
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6.1: 71

In Exercises 71 - 74, find the inverse of the function from the 'procedural perspective' discussed in Example 6.1.5 and graph the function and its inverse on the same set of axes.

71. $f(x) = 3^{x+2} - 4$

$y = 3^{x+2} - 4$

$x = 3^{y+2} - 4$

$3^{y+2} = x + 4$

$\log_3\left(3^{y+2}\right) = \log_3(x+4)$

$y + 2 = \log_3(x+4)$

$y = -2 + \log_3(x+4)$

$\boxed{f^{-1}(x) = -2 + \log_3(x+4)}$

Procedural perspective
begin with x
1) add 2
2) put into exponent of 3
3) subtract 4

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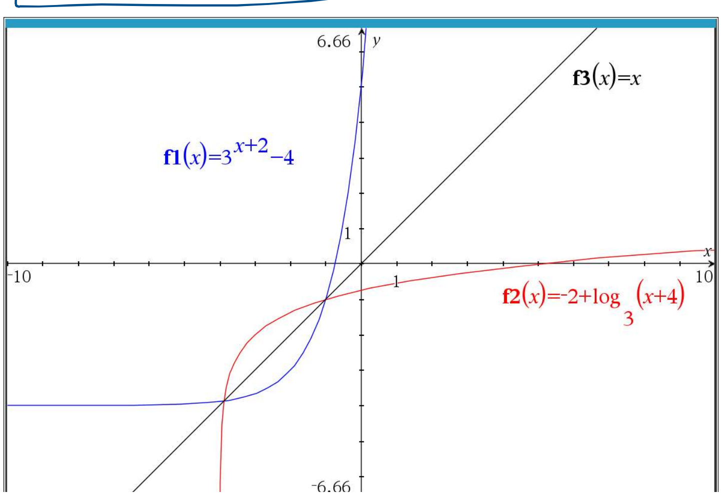
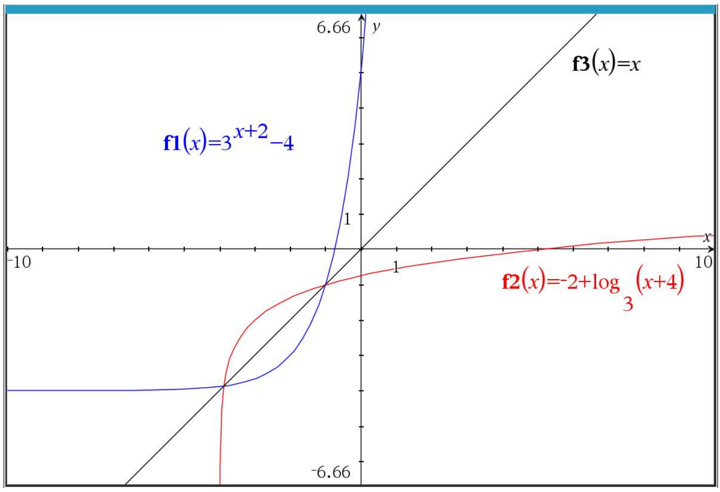
1) add 4
$x + 4$
2) $\log_3(x+4)$
3) $\log_3(x+4) - 2$





6.2: 5

In Exercises 1 - 15, expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers.

5. $\ln\left(\frac{\sqrt{z}}{xy}\right)$

$= \ln(\sqrt{z}) - \ln(xy)$

$= \ln(z^{\frac{1}{2}}) - \ln(xy)$

$= \boxed{\left(\frac{1}{2}\right)\ln(z) - \ln(x) - \ln(y)}$

6.2:30

In Exercises 30 - 33, use the appropriate change of base formula to convert the given expression to an expression with the indicated base.

30. 7^{x-1} to base e

Let $y = 7^{x-1}$

convert to ln form

$\log_7(y) = x-1$

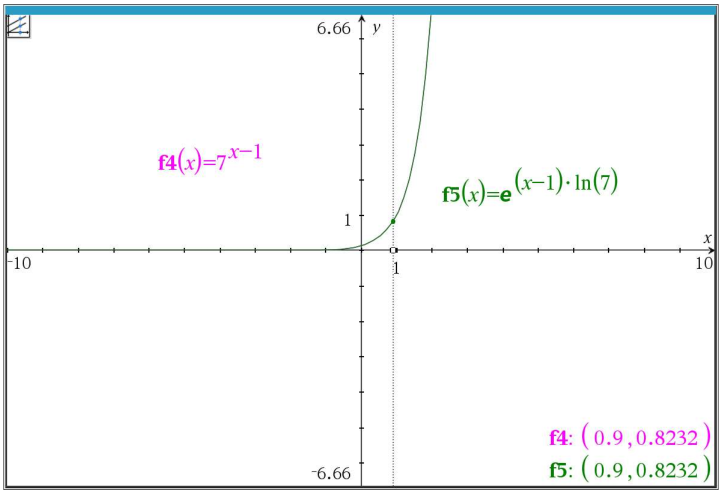
supplied

Theorem 6.7. (Change of Base Formulas) Let $a, b > 0$, $a, b \neq 1$.

- $a^x = b^{x \log_b(a)}$ for all real numbers x.
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$ for all real numbers $x > 0$.

$7^{x-1} = \boxed{e^{(x-1)\ln(7)}}$

$$\boxed{7^{x-1} = e^{(x-1)\ln(7)}}$$



6.2: 42

42. Give numerical examples to show that, in general,

(a) $\log_b(x+y) \neq \log_b(x) + \log_b(y)$

(b) $\log_b(x-y) \neq \log_b(x) - \log_b(y)$

(c) $\log_b\left(\dfrac{x}{y}\right) \neq \dfrac{\log_b(x)}{\log_b(y)}$

(a) $\log_2(2+2) \stackrel{?}{=} \log_2(2) + \log_2(2)$

$\log_2(4) \stackrel{?}{=} 1 + 1$

$2 \stackrel{?}{=} 2$ not a counter example

$\log_2(2+6) \stackrel{?}{=} \log_2(2) + \log_2(6)$

$\log_2(8) \stackrel{?}{=} 1 + \log_2(6)$

$3 \stackrel{?}{=} 1 + \log_2 6$

$3 = 1 + \log_2(6)$

$\Leftrightarrow \log_2(6) = 2$

$\Leftrightarrow 2^2 = 6$

$\Leftrightarrow 4 = 6$

This is false

$\therefore$ The rule fails

$1+\log_2(6)$	$\log_2(12)$
$\left(\log_2(12)\right)$ ▸Decimal	3.58496

$3,58496 \neq 3$

$$f(x) = \log\left(\frac{1}{x^2-16}\right)$$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$ not in domain

Let $g(x) = \dfrac{1}{x^2-16} > 0$

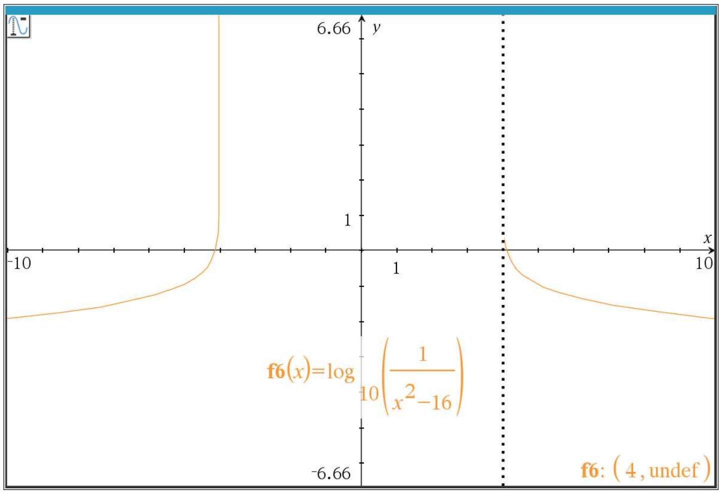
$$\frac{1}{(x+4)(x-4)}$$

$$\boxed{\begin{array}{l}\text{domain of } f(x) \\ (-\infty, -4) \cup (4, \infty)\end{array}}$$

$$g(-5) = \frac{1}{(-5)^2 - 16} = \frac{1}{25-16} > 0$$

$g(0) = \frac{1}{0^2-16} < 0$

$g(5) = g(-5) > 0$



6.3

Steps for Solving an Equation involving Exponential Functions

1. Isolate the exponential function.
2. (a) If convenient, express both sides with a common base and equate the exponents.
   (b) Otherwise, take the natural log of both sides of the equation and use the Power Rule.

solve $3^x = 9^{x+1}$

$3^x = (3^2)^{x+1}$

$3^x = 3^{2x+2}$

$\Rightarrow x = 2x+2$

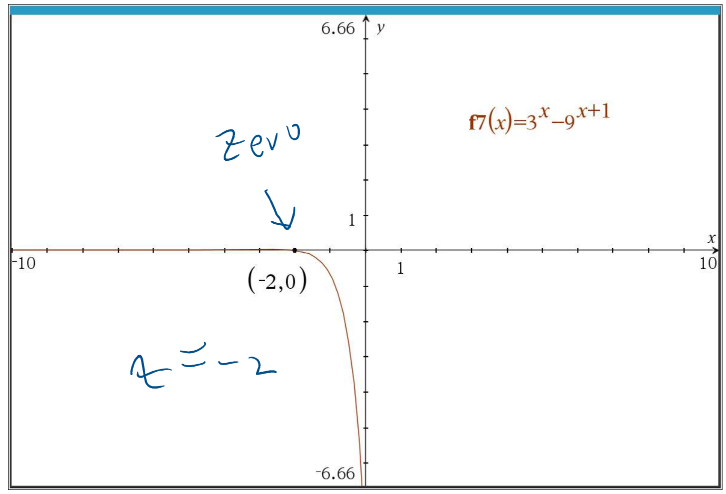
$\boxed{x = -2}$

check $3^{-2} \stackrel{?}{=} 9^{-2+1}$

$3^{-2} \stackrel{?}{=} 9^{-1}$

$3^{-} = 9$

$\frac{1}{9} = \frac{1}{9}$ ✓



solve $2^{x} = 5^{3x}$

$\ln(2^{x}) = \ln(5^{3x})$

$x\ln(2) = 3x\ln(5)$

$3x\ln(5) - x\ln(2) = 0$

$x(3\ln(5) - \ln(2)) = 0$

$x = 0$ or $3\ln(5) - \ln(2) = 0$

$3\cdot\ln(5)-\ln(2)$	$\ln\left(\frac{125}{2}\right)$
$\left(\ln\left(\frac{125}{2}\right)\right)$ ▸Decimal	4.13517

$\neq 0$

$\therefore \boxed{x = 0}$

check $2^{0} \stackrel{?}{=} 5^{(3)(0)}$

check $2^0 = 5$

$1 \overset{?}{=} 5^0$

$1 = 1$ ✓

6.4

Memorize

Steps for Solving an Equation involving Logarithmic Functions

1. Isolate the logarithmic function.
2. (a) If convenient, express both sides as logs with the same base and equate the arguments of the log functions.

   (b) Otherwise, rewrite the log equation as an exponential equation.

solve

2. $2 - \ln(x - 3) = 1$

$\ln(x-3) = 2 - 1$

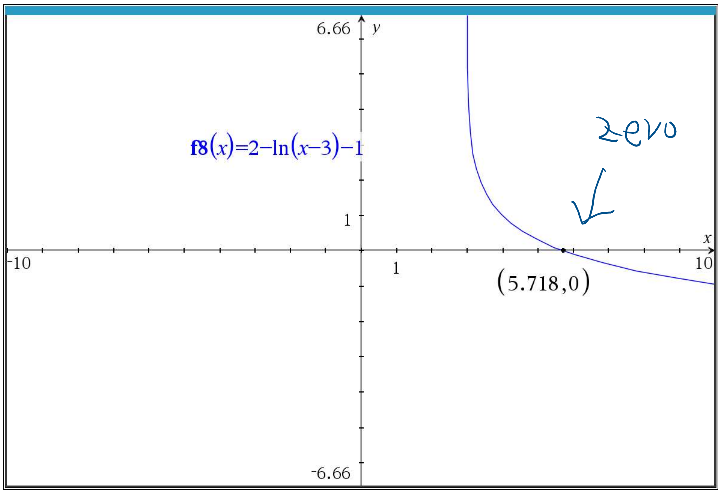
$\ln(x-3) = 1$

$\ln(x-3) = \ln(e)$

$\ln(e) = \log_e e^1 = 1$

$x - 3 = e$

$\boxed{x = e + 3}$



$(e^1+3)\blacktriangleright$Decimal 5.71828

exp form

2. $2-\ln(x-3)=1$

$\ln(x-3)=1$

$e^{\ln(x-3)}=e^1$

$x-3=e$

$\boxed{x=3+e}$

$e^1=x-3$

$e=x-3$

$\boxed{x=e+3}$

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1. Convert $3^4=81$ to log form.

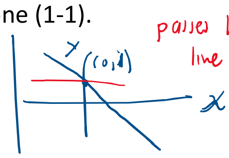
$\log_3(81)=4$

2. Convert $\log_2(8)=3$ to exponential form.

$2^3=8$

Let $f(x)=-x+1$.
Show $f(x)$ is one-to-one (1-1).

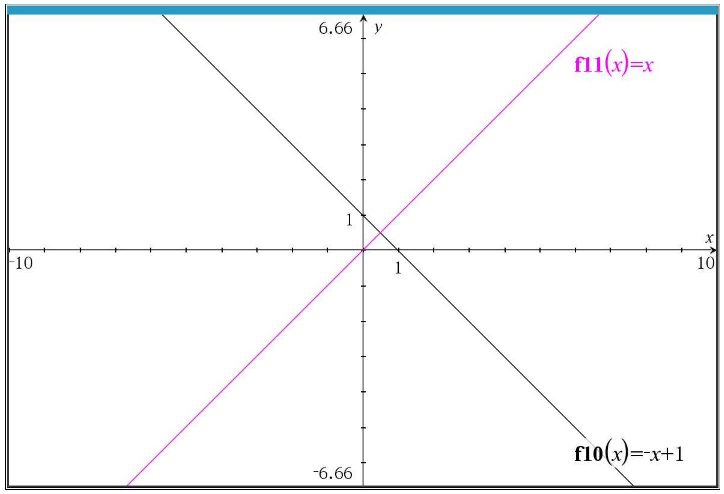
Let $f(c)=f(d)$
$-c+1=-d+1$
$-c=-d$
$c=d$ ✓



passes horizontal line test

4. Find $f^{-1}(x)$ for #3.

$y=-x+1$
$x=-y+1$
$y=1-x \Rightarrow \boxed{f^{-1}(x)=1-x}$



5 .Verify your inverse by calculating $(f \circ f^{-1})(x)$.

$(f \circ f^{-1})(x) = f(1-x)$
$= -(1-x)+1 = -1+x+1 = x$ ✓

6.Solve analytically: $2^{x^2} = 6$. Give exact answer.

$\ln\left(2^{x^2}\right) = \ln(6)$
$x^2 \ln(2) = \ln(6)$
$x^2 = \dfrac{\ln(6)}{\ln(2)}$
$\boxed{x = \pm\sqrt{\dfrac{\ln(6)}{\ln(2)}}}$

7.Solve #6, graphically. Sketch labeled graph.

$\dfrac{\sqrt{\ln(6)}}{\sqrt{\ln(2)}}$ ▸Decimal 1.60778