10-28-25 MTH 167-002N

5.2 Inverse Functions

5.2.1 Exercises

page 391 (403): 1, 5, 11, 17, 23

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6 Exponential and Logarithmic Functions

6.1 Introduction to Exponential and Logarithmic Functions

6.1.1 Exercises

page 429 (441): 1, 5, 16, 26, 45, 56, 71, 75

6.2 Properties of Logarithms

6.2.1 Exercises

page 445 (457): 1, 5, 10, 16, 23, 30, 42

Exam 2		stem & leaf		
38.73684	mean			A-0
37	median			B-0
19.83575	st. dev	7	4	C-1
11	min	6	249	D-3
74	max	5	8	F- 15
19	count	4	1569	
		3	567	
		2	58	
		1	12257	

Exam 1		stem & leaf		
51.08333	mean			A-0
55.5	median	8	5	B-1
19.59361	st. dev	7	1246	C-4
17	min	6	000189	D-6
85	max	5	56	F- 13
24	count	4	23349	
		3		
		2	4568	
		1	78	

5.2: 5

In Exercises 1 - 20, show that the given function is one-to-one and find its inverse. Check your answers algebraically and graphically. Verify that the range of f is the domain of f^{-1} and vice-versa.

5. $f(x) = \sqrt{3x-1} + 5$

1-1 Assume $f(c) = f(d)$ show $c = d$

$\sqrt{3c-1} + 5 = \sqrt{3d-1} + 5$

$\sqrt{3c-1} = \sqrt{3d-1}$

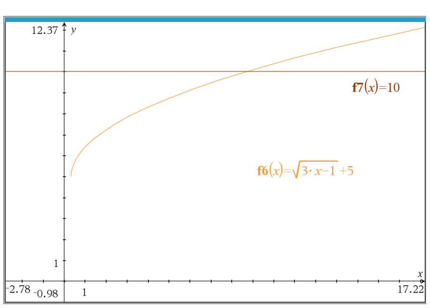
$3c - 1 = 3d - 1$

$3c = 3d$

$c = d$ ✓

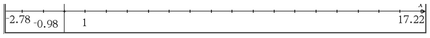
$\therefore f$ is 1-1

Horizontal line test



Graph passes the horizontal Line test

$\therefore f^{-1}$ exists   Replace $f^{-1}(x)$ with y



$\therefore f^{-1}$ exists Replace $f^{-1}(x)$ with y

$y = \sqrt{3x-1} + 5$ switch x, y

$x = \sqrt{3y-1} + 5$ solve for y

$\sqrt{3y-1} = x - 5$

$3y - 1 = (x-5)^2$

$3y = 1 + (x-5)^2$

$y = \dfrac{(x-5)^2 + 1}{3}$ replace y by $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \frac{(x-5)^2+1}{3}}$$

$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f\left(\dfrac{(x-5)^2+1}{3}\right)$

$= \sqrt{3\left(\dfrac{(x-5)^2+1}{3}\right) - 1} + 5$

$= \sqrt{(x-5)^2 + 1 - 1} + 5$

$= \sqrt{(x-5)^2} + 5$

$= (x-5) + 5$

$= x$ ✓

$f(x) = \sqrt{3x-1} + 5$

$3x - 1 \geq 0$ to avoid negative values under the radical

$3x \geq 1$

$$\boxed{x \geq \frac{1}{3}}$$

$$\boxed{x \geq \frac{1}{3}}$$

domain $\{x \mid x \geq \frac{1}{3}\}$

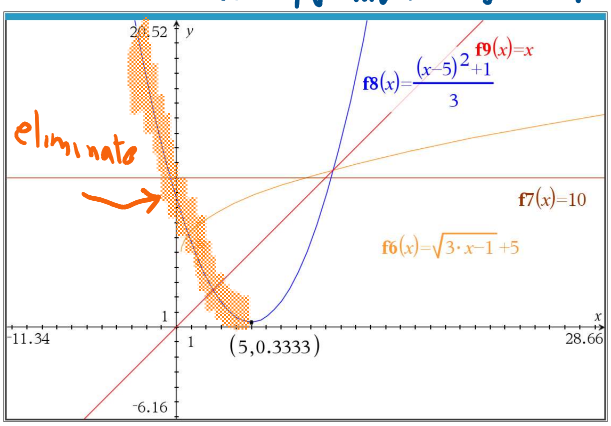
$= [\frac{1}{3}, \infty)$

range $= [5, \infty)$

$$\boxed{f^{-1}(x) = \frac{(x-5)^2+1}{3}}$$ for $x \geq 5$

range $[\frac{1}{3}, \infty)$

domain $[5, \infty)$, because we must delete the left half of the parabola to make f^{-1} 1-1

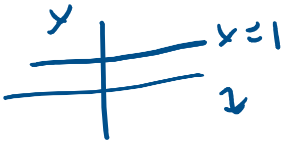


6.1

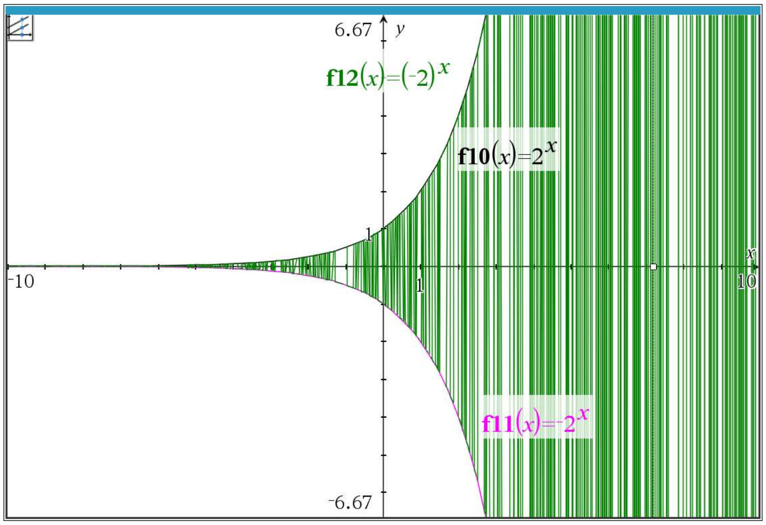
memorize

Definition 6.1. A function of the form $f(x) = b^x$ where b is a fixed real number, $b > 0$, $b \neq 1$ is called a **base b exponential function**.

let $b = 1 \Rightarrow f(x) = 1^x = 1$ all x



To avoid this mess (see graph), we have b >0.

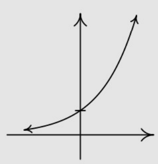


Memorize

Theorem 6.1. Properties of Exponential Functions: Suppose $f(x) = b^x$.

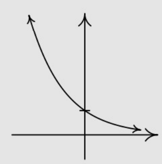
- The domain of f is $(-\infty, \infty)$ and the range of f is $(0, \infty)$.
- $(0, 1)$ is on the graph of f and $y = 0$ is a horizontal asymptote to the graph of f.
- f is one-to-one, continuous and smooth[a]

- If $b > 1$:
  - f is always increasing
  - As $x \to -\infty$, $f(x) \to 0^+$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



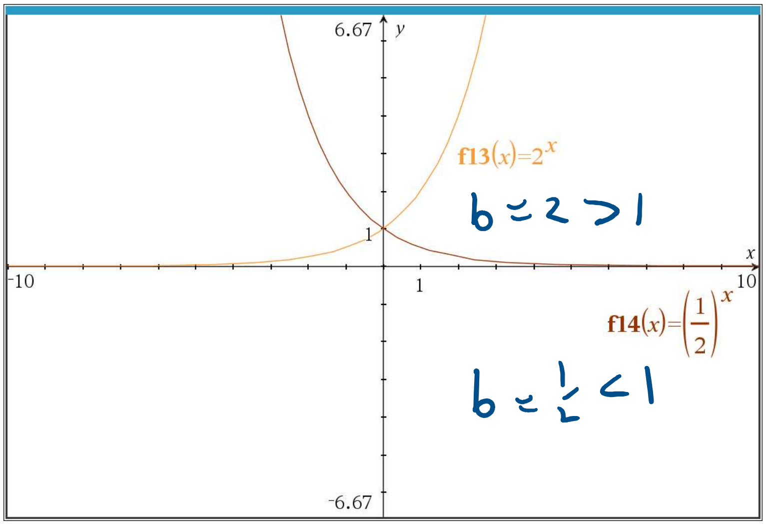
$y = b^x$, $b > 1$

- If $0 < b < 1$:
  - f is always decreasing
  - As $x \to -\infty$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to 0^+$
  - The graph of f resembles:



$y = b^x$, $0 < b < 1$

[a] Recall that this means the graph of f has no sharp turns or corners.



$f(x) = b^x$ is 1-1 $\Rightarrow$ f^{-1} exists

memorize

Definition 6.2. The inverse of the exponential function $f(x) = b^x$ is called the **base b logarithm function**, and is denoted $f^{-1}(x) = \log_b(x)$ We read '$\log_b(x)$' as 'log base b of x.'

Memorize

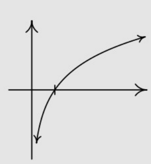
Definition 6.3. The **common logarithm** of a real number x is $\log_{10}(x)$ and is usually written $\log(x)$. The **natural logarithm** of a real number x is $\log_e(x)$ and is usually written $\ln(x)$.

Memorize

Theorem 6.2. Properties of Logarithmic Functions: Suppose $f(x) = \log_b(x)$.

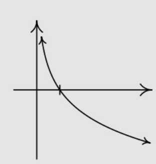
- The domain of f is $(0, \infty)$ and the range of f is $(-\infty, \infty)$.
- $(1, 0)$ is on the graph of f and $x = 0$ is a vertical asymptote of the graph of f.
- f is one-to-one, continuous and smooth
- $b^a = c$ if and only if $\log_b(c) = a$. That is, $\log_b(c)$ is the exponent you put on b to obtain c.
- $\log_b(b^x) = x$ for all x and $b^{\log_b(x)} = x$ for all $x > 0$

- If $b > 1$:
  - f is always increasing
  - As $x \to 0^+$, $f(x) \to -\infty$
  - As $x \to \infty$, $f(x) \to \infty$
  - The graph of f resembles:



$y = \log_b(x)$, $b > 1$

- If $0 < b < 1$:
  - f is always decreasing
  - As $x \to 0^+$, $f(x) \to \infty$
  - As $x \to \infty$, $f(x) \to -\infty$
  - The graph of f resembles:



$y = \log_b(x)$, $0 < b < 1$

convert $2^3 = 8$ to log form

$\log_2 8 = 3$

$10^2 = 100$

$\log_{10}(100) = 2$

$\log_{10}(1000) = 3$

$\log_{10}(1{,}000{,}000) = 6$

count the zero

$\log(100) = \ln 100 = 4.6052$

Scientific Notebook inteprets log as natural log

$\log_{10}(100) = 2$

TI-nspire interprets log as common log

I typed log (100) and my input was converted to base 10

$\log_{10}(100)$	2

6.2

memorize

Theorem 6.3. (Inverse Properties of Exponential and Logarithmic Functions)
Let $b > 0$, $b \neq 1$.

- $b^a = c$ if and only if $\log_b(c) = a$
- $\log_b(b^x) = x$ for all x and $b^{\log_b(x)} = x$ for all $x > 0$

Memorize

Theorem 6.4. (One-to-one Properties of Exponential and Logarithmic Functions)
Let $f(x) = b^x$ and $g(x) = \log_b(x)$ where $b > 0$, $b \neq 1$. Then f and g are one-to-one and

- $b^u = b^w$ if and only if $u = w$ for all real numbers u and w.
- $\log_b(u) = \log_b(w)$ if and only if $u = w$ for all real numbers $u > 0$, $w > 0$.

solve $2^x = 8$

$2^x = 2^3$

$\Rightarrow x = 3$

Memorize

Theorem 6.5. (Algebraic Properties of Exponential Functions) Let $f(x) = b^x$ be an exponential function ($b > 0$, $b \neq 1$) and let u and w be real numbers.

- **Product Rule:** $f(u + w) = f(u)f(w)$. In other words, $b^{u+w} = b^u b^w$
- **Quotient Rule:** $f(u - w) = \dfrac{f(u)}{f(w)}$. In other words, $b^{u-w} = \dfrac{b^u}{b^w}$
- **Power Rule:** $(f(u))^w = f(uw)$. In other words, $(b^u)^w = b^{uw}$

example $2^{3+2} = 2^5 = (2)(2)(2)(2)(2) = 32$

$= (2^3)(2^2)$

$(2^3)^2 = (2^3)(2^3) = ((2)(2)(2))((2)(2)(2))$

$= 2^6$

Memorize

Theorem 6.6. (Algebraic Properties of Logarithmic Functions) Let $g(x) = \log_b(x)$ be a logarithmic function ($b > 0$, $b \neq 1$) and let $u > 0$ and $w > 0$ be real numbers.

- **Product Rule:** $g(uw) = g(u) + g(w)$. In other words, $\log_b(uw) = \log_b(u) + \log_b(w)$
- **Quotient Rule:** $g\left(\frac{u}{w}\right) = g(u) - g(w)$. In other words, $\log_b\left(\frac{u}{w}\right) = \log_b(u) - \log_b(w)$
- **Power Rule:** $g(u^w) = wg(u)$. In other words, $\log_b(u^w) = w\log_b(u)$

Prove the product rule

prove $\log_b(uw) = \log_b(u) + \log_b(w)$

Let $A = \log_b(u)$, let $B = \log_b(w)$

$b^A = u$, $b^B = w$

$\log_b(uw) = \log_b(b^A b^B)$

$= \log_b(b^{A+B}) = \boxed{A + B}$

$\log_b(u) + \log_b(w) = \boxed{A + B}$

Theorem 6.7. (Change of Base Formulas) Let $a, b > 0$, $a, b \neq 1$.

- $a^x = b^{x \log_b(a)}$ for all real numbers x.
- $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$ for all real numbers $x > 0$.

convert $\log_2 x$ to $\ln$

Let $\boxed{y = \log_2 x}$

$2^y = x$

Let $\boxed{y = \log_2 x}$

$$2^y = x$$

$$\ln(2^y) = \ln(x)$$

$$y \ln(2) = \ln(x)$$

$$y = \frac{\ln(x)}{\ln(2)}$$

$$\log_2(x) = \frac{\ln(x)}{\ln(2)} = \frac{\log(x)}{\log(2)}$$