5.2 Inverse Functions

5.2.1 Exercises

page 391 (403): 1, 5, 11, 17, 23

6 Exponential and Logarithmic Functions

6.1 Introduction to Exponential and Logarithmic Functions

6.1.1 Exercises

page 429 (441): 1, 5, 16, 26, 45, 56, 71, 75

Took exam 2

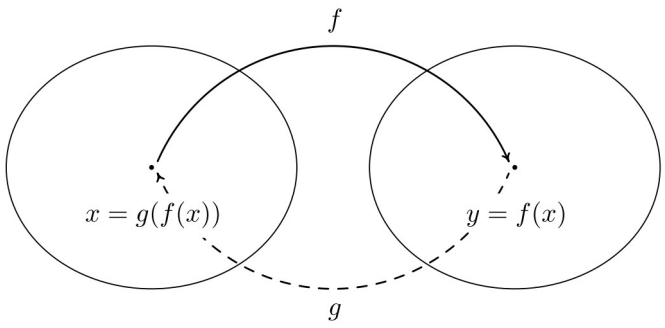
5.1

Memorize

Definition 5.2. Suppose f and g are two functions such that

1. $(g \circ f)(x) = x$ for all x in the domain of f **and**
2. $(f \circ g)(x) = x$ for all x in the domain of g

then f and g are **inverses** of each other and the functions f and g are said to be **invertible**.



memorize

Theorem 5.2. Properties of Inverse Functions: Suppose f and g are inverse functions.

- The range[a] of f is the domain of g and the domain of f is the range of g
- $f(a) = b$ if and only if $g(b) = a$
- (a, b) is on the graph of f if and only if (b, a) is on the graph of g

[a] Recall this is the set of all outputs of a function.

Memorize

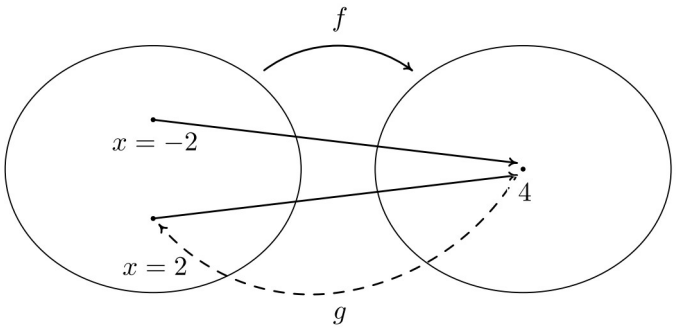
Theorem 5.3. Uniqueness of Inverse Functions and Their Graphs : Suppose f is an invertible function.

- There is exactly one inverse function for f, denoted f^{-1} (read f-inverse)
- The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ across the line $y = x$.

Let $f(x) = x^2$

$f(-2) = (-2)^2 = 4$

$f(2) = 2^2 = 4$



If two inputs of f(x) have the same output, f(x) does not have an inverse.

Memorize

1-1

Definition 5.3. A function f is said to be **one-to-one** if f matches different inputs to different outputs. Equivalently, f is one-to-one if and only if whenever $f(c) = f(d)$, then $c = d$.

Memorize

Theorem 5.4. The Horizontal Line Test: A function f is one-to-one if and only if no horizontal line intersects the graph of f more than once.

Let $f(x) = 2x - 4$

is f 1-1?

Assume $f(c) = f(d)$

$\Rightarrow 2c - 4 = 2d - 4$

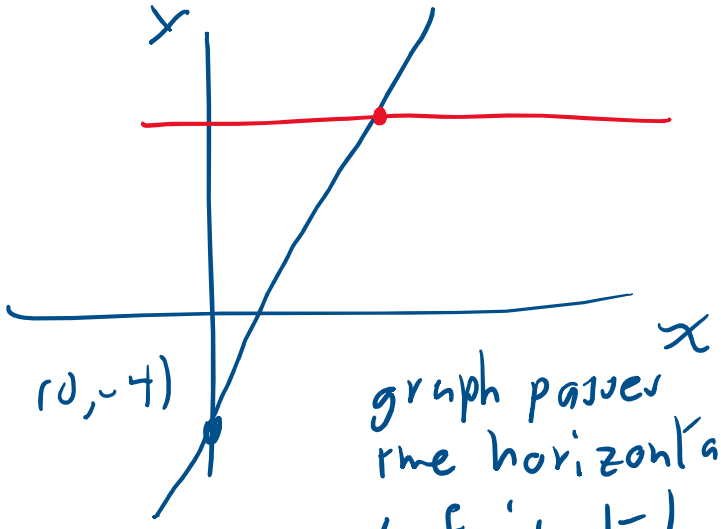
$2c = 2d$

$c = d$

$2c = 2d$

$c = d$

$\therefore f$ is 1-1



graph passes the horizontal line test

$\therefore f$ is 1-1

Let $f(x) = x^2$

is f 1-1?

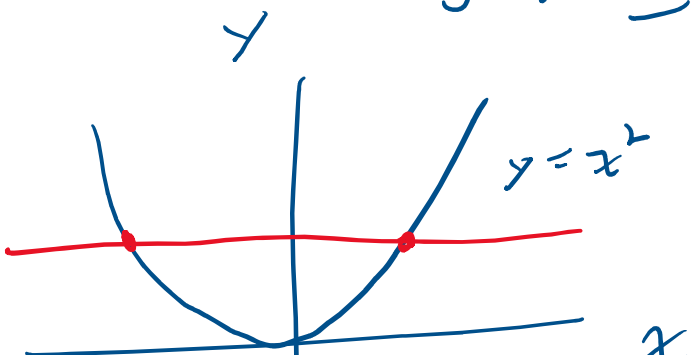
Assume $f(c) = f(d)$

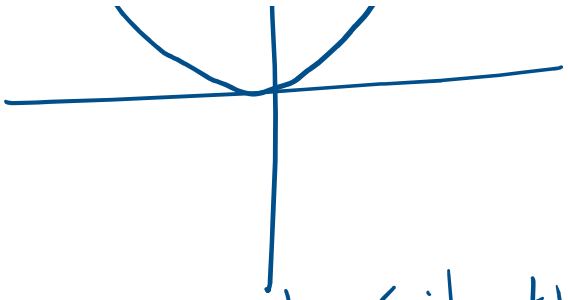
$c^2 = d^2$

$c = \pm d$

Since it is not necessary that $c = d$

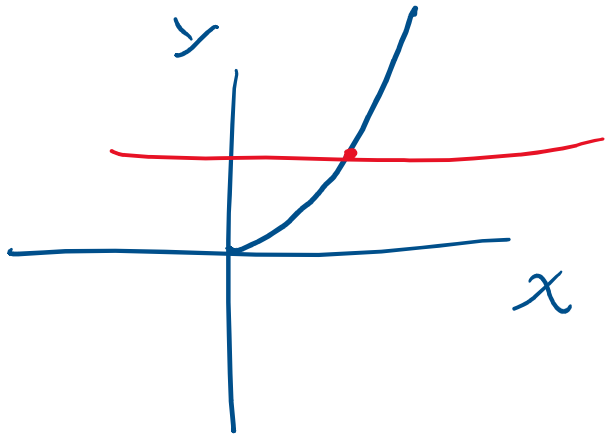
f is <u>not</u> 1-1





graph fails the horizontal line test
∴ f is not 1-1

Let $f(x) = x^2$ on $[0, \infty)$



Memorize

Theorem 5.5. Equivalent Conditions for Invertibility: Suppose f is a function. The following statements are equivalent.

- f is invertible
- f is one-to-one
- The graph of f passes the Horizontal Line Test

Memorize

Steps for finding the Inverse of a One-to-one Function

1. Write $y = f(x)$
2. Interchange x and y
3. Solve $x = f(y)$ for y to obtain $y = f^{-1}(x)$

$f(x) = 2x - 4$
we know that f is 1-1
∴ $f^{-1}(x)$ exists

$\therefore f^{-1}(x)$ exists

$y = 2x - 4$ switch x, y

$x = 2y - 4$ solve for y

$2y = x + 4$

$y = \frac{x+4}{2}$

$$\boxed{f^{-1}(x) = \frac{x+4}{2}}$$

check

$(f \circ f^{-1})(x)$

$= f(f^{-1}(x))$

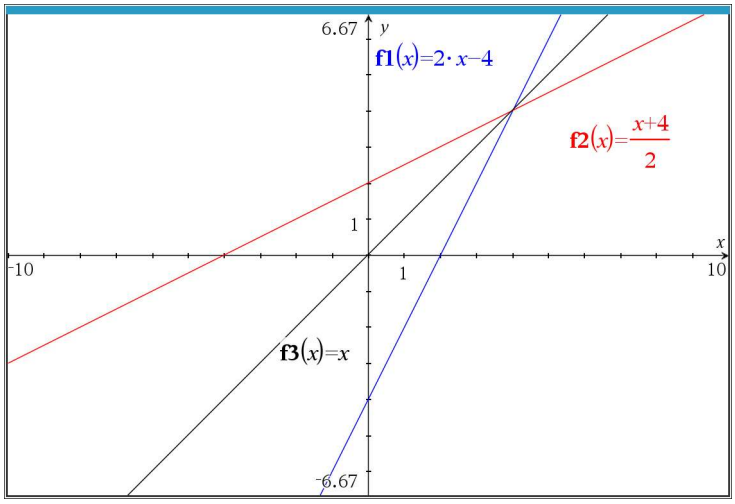
$= f\left(\frac{x+4}{2}\right)$

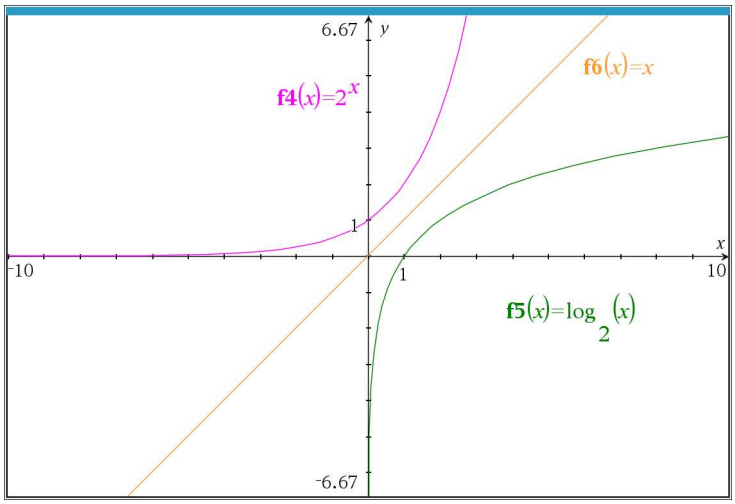
$= 2\left(\frac{x+4}{2}\right) - 4$

$= x + 4 - 4$

$= x$ $\surd$

The graph of f(x) and $f^{-1}(x)$ are mirror images or reflections across the line y = x.


6.67
y
f1(x)=2·x−4
f2(x)=(x+4)/2
1
x
-10
1
10
f3(x)=x
-6.67


6.67
y
f6(x)=x
f4(x)=2^x
1
x
-10
1
10
f5(x)=log2(x)
-6.67