10-22-25 MTH 167-002N

After class Notes

3.3

In Exercises 35 - 44, find the real solutions of the polynomial equation. (See Example 3.3.7.)

37. $x^3 + 6 = 2x^2 + 5x$

$$x^3 - 2x^2 - 5x + 6 = 0$$

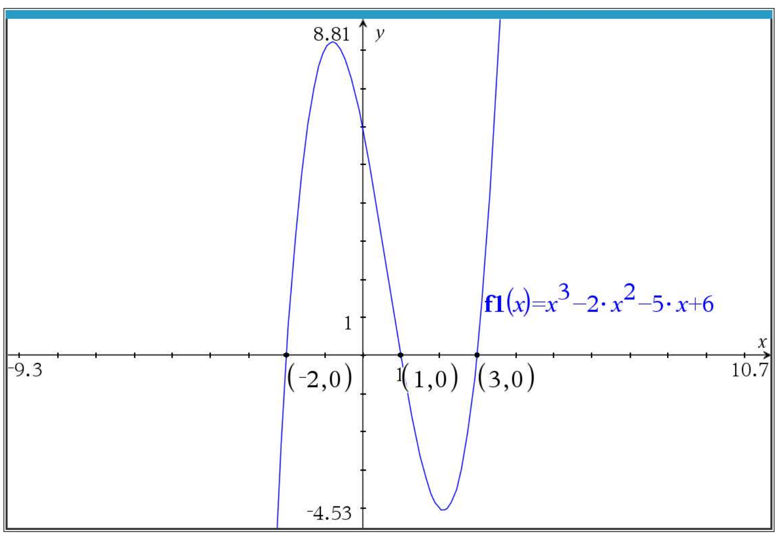
Try rational zeros theorem

Theorem 3.9. Rational Zeros Theorem: Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$, and a_0, a_1, $\ldots a_n$ are integers. If r is a rational zero of f, then r is of the form $\pm\frac{p}{q}$, where p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.

$$p \mid 6 \Rightarrow p = \pm 1, \pm 2, \pm 3, \pm 6$$

$$q \mid 1 \Rightarrow q = \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$



$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$\therefore$ $\boxed{x = -2 \text{ is a zero}}$

$$\begin{array}{r|rrr} -2 & 1 & -4 & 3 \\ & & -2 & 12 \\ \hline & 1 & -6 & \boxed{15} \end{array}$$

$$\begin{array}{r|rrr} & & -2 & 12 \\ \hline & 1 & -6 & (15) \end{array}$$

$15 \neq 0 \therefore$ mult 1.

$$\begin{array}{r|rrr} 1 & 1 & -4 & 3 \\ & & 1 & -3 \\ \hline & 1 & -3 & (0) \end{array}$$

$x = 1$ is a zero mult. 1

$$\begin{array}{r|rr} 3 & 1 & -3 \\ & & 3 \\ \hline & 1 & (0) \end{array}$$

$x = 3$ is a zero mult. 1

$z = 3 + 4i$

$\Rightarrow \bar{z} = 3 - 4i$

3.4

In Exercises 49 - 53, create a polynomial f with real number coefficients which has all of the desired characteristics. You may leave the polynomial in factored form.

52. • f is degree 5.
    • $x = 6$, $x = i$ and $x = 1 - 3i$ are zeros of f $\Rightarrow x = -i, x = 1 + 3i$ are zeros
    • as $x \to -\infty$, $f(x) \to \infty$

$f(x) = a(x-6)(x-i)(x+i)(x-(1-3i))(x-(1+3i))$

$= a(x-6)(x^2+1)((x-1)+3i)((x-1)-3i)$

$f(x) = a(x-6)(x^2+1)((x-1)^2+9)$

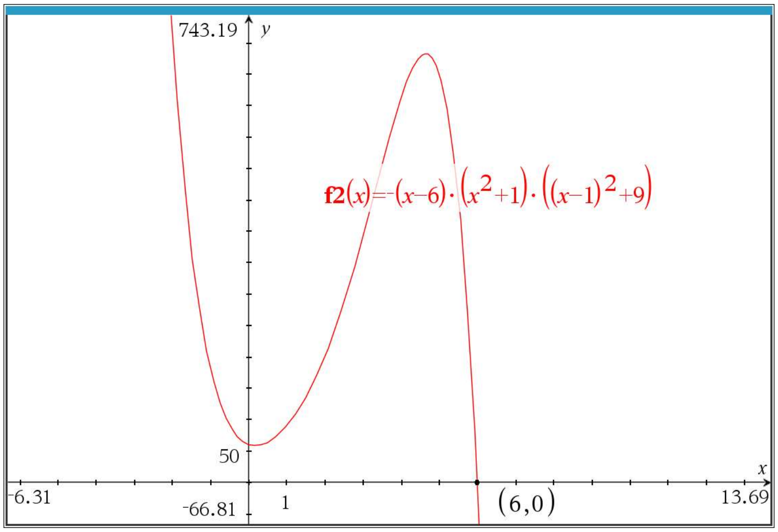
$x < 0 \Rightarrow f(x) = a(-)(+)(+) = -a(+) \Rightarrow a < 0$

Let $a = -1$

$f(x) = -(x-6)(x^2+1)((x-1)^2+9)$

$f(x) = -(x-6)(x^2+1)((x-1)^2+9)$

Note, we could use any negative number for a .



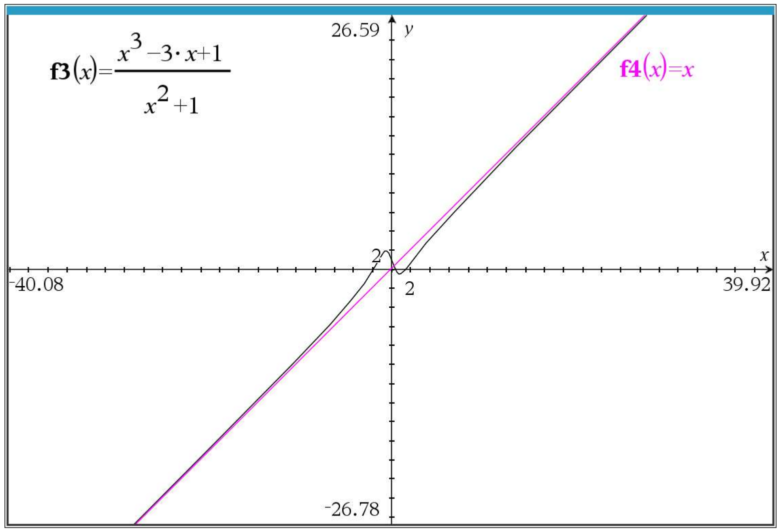
$\text{solve}\left(-(x-6)\cdot(x^2+1)\cdot((x-1)^2+9)=0,x\right)$	$x=6$
$\text{cSolve}\left(-(x-6)\cdot(x^2+1)\cdot((x-1)^2+9)=0,x\right)$	$x=1+3\cdot i$ or $x=1-3\cdot i$ or $x=i$ or $x=-i$ or $x=6$

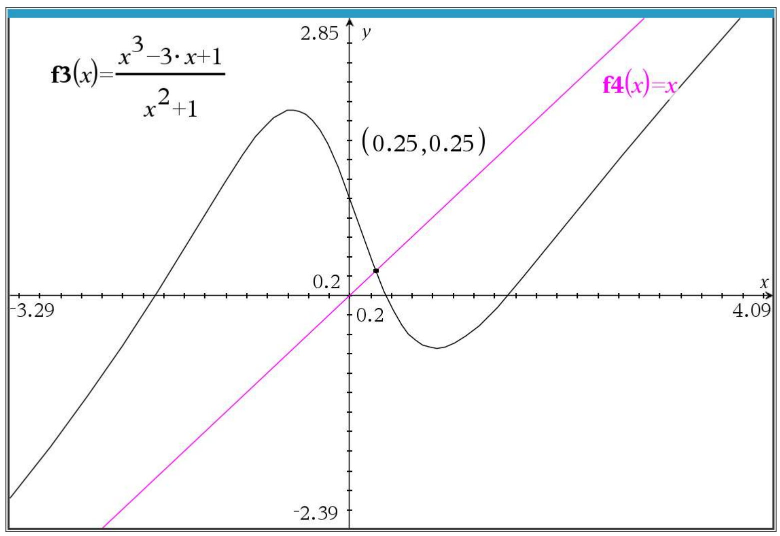
$$x + \frac{-4x+1}{x^2+1}$$

$$x^2+1 \overline{\big) x^3 + 0x^2 - 3x + 1}$$

$$\frac{x^3 \qquad + x}{\qquad -4x + 1}$$

$\text{polyQuotient}\left(x^3-3\cdot x+1,x^2+1\right)$	x
$\text{polyRemainder}\left(x^3-3\cdot x+1,x^2+1\right)$	$1-4\cdot x$





Note: The asymptote intersect the graph of the function at $\left(\frac{1}{4}, \frac{1}{4}\right)$