10-21-25 MTH 167-002N

5 Further Topics in Functions
5.1 Function Composition
5.1.1 Exercises
page 369 (381): 1, 9, 13, 17, 22, 31, 39, 62

Exam 2, Thursday, 10/23/25, 2.2 - 2.4, 3.1 - 3.4, 4.1 - 4.3, 5.1

---

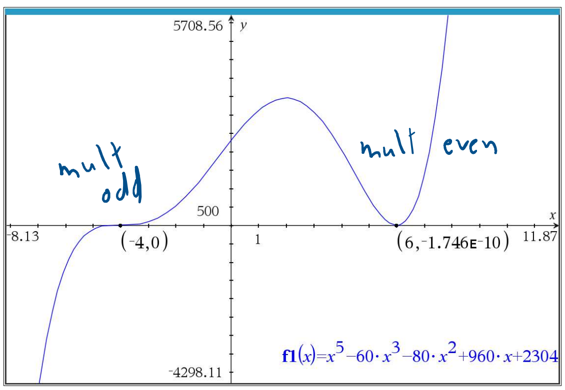
14 class meetings before final exam
24 required textbook sections
24/14=1.7143 sections per class meeting

After class notes

3.3: 31

In Exercises 31 - 33, use your calculator,[9] to help you find the real zeros of the polynomial. State the multiplicity of each real zero.

31. $f(x) = x^5 - 60x^3 - 80x^2 + 960x + 2304$



$$\begin{array}{r|rrrrrr} -4 & 1 & 0 & -60 & -80 & 960 & 2304 \\ & & -4 & 16 & 176 & -384 & -2304 \\ \hline & 1 & -4 & -44 & 96 & 576 & (0) \end{array}$$

$f(x) = (x+4)(x^4 - 4x^3 - 44x^2 + 96x + 576)$

$$\begin{array}{r|rrrrr} -4 & 1 & -4 & -44 & 96 & 576 \\ & & -4 & 32 & 48 & -576 \\ \hline & 1 & -8 & -12 & 144 & (0) \end{array}$$

$f(x) = (x+4)^2(x^3 - 8x^2 - 12x + 144)$

$f(x) = (x+4)^2(x^3 - 8x^2 - 12x + 144)$

$$\begin{array}{r|rrrr} -4 & 1 & -8 & -12 & 144 \\ & & -4 & 48 & -144 \\ \hline & 1 & -12 & 36 & |0) \end{array}$$

$f(x) = (x+4)^3(x^2 - 12 + 36)$

$x^2 - 12 + 36 = (x-6)(x-6)$

$$\boxed{f(x) = (x+3)^3(x-6)^2}$$

real zeros $x = -3$ mult 3

6 mult 2

4.1.1 Exercises

In Exercises 1 - 18, for the given rational function f:

- Find the domain of f.
- Identify any vertical asymptotes of the graph of $y = f(x)$.
- Identify any holes in the graph.
- Find the horizontal asymptote, if it exists.
- Find the slant asymptote, if it exists.
- Graph the function using a graphing utility and describe the behavior near the asymptotes.

3. $f(x) = \dfrac{x}{x^2 + x - 12}$

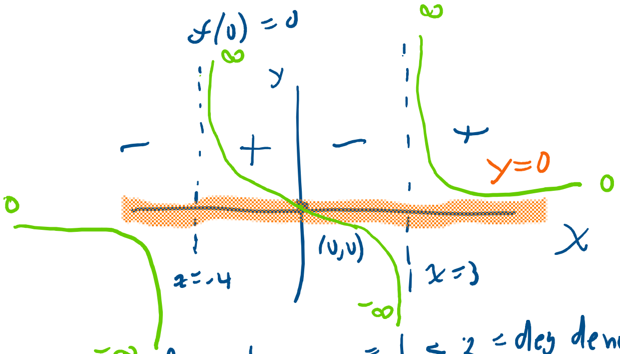
domain $= \{x \mid x \neq -4, 3\}$

set $x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, 3$ not in domain

$f(0) = 0$



$f(x) = \dfrac{x}{(x+4)(x-3)}$

vert. asymptotes

$x = -4,\ x = 3$

No hole

deg of num $= 1 < 2 =$ deg denom

$\Rightarrow$ horizontal asymptote $\boxed{y=0}$

$\therefore$ No slant asymptote

$f(-5) = \dfrac{-5}{(-5+4)(-5-3)} = \dfrac{(-)}{(-)(-)} < 0$

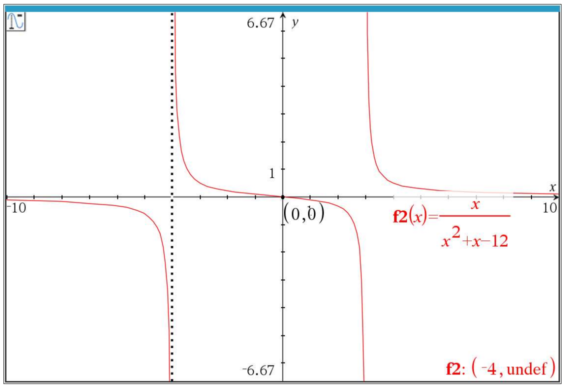
$f(-1) = \dfrac{-1}{(-1+4)(-1-3)} = \dfrac{(-)}{(+)(-)} > 0$

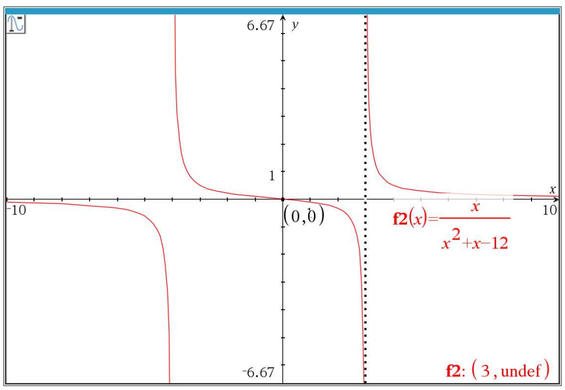
$f(1) = \dfrac{1}{\quad} = \dfrac{(+)}{\quad} < 0$

$(-1+4)(-1-3) \quad (+)(-)$

$f(1) = \dfrac{1}{(1+4)(1-3)} = \dfrac{(+)}{(+)(-)} < 0$

$f(4) = \dfrac{4}{(4+4)(4-3)} = \dfrac{(+)}{(+)(+)} > 0$





4.2: 5

4.2.1 Exercises

In Exercises 1 - 16, use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

5. $f(x) = \dfrac{2x-1}{-2x^2-5x+3}$

$= \dfrac{2x-1}{-(2x^2+5x-3)}$

$= \dfrac{2x-1}{-(2x-1)(x+3)}$

$2x-1=0$

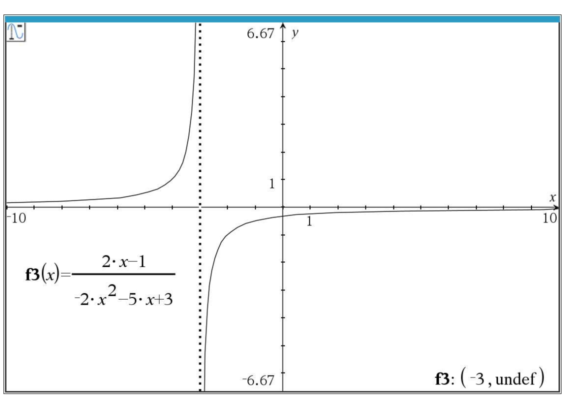
$\Rightarrow 2x = 1$

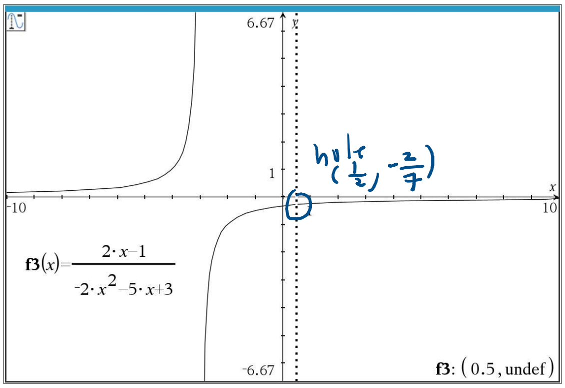
$\boxed{x = \frac{1}{2}}$

zero?

No hole at $x = \frac{1}{2}$

$\therefore$ No x-intercept





$f(x) = \frac{-1}{x+3}$ if $x \neq \frac{1}{2}$

$f(\frac{1}{2}) = -\frac{1}{\frac{1}{2}+3}$

$= -\frac{1}{\frac{1}{2}+\frac{6}{2}}$

$= -\frac{1}{\frac{7}{2}} = -\frac{2}{7}$

2.1: 29

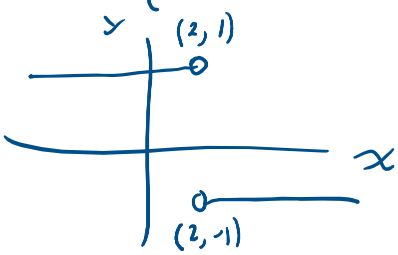
In Exercises 22 - 33, graph the function. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

29. $f(x) = \dfrac{|2-x|}{2-x}$

$|2-x| = \begin{cases} 2-x & \text{if } 2-x \geq 0 \iff x \leq 2 \\ -(2-x) & \text{if } 2-x < 0 \iff x > 2 \end{cases}$

$f(x) = \begin{cases} \frac{2-x}{2-x} & \text{if } x < 2, \text{ because } x=2 \Rightarrow f(x) \text{ not defined} \\ \frac{-(2-x)}{2-x} & \text{if } x > 2 \end{cases}$

$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ -1 & \text{if } x > 2 \\ \text{not defined} & \text{if } x = 2 \end{cases}$



domain $(-\infty, 2) \cup (2, \infty)$

range $= \{1, -1\}$

constant $(-\infty, 2), (2, \infty)$

Determine if $x^2 + 1$ is a factor of $p(x) = x^4 + 2x^3 - x^2 + 3x + 6$.

$$x^2+2x-2+\frac{x+8}{x^2+1}$$

$$\begin{array}{r} x^2+1 \,\big)\, x^4 + 2x^3 - x^2 + 3x + 6 \\ x^4 + 0x^3 + x^2 \\ \hline 2x^3 - 2x^2 + 3x \\ 2x^3 \quad 0x^2 \quad + 2x \\ \hline -2x^2 + x + 6 \\ -2x^2 \quad 0x \quad -2 \\ \hline x + 8 \end{array}$$

$$\frac{-2x^2 \quad -2}{x+8}$$

Because $\exists$ a non-zero remainder, there exists,

x^2+1 is <u>not</u> a factor of $p(x)$

5.1: 22

In Exercises 13 - 24, use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

- $(g \circ f)(x)$
- $(f \circ g)(x)$
- $(f \circ f)(x)$

22. $f(x) = \dfrac{3x}{x-1}$, $g(x) = \dfrac{x}{x-3}$

$$g(\text{input}) = \frac{\text{input}}{\text{input}-3}$$

$$(g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{3x}{x-1}\right) = \frac{\frac{3x}{x-1}}{\frac{3x}{x-1} - 3} = \frac{\frac{3x}{x-1}}{\frac{3x}{x-1} \frac{-3(x-1)}{x-1}}$$

$1 \notin$ domain

$$\frac{3x}{x-1} - 3 = 0$$

$$\frac{3x}{x-1} = 3$$

$$\frac{x}{x-1} = 1$$

$$x = x-1$$

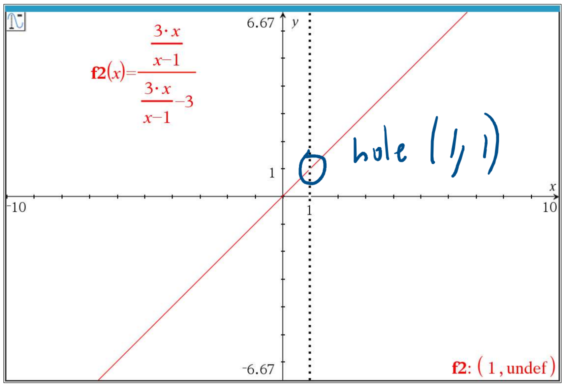
$$0 \neq -1$$

$\therefore$ no solution

domain $= \{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$

$$= \frac{\frac{3x}{x-1}}{\frac{3x-3x+3}{x-1}} = \frac{3x}{3}$$

$$= x$$



$$\frac{\frac{3x}{x-1}}{\frac{3x}{x-1}-3} \qquad (3x/(x-1))/[(3x/(x-1))-3]$$

5.1: 39

In Exercises 31 - 40, write the given function as a composition of two or more non-identity functions. (There are several correct answers, so check your answer using function composition.)

39. $v(x) = \dfrac{2x+1}{3-4x}$

$v(x) = v(x) \circ I(x)$

Find $f(x)$, $g(x)$ such that $v(x) = (f \circ g)(x)$

$\boxed{\text{Let } g(x) = 2x}$

$v(x) = \dfrac{g(x)+1}{3-2g(x)}$

$\boxed{f(x) = \dfrac{x+1}{3-2x}}$

$(f \circ g)(x) = f(g(x))$

$= f(2x)$

$= \dfrac{2x+1}{3-2(2x)} = \dfrac{2x+1}{3-4x} = v(x)$

$f(x) = \dfrac{2x}{7-4x}$

$f(x) = \frac{2x}{7-4x}$

$g(x) = x+1$

$(f \circ g)(x) = f(g(x))$

$= f(x+1)$

$= \frac{2(x+1)}{7-4(x+1)}$

$= \frac{2x+2}{7-4x-4} = \frac{2x+2}{3-4x}$ 1

$f(x) = x^4 - 5x^3 + 2x^2 + 8x - 8$

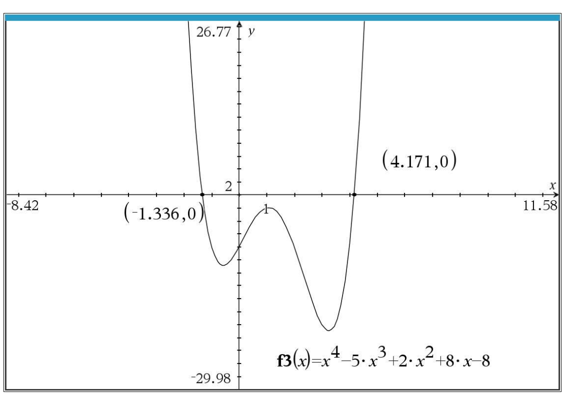
Find rational zeros

If $\frac{p}{q}$ is a rational zero of $f(x)$

then $p \mid (-8) \Rightarrow p = \pm 1, \pm 2, \pm 4, \pm 8$

$q \mid 1 \qquad q = \pm 1$

$\boxed{\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8}$ list of all possible rational zeros



-1.336 and 4.171 are not in our list

$\therefore$ No rational zeros

and $x \approx -1.386$

$x \approx 4.171$

are real, irrational zeros

4.3: 25

25. Working together, Daniel and Donnie can clean the llama pen in 45 minutes. On his own, Daniel can clean the pen in an hour. How long does it take Donnie to clean the llama pen on his own?

25. Working together, Daniel and Donnie can clean the llama pen in 45 minutes. On his own, Daniel can clean the pen in an hour. How long does it take Donnie to clean the llama pen on his own?

Think: $d = rt$

r_{Dan} = Daniel's rate

r_{Don} = Donnie's rate

t_{Dan} = Daniel's time

t_{Don} = Donnie's time

$$1 \text{ pen} = r_{Dan}(45 \text{ min}) + r_{Don}(45 \text{ min})$$

$$1 \text{ pen} = r_{Dan}(60 \text{ min})$$

$$\boxed{r_{Dan} = \frac{1 \text{ pen}}{60 \text{ min}}}$$

Find r_{Don}

$$1 \text{ pen} = \left(1 \frac{\text{pen}}{60 \text{ min}}\right)(45 \text{ min}) + r_{Don}(45 \text{ min})$$

solve for r_{Don}

$$r_{Don}(45 \text{ min}) = 1 \text{ pen} - (1 \text{ pen})\frac{45}{60}$$

$$r_{Don} = \frac{\left(1 - \frac{45}{60}\right)}{45} \frac{\text{pen}}{\text{min}}$$

$$r_{Don} = \frac{\frac{60-45}{60}}{45} \frac{\text{pen}}{\text{min}}$$

$$r_{Don} = \frac{15}{(60)(45)} \frac{\text{pen}}{\text{min}}$$

$$r_{Don} = \frac{1}{(60)(3)} \frac{\text{pen}}{\text{min}}$$

$$\boxed{r_{Don} = \frac{1}{180} \frac{\text{pen}}{\text{min}}}$$

Donnie cleans 1 llama pen in 180 min

$\boxed{\text{Donnie cleans 1 llama per in 180 min or 3 hr.}}$

2.4: 18

2.4.1 Exercises

In Exercises 1 - 32, solve the inequality. Write your answer using interval notation.

18. $16x^2 + 8x + 1 > 0$

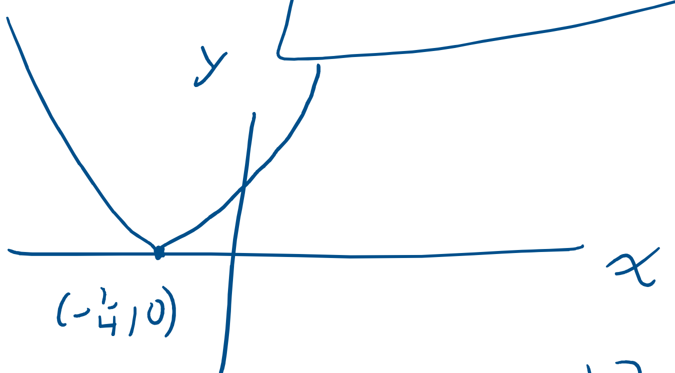
$(4x+1)(4x+1) > 0$

Find zeros

$4x+1=0$

$4x=-1$

$\boxed{x=-\frac{1}{4} \quad \text{mult. } 2}$



solution $\{x \mid x \neq -\frac{1}{4}\}$

$=(-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

algebraic method

$(4x+1)^2 > 0$

if $4x+1 \neq 0$

if $4x+1 \neq 0$

$\Leftrightarrow$ if $x \neq -\frac{1}{4}$

4.2: 5

In Exercises 1 - 16, use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

5. $f(x) = \dfrac{2x-1}{-2x^2-5x+3}$

$= \dfrac{2x-1}{-(2x^2+5x-3)}$

$= \dfrac{2x-1}{-(2x-1)(x+3)}$

$2x-1=0$

$\Rightarrow 2x=1$

$\boxed{x=\frac{1}{2}}$

hole

$x+3=0$

$\Rightarrow \boxed{x=-3}$

asymptote

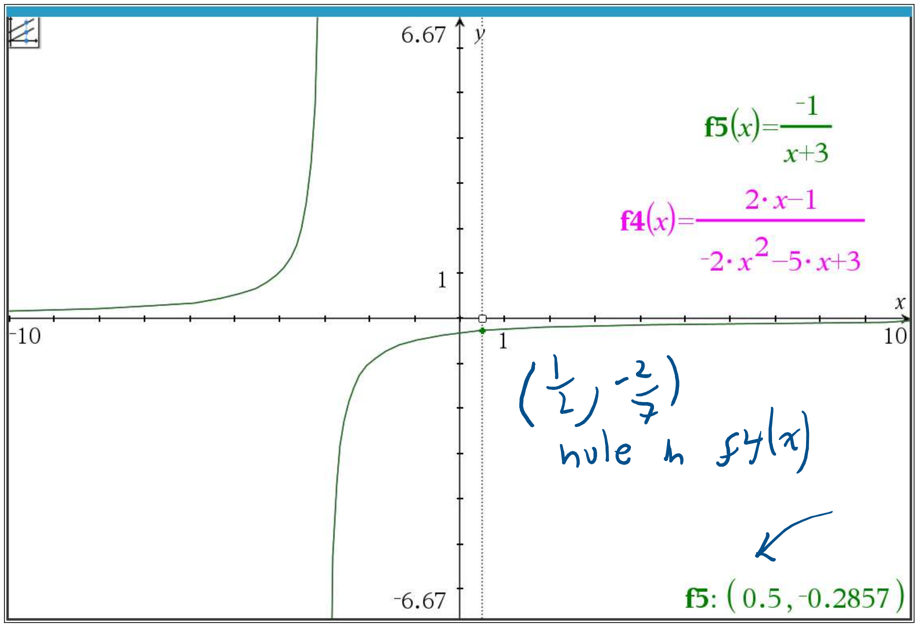
If $x \neq -\frac{1}{2}$, $f(x) = \dfrac{-1}{x+3}$

$x \to -3$ from left $\Rightarrow f(x) = \dfrac{-1}{\text{small neg}} > 0$

$\Rightarrow f(x) \to \infty$

$x \to -3$ from right $\Rightarrow f(x) = \dfrac{-1}{\text{small pos}} < 0$

$\Rightarrow f(x) \to -\infty$



$\left(\frac{1}{2}, -\frac{2}{7}\right)$ hole in $f4(x)$

Trace both $f_4(x)$ vanished

$$f5\left(\tfrac{1}{2}\right) = \frac{-1}{\frac{1}{2}+3} = \frac{-1}{\frac{1}{2}+\frac{6}{2}} = \frac{-1}{\frac{7}{2}} = -\frac{2}{7}$$

Complete the square

$$f(x) = 3x^2 + 7x + 2$$
$$= (3x^2 + 7x) + 2$$
$$= 3\left(x^2 + \frac{7x}{3}\right) + 2$$
$$= 3\left(x^2 + \frac{7x}{3} + \frac{49}{36} - \frac{49}{36}\right) + 2$$
$$= 3\left(x^2 + \frac{7x}{3} + \frac{49}{36}\right) - \frac{(3)(49)}{36} + 2$$
$$= 3\left(x + \frac{7}{6}\right)^2 - \frac{49}{12} + \frac{2(12)}{12}$$
$$= 3\left(x + \frac{7}{6}\right)^2 - \frac{49}{12} + \frac{24}{12}$$

$$\frac{1}{2}\left(\frac{7}{3}\right) = \frac{7}{6}$$
$$\left(\frac{7}{6}\right)^2 = \frac{49}{36}$$

$f(x) = 3\left(x + \frac{7}{6}\right)^2 - \quad 25$

$$\boxed{f(x) = 3\left(x + \frac{7}{6}\right)^2 - \frac{25}{12}}$$

$$f(x) = 3\left(x - \left(-\frac{7}{6}\right)\right)^2 - \frac{25}{12}$$

$$f(x) = a(x - h)^2 + k$$

$$h = -\frac{7}{6}$$

$$k = -\frac{25}{12}$$

$$\text{vertex} = (h, k) = \left(-\frac{7}{6}, -\frac{25}{12}\right)$$

If $z = a + bi$

conjugate of $z = \bar{z} = a - bi$