**4.2 Graphs of Rational Functions**
4.2.1 Exercises
page 333 (345): 5, 14
**4.3 Rational Inequalities and Applications**
4.3.2 Exercises
page 353(365): 2, 5, 13, 23, 25, 38

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**5 Further Topics in Functions**
**5.1 Function Composition**
5.1.1 Exercises
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Exam 2, Thursday, 10/23/25, 2.2 - 2.4, 3.1 - 3.4, 4.1 - 4.3, 5.1

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After class notes

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2.2.1 Exercises

In Exercises 1 - 15, solve the equation.

14. $|x| = 12 - x^2$

$x = 12 - x^2$ or $x = -(12 - x^2)$

$x^2 + x - 12 = 0$ | $x = -12 + x^2$

$(x-3)(x+4) = 0$ | $x^2 - x - 12 = 0$

$x = 3, -4$ | $x = -3, 4$

check: $|-3| \stackrel{?}{=} 12 - (-3)^2$ $\qquad$ $|3| \stackrel{?}{=} 12 - 3^2$

$3 \stackrel{?}{=} 12 - 9$ $\qquad$ $3 \stackrel{?}{=} 12 - 9$

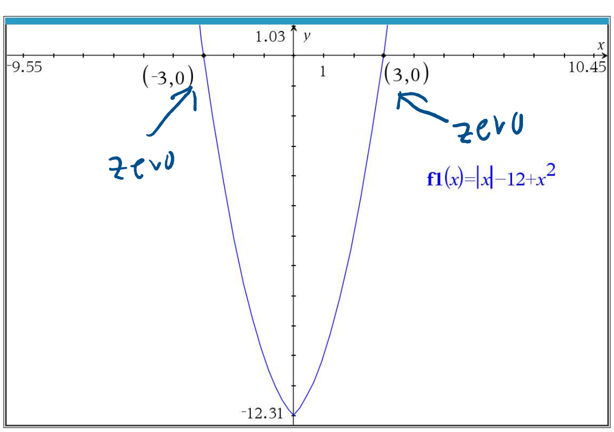
$3 = 3 \checkmark$ $\qquad$ $3 = 3 \checkmark$

$|-4| \stackrel{?}{=} 12 - (4)^2$ $\qquad$ $|4| \stackrel{?}{=} 12 - 4^2$

$4 \stackrel{?}{=} 12 - 16$ $\qquad$ $4 \stackrel{?}{=} 12 - 16$

$4 \neq -4$ $\qquad$ $4 \neq -4$

$x = \pm 4$ are extraneous solution



2.2: 8

8. $\frac{2}{3}|5-2x| - \frac{1}{2} = 5$

$$(6)\left(\frac{2}{3}\right)|5-2x| - 6\left(\frac{1}{2}\right) = (6)(5)$$

$$4|5-2x| - 3 = 30$$

$$4|5-2x| = 33$$

$$|5-2x| = \frac{33}{4}$$

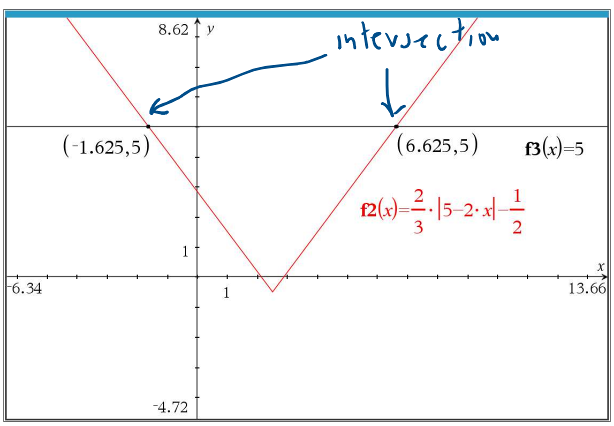
$$5-2x = \frac{33}{4} \quad \text{or} \quad 5-2x = -\frac{33}{4}$$

$$20-8x = 33 \qquad 20-8x = -33$$

$$8x = -13 \qquad 8x = 53$$

$$\boxed{x = -\frac{13}{8}} \qquad \boxed{x = \frac{53}{8}}$$

-13/8=-1.625
53/8=6.625




2.2: 29

In Exercises 22 - 33, graph the function. Find the zeros of each function and the $x$- and $y$-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

29. $f(x) = \dfrac{|2-x|}{2-x}$

$$|2-x| = \begin{cases} 2-x & \text{if } 2-x \geq 0 \iff x \leq 2 \\ -2+x & \text{if } 2-x < 0 \iff x > 2 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{2-x}{2-x} & \text{if } x < 2 \\ \dfrac{-(2-x)}{2-x} & \text{if } x > 2 \end{cases}$$

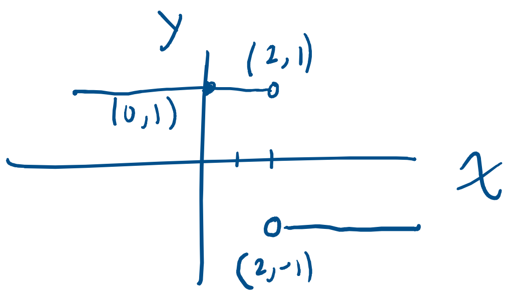
$$1 \quad \text{if } x < 2$$

$\left( -\frac{(2-x)}{2-x} \text{ if } x > 2 \right.$

$$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ -1 & \text{if } x > 2 \\ \text{not defined} & \text{if } x = 2 \end{cases}$$

domain $= (-\infty, 2) \cup (2, \infty)$

range $= \{-1, 1\}$




local and global max $= 1$ on $(-\infty, 2)$

local min $= 1$ on $(-\infty, 2)$

local and global min $= -1$ on $(2, \infty)$

local max $= -1$ on $(2, \infty)$

No zeros

y-intercept $(0,1)$ or $y = 1$

no increase or decrease

constant on $(-\infty, 2)$ and on $(2, \infty)$

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4.2: 5

In Exercises 1 - 16, use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

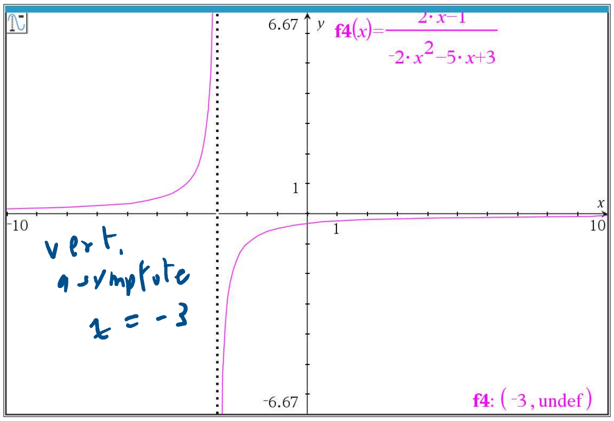
5. $f(x) = \dfrac{2x-1}{-2x^2-5x+3}$

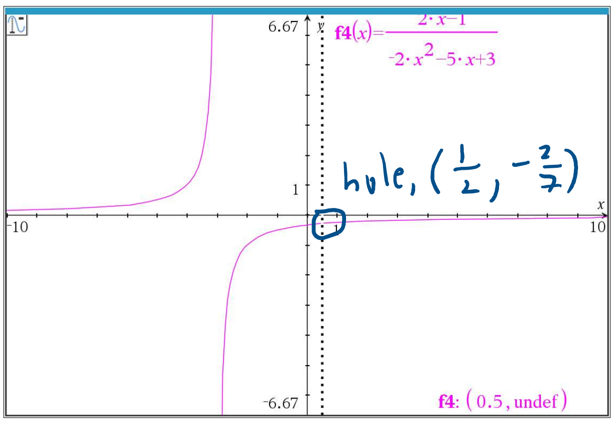
$-2x^2 - 5x + 3$

$= -(2x^2 + 5x - 3)$

$= -(2x-1)(x+3)$

$f(x) = -\dfrac{2x-1}{(2x-1)(x+3)} = -\dfrac{1}{x+3}$




If $x \neq -3$

then $f(x) = -\dfrac{1}{x+3}$

$\Rightarrow f\left(\frac{1}{2}\right) = -\dfrac{1}{\frac{1}{2}+3}$

$= -\dfrac{1}{\frac{1}{2}+\frac{6}{2}} = -\dfrac{1}{\frac{7}{2}}$

$= \boxed{-\frac{2}{7}}$

4.3: 17

In Exercises 7 - 20, solve the rational inequality. Express your answer using interval notation.

17. $\dfrac{-x^3+4x}{x^2-9} \geq 4x$

$$\frac{-x^3+4x}{x^2-9} - 4x \geq 0$$

$$\frac{-x^3+4x}{x^2-9} - \frac{4x(x^2-9)}{x^2-9} \geq 0$$

$$\frac{-x^3+4x-4x^3+36x}{x^2-9} \geq 0$$

$$\frac{-5x^3+40x}{\phantom{x}} \geq 0$$

$$\boxed{\frac{-5x^3+40x}{x^2-9} \geq 0}$$

let $f(x) = \dfrac{-5x^3+40x}{x^2-9}$

solve $f(x) \geq 0$

$$f(x) = \frac{5x(-x^2+8)}{x^2-9}$$

$$f(x) = \frac{5x(-x^2+8)}{(x+3)(x-3)}$$

domain $= \{x \mid x \neq \pm 3\}$

zero $\boxed{x=0}$

$-x^2+8=0$

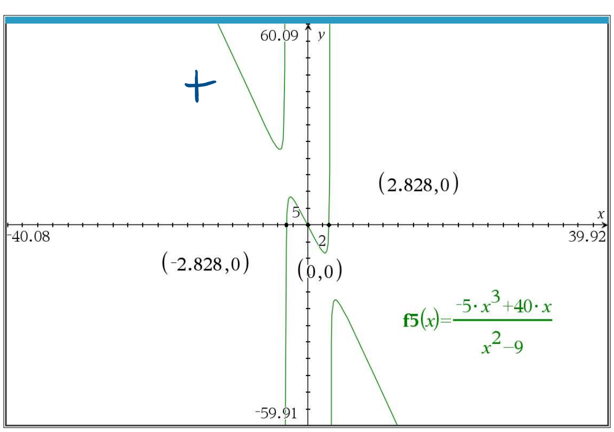
$x^2=8$

$x=\pm\sqrt{8}$

$\boxed{x=\pm 2\sqrt{2}}$

$+ \;\vdots\; + \quad + \quad - \quad + \;\vdots\; -$

$-3 \quad -2\sqrt{2} \quad 0 \quad 2\sqrt{2} \quad 3$

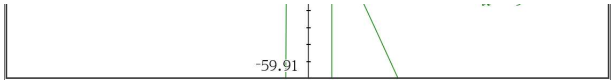


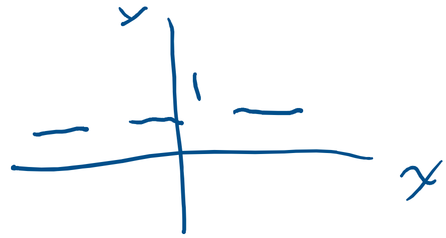

solution set

$(-\infty,-3) \cup [-2\sqrt{2}, 0] \cup (2\sqrt{2}, 3)$

$Y_1(x) \geq 0$

$y_1$

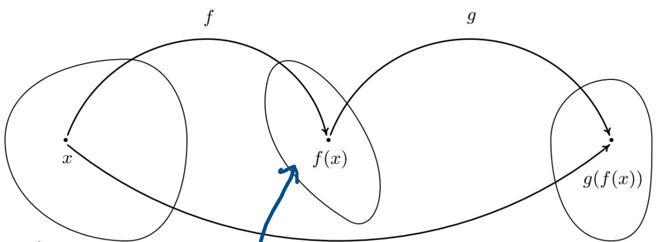



5.1

Memorize

**Definition 5.1.** Suppose $f$ and $g$ are two functions. The **composite** of $g$ with $f$, denoted $g \circ f$, is defined by the formula $(g \circ f)(x) = g(f(x))$, provided $x$ is an element of the domain of $f$ and $f(x)$ is an element of the domain of $g$.




domain of f

domain of g

When finding the domain of a composite function,
look at the unsimplified form.

$$f(x) = \frac{x-4}{x-4} = 1 \quad \text{if } x \neq 4$$

domain of $f = \{x \mid x \neq 4\}$

domain of $g(x) = 1 \quad = \mathbb{R} = (-\infty, \infty)$

Memorize

**Theorem 5.1. Properties of Function Composition:** Suppose $f$, $g$, and $h$ are functions.

- $h \circ (g \circ f) = (h \circ g) \circ f$, provided the composite functions are defined.
- If $I$ is defined as $I(x) = x$ for all real numbers $x$, then $I \circ f = f \circ I = f$.

In general, is $f \circ g = g \circ f$ ?

let $f(x) = 3 + x$

let $g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 3 + \sqrt{x}$

let $g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 3 + \sqrt{x}$

$(g \circ f)(x) = g(f(x)) = g(3 + x) = \sqrt{3+x}$

$3 + \sqrt{x} \neq \sqrt{3+x}$

e.g. $3 + \sqrt{1} \overset{?}{=} \sqrt{3+1}$

$3 + 1 \overset{?}{=} \sqrt{4}$

$4 \neq 2$

$\therefore$ in general $f \circ g \neq g \circ f$

5.1

In Exercises 1 - 12, use the given pair of functions to find the following values if they exist.

- $(g \circ f)(0)$
- $(f \circ g)(-1)$
- $(f \circ f)(2)$
- $(g \circ f)(-3)$
- $(f \circ g)\left(\frac{1}{2}\right)$
- $(f \circ f)(-2)$

2. $f(x) = 4 - x$, $g(x) = 1 - x^2$

$(g \circ f)(0) = g(f(0)) = g(4 - 0) = g(4)$

$= 1 - 4^2 = 1 - 16 = \boxed{-15}$

$(g \circ f)(x) = g(f(x)) = g(4 - x)$

$= 1 - (4 - x)^2$

$\Rightarrow (g \circ f)(0) = 1 - (4 - 0)^2 = 1 - 16 = -15$

a

5.1

In Exercises 13 - 24, use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

- $(g \circ f)(x)$
- $(f \circ g)(x)$
- $(f \circ f)(x)$

23. $f(x) = \dfrac{x}{2x+1}$, $g(x) = \dfrac{2x+1}{x}$

$(g \circ f)(x) = g(f(x))$

$$(g\circ f)(x) = g(f(x))$$

$$= g\left(\frac{x}{2x+1}\right)$$

$$\boxed{(g\circ f)(x) = \frac{2\left(\frac{x}{2x+1}\right)+1}{\frac{x}{2x+1}}}$$

$(g\circ f)(-\frac{1}{2})$ not defined

$(g\circ f)(0)$ not defined

domain of $(g\circ f)(x) = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$

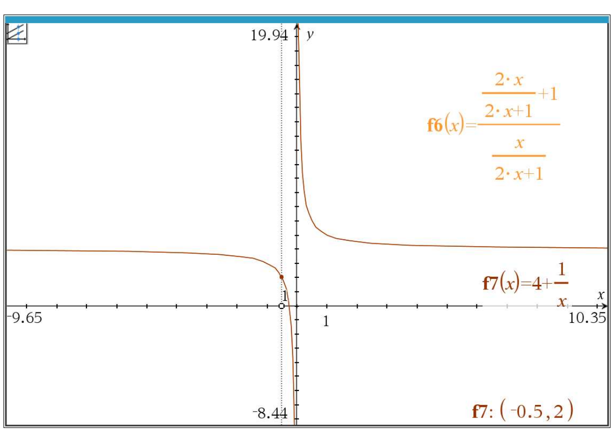
$$(g\circ f)(x) = 2 + \frac{1}{\frac{x}{2x+1}}$$

$$(g\circ f)(x) = 2 + \frac{2x+1}{x}$$

$$= 2 + \frac{2x}{x} + \frac{1}{x}$$

$$= 2 + 2 + \frac{1}{x} \quad \text{for } x \neq 0$$

$$= 4 + \frac{1}{x} \quad \text{for } x \neq 0$$



Your name MTH 167-002N quiz 4

1. Let $f(x) = x^2 - 5x + 6$

write $f(x)$ in vertex form
by completing the square
Find and plot the vertex
calculate and plot the y-intercept
calculate and plot a 3rd point by symmetry.
Graph the parabola by connecting the 3 points

goal $f(x) = a(x-h)^2 + k$, vertex $= (h, k)$

$$\begin{aligned} f(x) &= x^2 - 5x + 6 \\ &= (x^2 - 5x) + 6 \\ &= \left(x^2 - 5x + \tfrac{25}{4} - \tfrac{25}{4}\right) + 6 \\ &= \left(x^2 - 5x + \tfrac{25}{4}\right) - \tfrac{25}{4} + 6 \\ &= \left(x - \tfrac{5}{2}\right)^2 - \tfrac{25}{4} + \tfrac{24}{4} \end{aligned}$$

$\left(\frac{1}{2}\right)(-5) = -\frac{5}{2}$

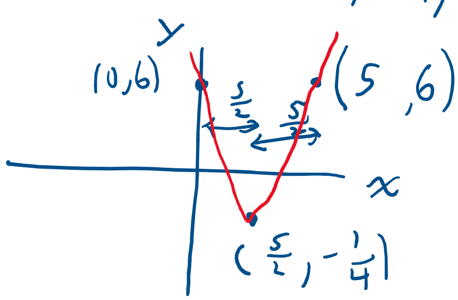
$\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$

$$\boxed{f(x) = \left(x - \tfrac{5}{2}\right)^2 - \tfrac{1}{4}}$$

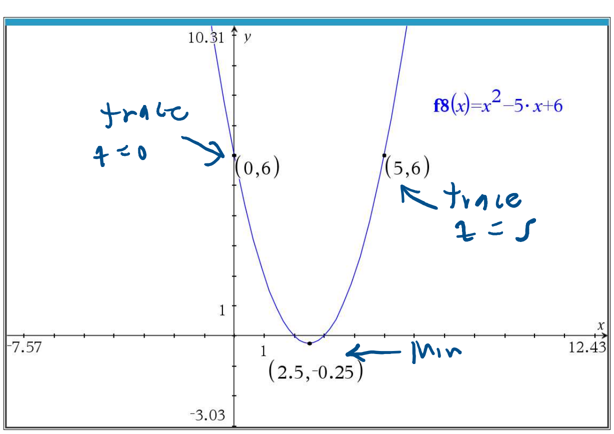
vertex $\left(\frac{5}{2}, -\frac{1}{4}\right)$

y-intercept

$f(0) = 0^2 - (5)(0) + 6 = 6$




calc



$x^2 + \underline{5}x$

want $p^2 + \underline{2}pq + q^2 = (p+q)^2$

$x \leftrightarrow p$

$x^2 + \underline{2q}x + q^2$

$5 = 2q$

$\Rightarrow q = \frac{5}{2} \Rightarrow q^2 = \frac{25}{4}$

motivation for completing the square algorithm

2. Solve analytically $|8x - 4| = 12$

$8x - 4 = 12$

$8x = 16$

$\boxed{x = 2}$

$8x - 4 = -12$

$8x = -8$

$\boxed{x = -1}$

check $|8(2) - 4| \stackrel{?}{=} 12$

$|16 - 4| \stackrel{?}{=} 12$

$|12| \stackrel{?}{=} 12$

$12 = 12$ ✓

$|8(-1) - 4| \stackrel{?}{=} 12$

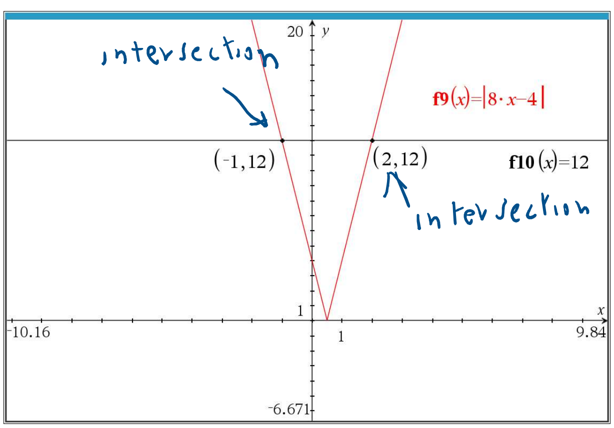
$|-8 - 4| \stackrel{?}{=} 12$

$|-12| \stackrel{?}{=} 12$

$12 = 12$ ✓

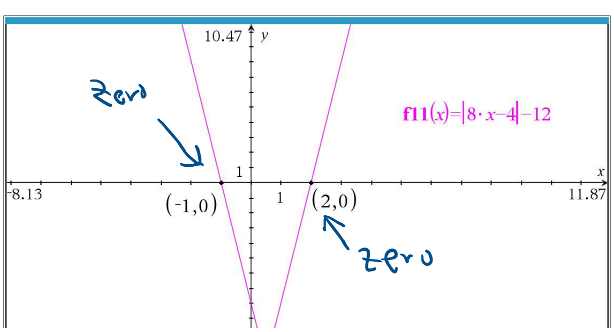
3. Solve graphically and sketch a labeled graph for #2

calc



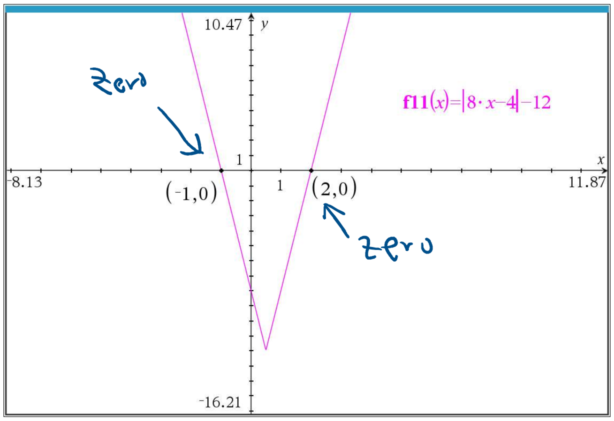

$\boxed{x \approx -1, 2}$

calc




$\boxed{x \approx -1, 2}$

Calc




$\boxed{x \approx -1, 2}$

4. Let $f(x) = \frac{(x-4)(x+3)}{x}$. $Solve \;\; f(x) \geq 0$

Write answer in interval notation.

First find zeros and points not in domain

$f(0)$ not defined

$f(4) = 0 = f(-3)$

$-$   $+$   $-$   $+$

$-3$   $0$   $4$

solution set

$\boxed{[-3,0) \cup [4, \infty)}$

$f(-4) = \frac{(-4-4)(-4+3)}{-4}$

$= \frac{(-)(-)}{(-)} < 0$

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$f(-1) = \frac{(-1-4)(-1+3)}{-1}$

$= \frac{(-)(+)}{(-)} > 0$

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$f(2) = \frac{(2-4)(2+3)}{2}$

$= \frac{(-)(+)}{(+)} < 0$

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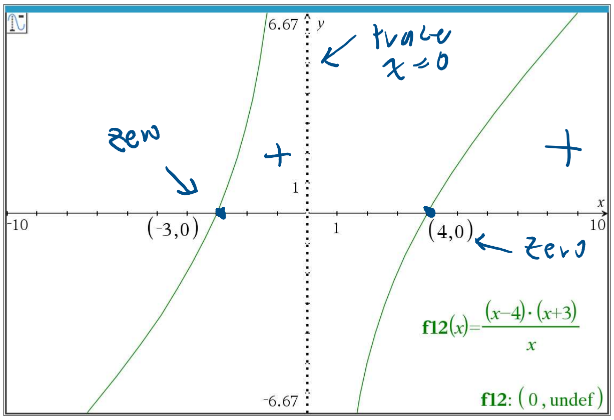
$f(5) = \frac{(5-4)(5+3)}{5}$

$= \frac{(+)(+)}{} > 0$

$$= \frac{(+)^5 (+)}{(+)} > 0$$

calc



6.67
y
trace
x = 0
zero
+
+
1
x
-10
(-3,0)
1
(4,0)
10
zero
f12(x)=(x-4)·(x+3)/x
-6.67
f12: (0, undef)