09-25-25 MTH 167-002N

2.3 Quadratic Functions
  2.3.1 Exercises
  page 200 (212): 1, 4, 11, 21, 26

2.4 Inequalities with Absolute Value and Quadratic Functions
  2.4.1 Exercises
  page 220 (232): 1, 8, 17, 34, 36

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3 Polynomial Functions

3.1 Graphs of Polynomials
  3.1.1 Exercises
  page 246 (258): 3, 7, 13, 21, 27

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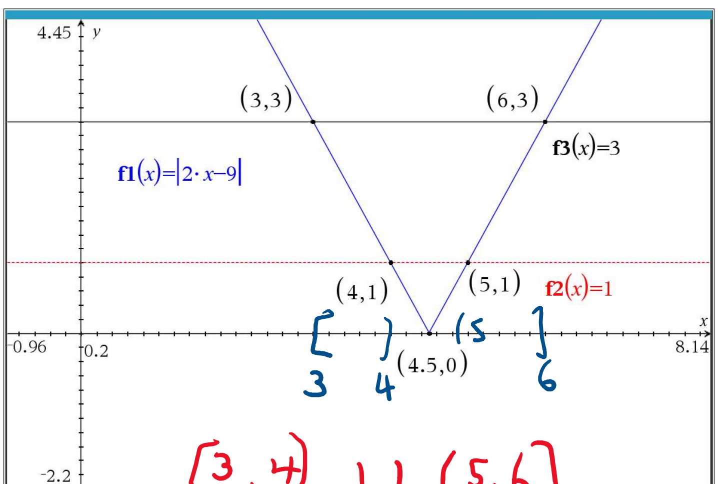
2.4: 8

In Exercises 1 - 32, solve the inequality. Write your answer using interval notation.

8. $1 < |2x - 9| \leq 3$



$[3, 4) \cup (5, 6]$

Is $3 \in$ solution set?

$1 \overset{?}{<} |2 \cdot 3 - 9| \overset{?}{\leq} 3$

$1 \overset{?}{<} |6 - 9| \overset{?}{\leq} 3$

$1 \overset{?}{<} |-3| \overset{?}{\leq} 3$

$1 < 3 \leq 3$ True

$\therefore 3 \in$ solution set

$\therefore 5 \in$ solution set

$1 < |2x - 9|$

$|2x-9| = 2x - 9$

if $2x - 9 \geq 0$

$\iff 2x \geq 9$

$\iff \boxed{x \geq \frac{9}{2}}$

$1 < 2x - 9$

$10 < 2x$

$\boxed{5 < x} \Rightarrow x \geq \frac{9}{2}$

$|2x-9| = -(2x-9)$

if $2x - 9 < 0$

$2x < 9$

$\boxed{x < \frac{9}{2}}$

$1 < -(2x-9)$

$1 < -2x + 9$

$1 + 2x < -2x + 2x + 9$

$1 + 2x < 9$

$2x < 8$

$\boxed{x < 4}$ $\quad 4 < \frac{9}{2}$

or

Do the same for the upper bound.

Quadratic inequalities

$f(x) = (x-4)(x+5)$

$= x^2 + 5x - 4x - 20$

$= x^2 + x - 20$ quadratic function

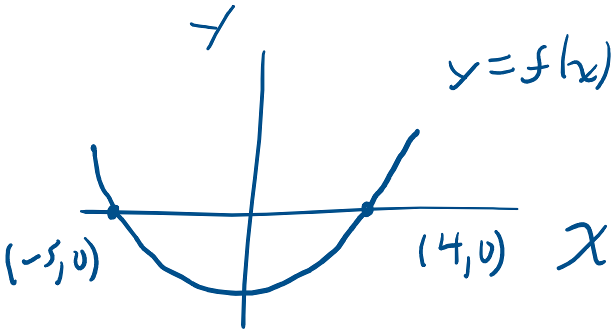
solve $f(x) > 0$

Find zero of $f(x)$

$(x-4)(x+5) = 0$

$x - 4 = 0$	$x + 5 = 0$
$x = 4$	$x = -5$

$x = 4 \quad | \quad x = -5$



$(-\infty, -5) \cup (4, \infty)$

$f(x) = (x-4)(x+5)$ [algebraic method]

solve $f(x) > 0$

$(x-4)(x+5) > 0$ [$ab > 0$

$[x-4>0 \text{ and } x+5>0]$ or $[x-4<0 \text{ and } x+5<0]$

$[x>4 \text{ and } x>-5]$ or $[x<4 \text{ and } x<-5]$

$x>4$

$x<-5$

solution set $= (-\infty, -5) \cup (4, \infty)$

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numeric test value method

$f(x) = (x-4)(x+5) = x^2 + x - 20$

numeric

$f(x) = (x-4)(x+5) = x^2 + x - 20$

Find zeros and plot them

$+ \qquad - \qquad +$

$-5 \qquad 4$

Pick a convenient test value in each interval

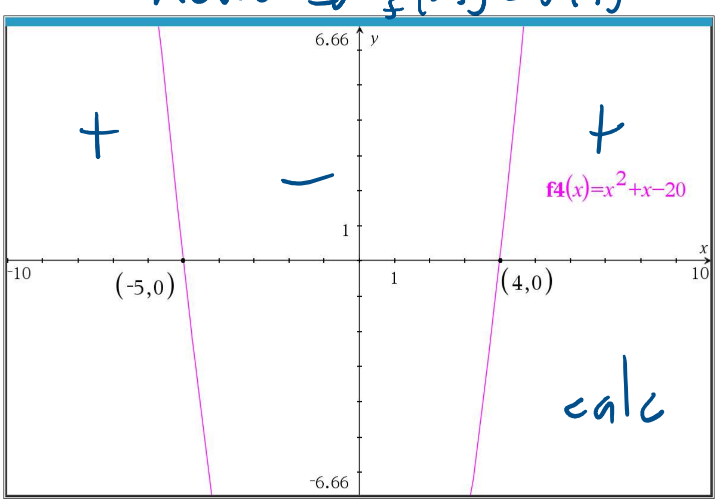
$f(-6) = (-6-4)(-6+5)$
$= (-)(-) > 0$

solution set
$(-\infty, -5) \cup (4, \infty)$

$f(0) = (0-4)(0+5)$
$= (-)(+) < 0$

$f(5) = (5-4)(5+5)$
$= (+)(+) > 0$

Above $\Rightarrow$ $f(-5) = f(4) = 0$



3.1
Memorize

Definition 3.1. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0,$$

where a_0, a_1, $\ldots$, a_n are real numbers and $n \geq 1$ is a natural number. The domain of a polynomial function is $(-\infty, \infty)$.

$0^0 = ?$

Memorize

Definition 3.2. Suppose f is a polynomial function.

$a_0 x^0 = a_0(1) = a_0$

- Given $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$, we say
  - The natural number n is called the **degree** of the polynomial f.
  - The term $a_n x^n$ is called the **leading term** of the polynomial f.
  - The real number a_n is called the **leading coefficient** of the polynomial f.
  - The real number a_0 is called the **constant term** of the polynomial f.
- If $f(x) = a_0$, and $a_0 \neq 0$, we say f has degree 0.
- If $f(x) = 0$, we say f has no degree.[a]

[a] Some authors say $f(x) = 0$ has degree $-\infty$ for reasons not even we will go into.

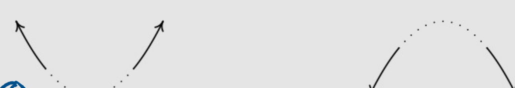
Memorize

End Behavior of functions $f(x) = ax^n$, n even.

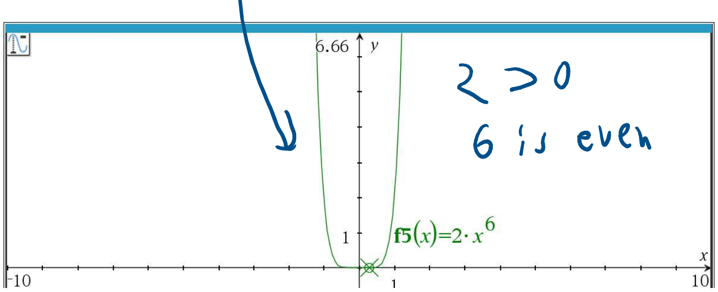
Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and n is an even natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:

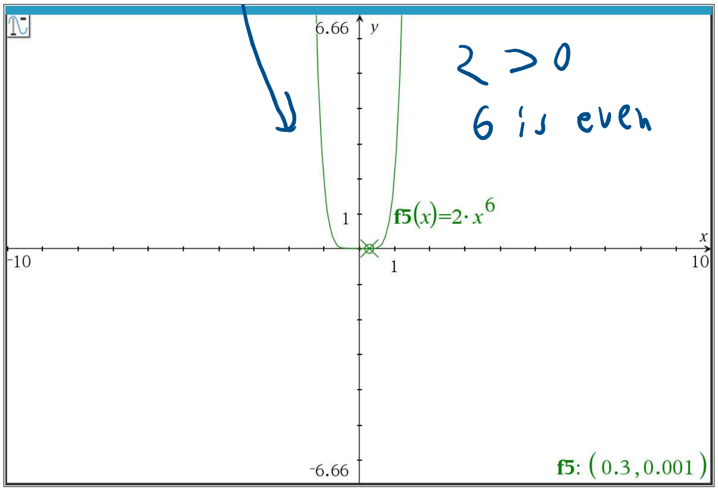


$a > 0$ $\qquad$ $a < 0$



$2 > 0$

6 is even



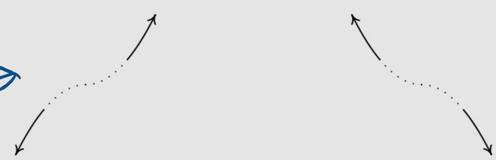
Memorize

End Behavior of functions $f(x) = ax^n$, n odd.

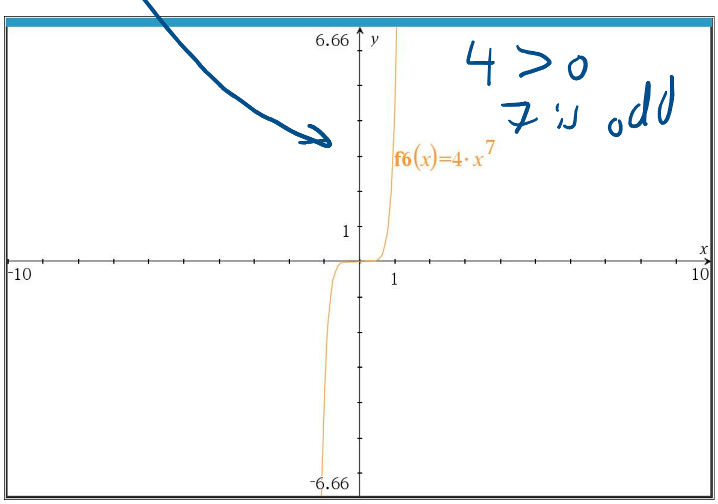
Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and $n \geq 3$ is an odd natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:

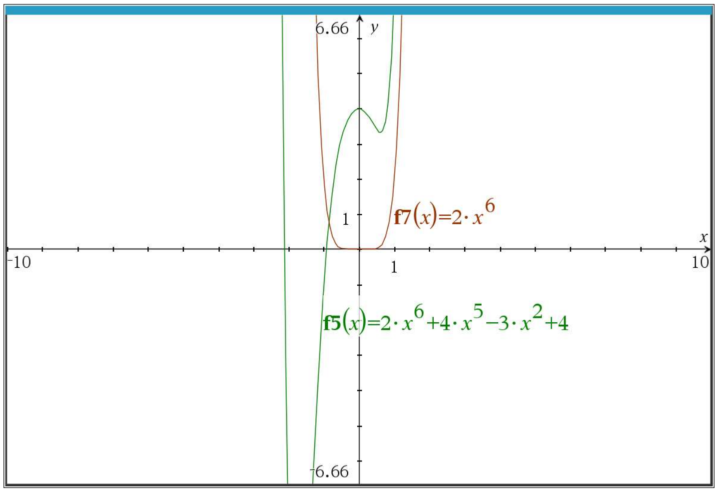


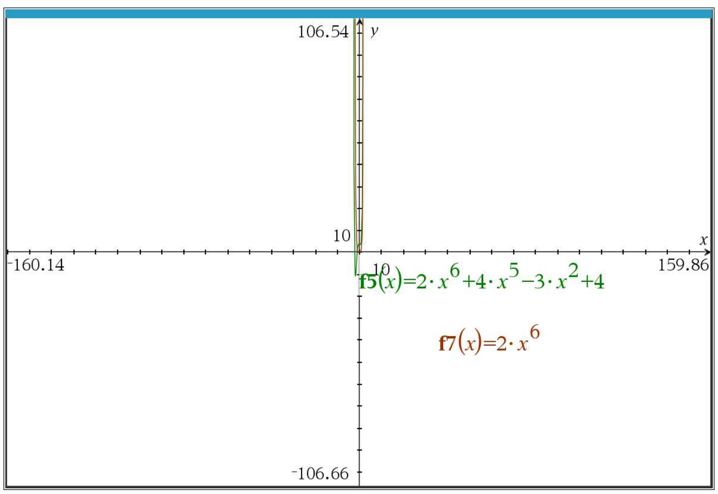
$a > 0$ $\qquad$ $a < 0$



Memorize

Theorem 3.2. End Behavior for Polynomial Functions: The end behavior of a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$ matches the end behavior of $y = a_n x^n$.





Memorize the procedure

Steps for Constructing a Sign Diagram for a Polynomial Function

Suppose f is a polynomial function.

1. Find the zeros of f and place them on the number line with the number 0 above them.
2. Choose a real number, called a **test value**, in each of the intervals determined in step 1.
3. Determine the sign of $f(x)$ for each test value in step 2, and write that sign above the corresponding interval.

Supplied

Theorem 3.1. The Intermediate Value Theorem (Zero Version): Suppose f is a continuous function on an interval containing $x = a$ and $x = b$ with $a < b$. If $f(a)$ and $f(b)$ have different signs, then f has at least one zero between $x = a$ and $x = b$; that is, for at least one real number c such that $a < c < b$, we have $f(c) = 0$.

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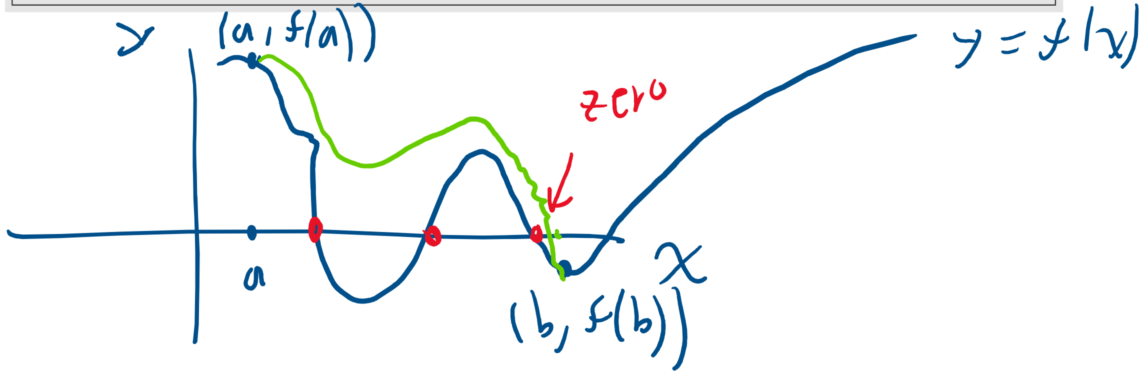
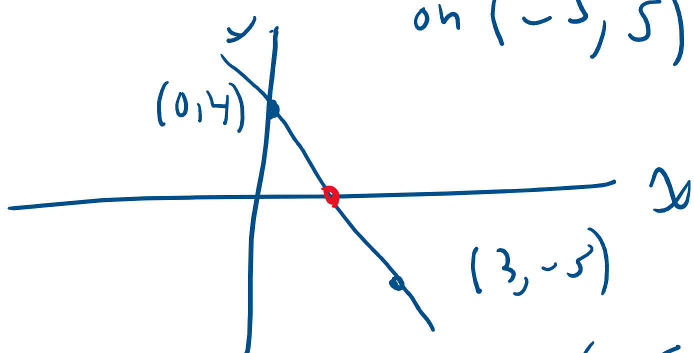


$f(a) > 0, \quad f(b) < 0$

$f(x) = -3x + 4$ on $(-5, 5)$



Is there a zero on $(-5, 5)$?

$f(-5) = -3(-5) + 4 = 19$
$f(5) = -3(5) + 4 = -11$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ opposite signs

$\therefore \exists\, c \in (-5, 5)$ such that $f(c) = 0$

$-3x + 4 = 0$

$3x = 4$

$x = \frac{4}{3} \in (-5, 5)$

Definition 3.3. Suppose f is a polynomial function and m is a natural number. If $(x-c)^m$ is a factor of $f(x)$ but $(x-c)^{m+1}$ is not, then we say $x = c$ is a zero of **multiplicity** m.

$f(x) = (x-2)^3 (x-4)^5$

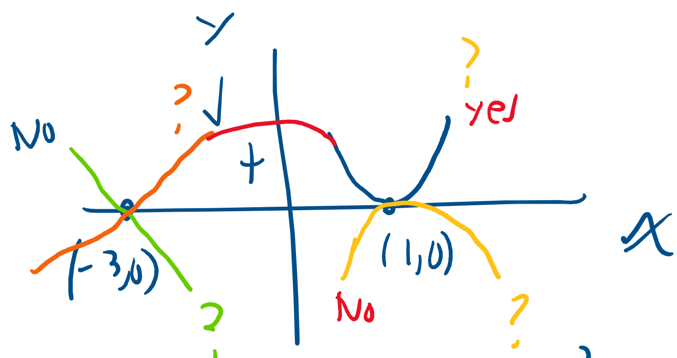
zero	multiplicity
$x=2$	3
$x=4$	5

Memorize

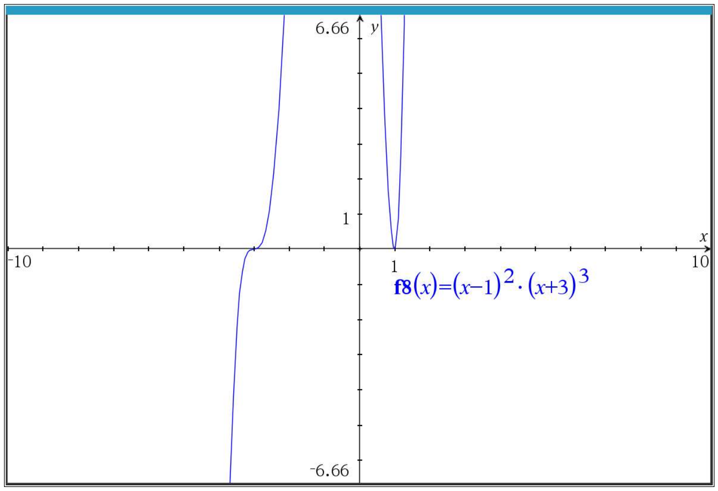
Theorem 3.3. The Role of Multiplicity: Suppose f is a polynomial function and $x = c$ is a zero of multiplicity m.

- If m is even, the graph of $y = f(x)$ touches and rebounds from the x-axis at $(c, 0)$.
- If m is odd, the graph of $y = f(x)$ crosses through the x-axis at $(c, 0)$.

$f(x) = (x-1)^2 (x+3)^3$



$f(0) = (0-1)^2 (0+3)^3$
$= (+)(+)$


6.66
y
1
x
-10
1
10
f8(x)=(x−1)^2·(x+3)^3
-6.66