

2.2 Absolute Value Functions**2.2.1 Exercises**

page 183 (165): 1, 2, 15, 17, 22, 29

2.3 Quadratic Functions**2.3.1 Exercises**

page 200 (212): 1, 4, 11, 21, 26

2.4 Inequalities with Absolute Value and Quadratic Functions**2.4.1 Exercises**

page 220 (232): 1, 8, 17, 34, 36

34 required textbook sections remaining

21 class meetings before final exam

 $34/21=1.619$

1-2 sections per class meetings

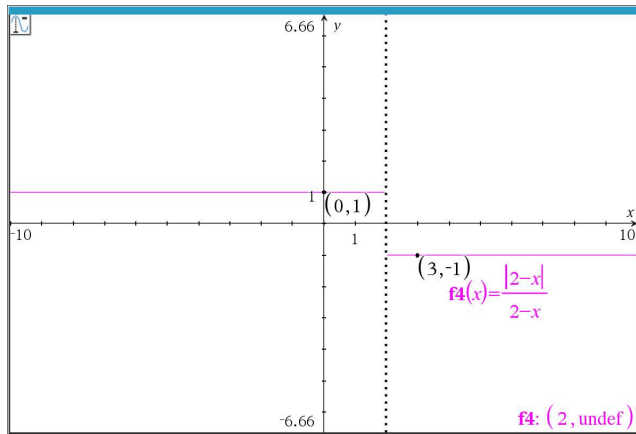
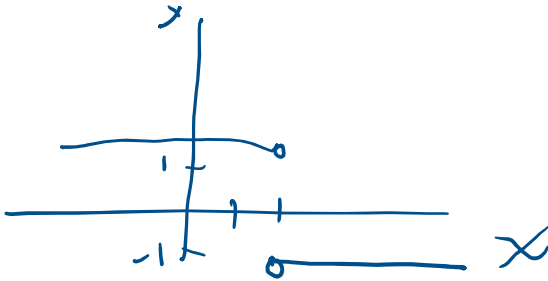
Exam 1

51.08333	mean			A-0
55.5	median	8	5	B-1
19.59361	st. dev	7	1246	C-4
17	min	6	000189	D-6
85	max	5	56	F-13
24	count	4	23349	
		3		
		2	4568	
		1	78	

2.2:29

In Exercises 22 - 33, graph the function. Find the zeros of each function and the x - and y -intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

$$29. f(x) = \frac{|2-x|}{2-x}$$



$$2-x > 0 \\ \Rightarrow |2-x| = 2-x$$

$$2-x \geq 0$$

$$\Rightarrow |2-x| = 2-x$$

$$2-x \geq 0 \Leftrightarrow x \leq 2$$

$$\Rightarrow f(x) = \frac{2-x}{2-x} = 1$$

$$2-x < 0 \Rightarrow |2-x| = -(2-x)$$

$$x > 2$$

$$f(x) = \frac{-(2-x)}{2-x} = -1$$

$$2-x \geq 0$$

$$\Rightarrow |2-x| = 2-x$$

$$2-x \geq 0 \Leftrightarrow x \leq 2$$

$$\Rightarrow f(x) = \frac{2-x}{2-x} = 1$$

$$2-x < 0 \Rightarrow |2-x| = -(2-x)$$

$$x > 2$$

$$f(x) = \frac{-(2-x)}{2-x} = -1$$

increasing - never

decreasing - never

constant on $(-\infty, 2)$ and on $(2, \infty)$

every point on $(-\infty, 2)$ is a local and global max

" " " $(2, \infty)$ is " " " min

2.3

Complete the square

Memorize the algorithm

$$f(x) = 2x^2 + 6x - 5$$

① group x -terms together

$$f(x) = (2x^2 + 6x) - 5$$

② Factor out the coefficient of x^2

$$f(x) = 2(x^2 + 3x) - 5$$

③ Take $\left(\frac{1}{2}\right)$ the coefficient of x

square it

- (3) Take $(\frac{1}{2})$ the coefficient of x
square it
Add and subtract this inside parentheses

$$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) = \frac{3}{4}$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$f(x) = 2\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4}\right) - 5$$

- (4) Take the negative constant
out of the parentheses

$$f(x) = 2\left(x^2 + 3x + \frac{9}{4}\right) - \left(\frac{9}{4}\right)k - 5$$

- (5) factor the perfect square

$$f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} - 5$$

$$\boxed{f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{19}{2}}$$

$$\begin{aligned} -\frac{9}{2} - \frac{10}{2} \\ = -\frac{19}{2} \end{aligned}$$

$$f(x) = 2\left(x - \left(-\frac{3}{2}\right)\right)^2 - \frac{19}{2}$$

$$f(x) = a(x - h)^2 + k$$

$$\text{vertex} = (h, k) = \left(-\frac{3}{2}, -\frac{19}{2}\right)$$

motivation

$$(p + q)^2 = p^2 + 2pq + q^2$$

$$f(x) = 2(x^2 + 3x) - 5$$

$$x = p$$

$$p^2 + 2pq + q^2 \rightarrow x^2 + 2qx + q^2$$

$$2qx = 3x$$

$$q = \frac{3}{2}$$

$$q^2 = \frac{9}{4}$$

Equation 2.4. Vertex Formulas for Quadratic Functions: Suppose a , b , c , h and k are real numbers with $a \neq 0$.

- If $f(x) = a(x - h)^2 + k$, the vertex of the graph of $y = f(x)$ is the point (h, k) .
- If $f(x) = ax^2 + bx + c$, the vertex of the graph of $y = f(x)$ is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

You can use the bottom formula to check the result of completing the square.

Memorize

Equation 2.5. The Quadratic Formula: If a , b and c are real numbers with $a \neq 0$, then the solutions to $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

memorize

Definition 2.7. If a , b and c are real numbers with $a \neq 0$, then the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is the quantity $b^2 - 4ac$.

memorize

Theorem 2.3. Discriminant Trichotomy: Let a , b and c be real numbers with $a \neq 0$.

- If $b^2 - 4ac < 0$, the equation $ax^2 + bx + c = 0$ has no real solutions.
- If $b^2 - 4ac = 0$, the equation $ax^2 + bx + c = 0$ has exactly one real solution.
- If $b^2 - 4ac > 0$, the equation $ax^2 + bx + c = 0$ has exactly two real solutions.

Graph a quadratic function by the following steps.

- 1) Complete the square to find and plot the vertex.
- 2) Calculate and plot the y-intercept
 $y = f(0)$

$$f(x) = ax^2 + bx + c$$

$$\Rightarrow f(0) = a(0^2) + (b)(0) + c$$

$$= c$$

$$\Rightarrow f(0) = a(0) + (b)(0) + c$$

$$f(0) = 0 + 0 + c$$

$$\boxed{f(0) = c}$$

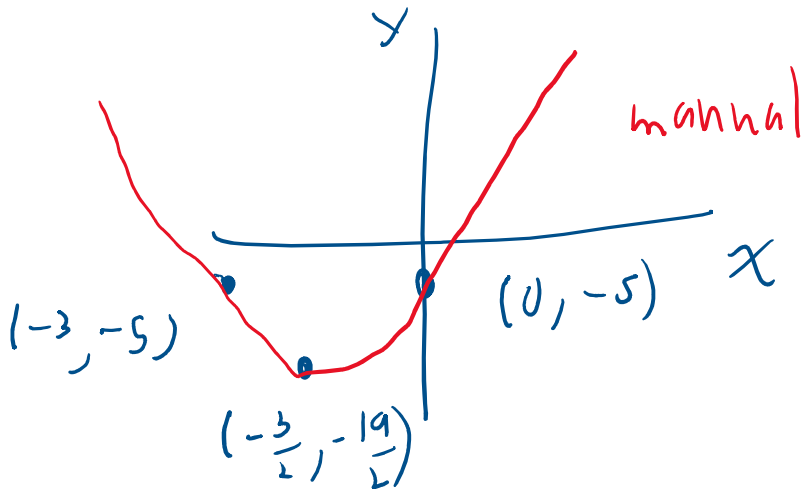
- 3) Find a 3rd point by symmetry with respect to the y-intercept.

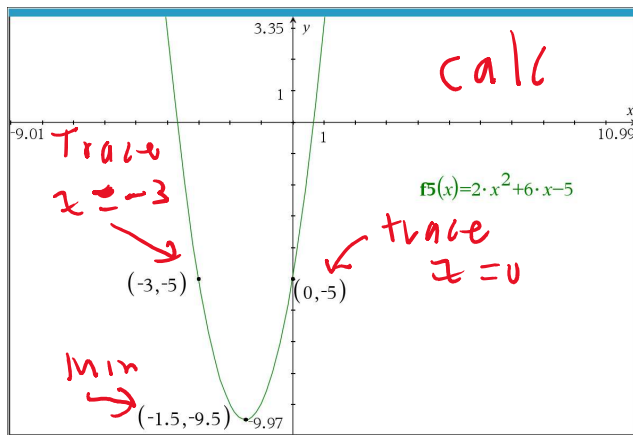
$$f(x) = 2x^2 + 6x - 5$$

$$f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{19}{2}$$

$$\text{vertex} = \left(-\frac{3}{2}, -\frac{19}{2}\right)$$

$$y\text{-intercept} = f(0) = -5$$





2.4

Graphical Interpretation of Equations and Inequalities

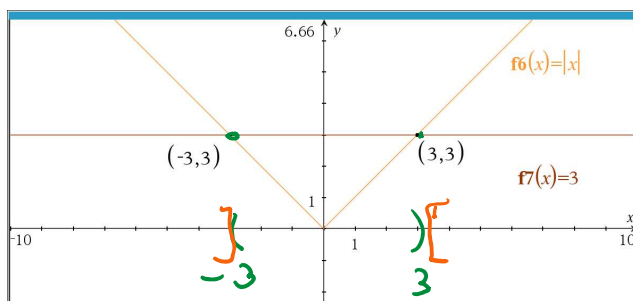
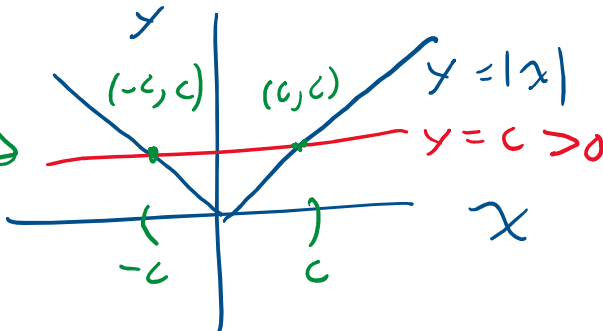
Suppose f and g are functions.

- The solutions to $f(x) = g(x)$ are the x values where the graphs of $y = f(x)$ and $y = g(x)$ intersect.
- The solution to $f(x) < g(x)$ is the set of x values where the graph of $y = f(x)$ is *below* the graph of $y = g(x)$.
- The solution to $f(x) > g(x)$ is the set of x values where the graph of $y = f(x)$ is *above* the graph of $y = g(x)$.

Memorize

Theorem 2.4. Inequalities Involving the Absolute Value: Let c be a real number.

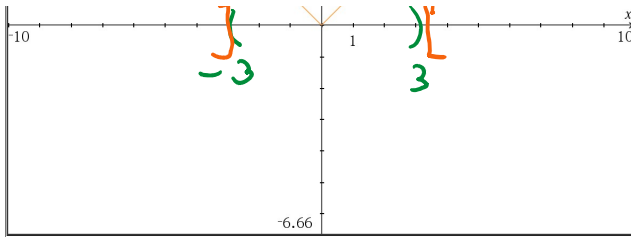
- For $c > 0$, $|x| < c$ is equivalent to $-c < x < c$.
- For $c > 0$, $|x| \leq c$ is equivalent to $-c \leq x \leq c$.
- For $c \leq 0$, $|x| < c$ has no solution, and for $c < 0$, $|x| \leq c$ has no solution.
- For $c \geq 0$, $|x| > c$ is equivalent to $x < -c$ or $x > c$.
- For $c \geq 0$, $|x| \geq c$ is equivalent to $x \leq -c$ or $x \geq c$.
- For $c < 0$, $|x| > c$ and $|x| \geq c$ are true for all real numbers.



$$|x| < 3 \quad (-3, 3)$$

$$\sqrt{|x| = 3}$$

$$x = \pm 3$$

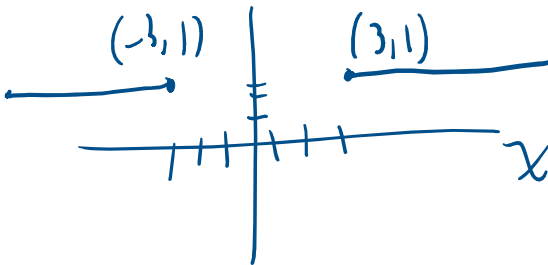


$$|x| > 3$$

$$|x| > 3$$

$$(-\infty, -3) \cup (3, \infty)$$

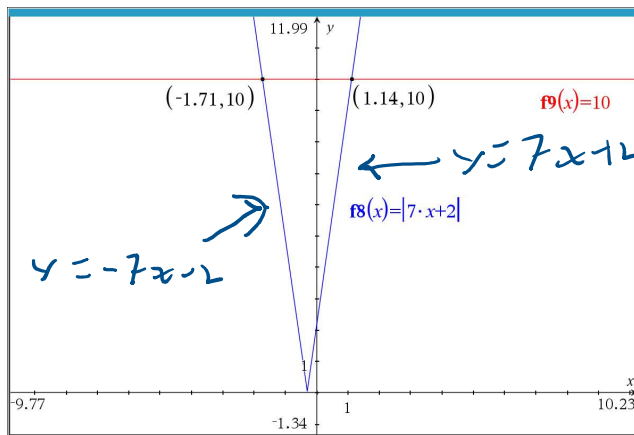
$$TI \quad y = |x| > 3$$



2.4

In Exercises 1 - 32, solve the inequality. Write your answer using interval notation.

2. $|7x + 2| > 10$



approximation

$$(-\infty, -1.71) \cup (1.14, \infty)$$

$$7x + 2 \geq 0 \Rightarrow |7x + 2| = 7x + 2$$

$$7x \geq -2$$

$$x \geq -\frac{2}{7}$$

