09-16-25 MTH 167-002N

1.7 Transformations

1.7.1 Exercises

page 140 (152): 1,4, 19, 24, 48, 56

2 Linear and Quadratic Functions

2.1 Linear Functions

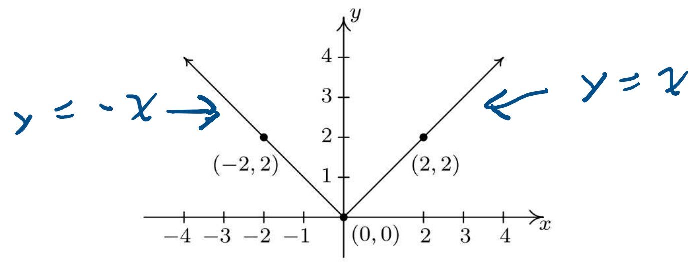
2.1.1 Exercises

page 163 (175): 5, 7, 15, 18, 26, 28, 32, 35, 45

Exam 1: 1.1-1.7, 2.1

1.7: 24

The complete graph of $y = f(x)$ is given below. In Exercises 19 - 27, use it and Theorem 1.7 to graph the given transformed function.



The graph for Ex. 19 - 27

24. $y = f(2x)$

Theorem 1.7. Transformations. Suppose f is a function. If $A \neq 0$ and $B \neq 0$, then to graph

$$g(x) = Af(Bx + H) + K$$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the left if $H > 0$ or right if $H < 0$.
2. Divide the x-coordinates of the points on the graph obtained in Step 1 by B. This results in a horizontal scaling, but may also include a reflection about the y-axis if $B < 0$.
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if $A < 0$.
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift up if $K > 0$ or down if $K < 0$.

$A = 1$ $\quad$ $H = 0$

$B = 2 \qquad k = 0$

$(-2,2) \qquad (0,0) \qquad (2,2)$

1. $(-2-0, 2) \qquad (0-0, 0) \qquad (2-0, 2)$

   $\boxed{(-2,2)} \qquad \boxed{(0,0)} \qquad \boxed{(2,2)}$

2. $\left(-\frac{2}{2}, 2\right) \qquad \left(\frac{0}{2}, 0\right) \qquad \left(\frac{2}{2}, 2\right)$

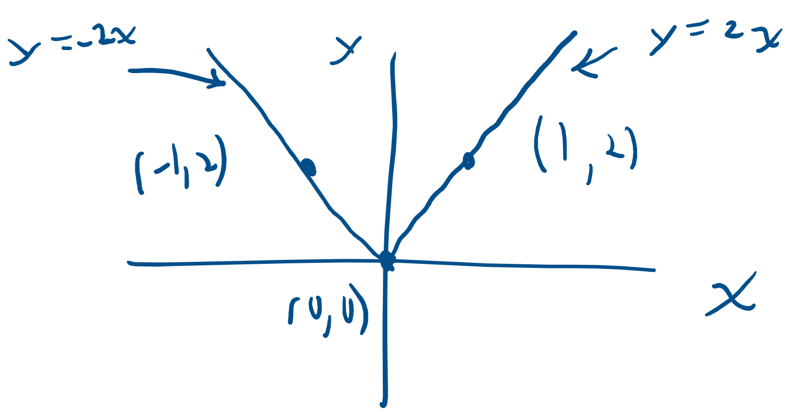
   $\boxed{(-1,2)} \qquad \boxed{(0,0)} \qquad \boxed{(1,2)}$

3. $(-1, (2)(1)) \qquad (0, (0)(1)) \qquad (1, (2)(1))$

   $\boxed{(-1,2)} \qquad \boxed{(0,0)} \qquad \boxed{(1,2)}$

4. $(-1, 2+0) \qquad (0, 0+0) \qquad (1, 2+0)$

   $\boxed{(-1,2)} \qquad \boxed{(0,0)} \qquad \boxed{(1,2)}$



Assume we have two straight lines. Find their equations.

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$g(x) = f(2x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$$

2.1: 35

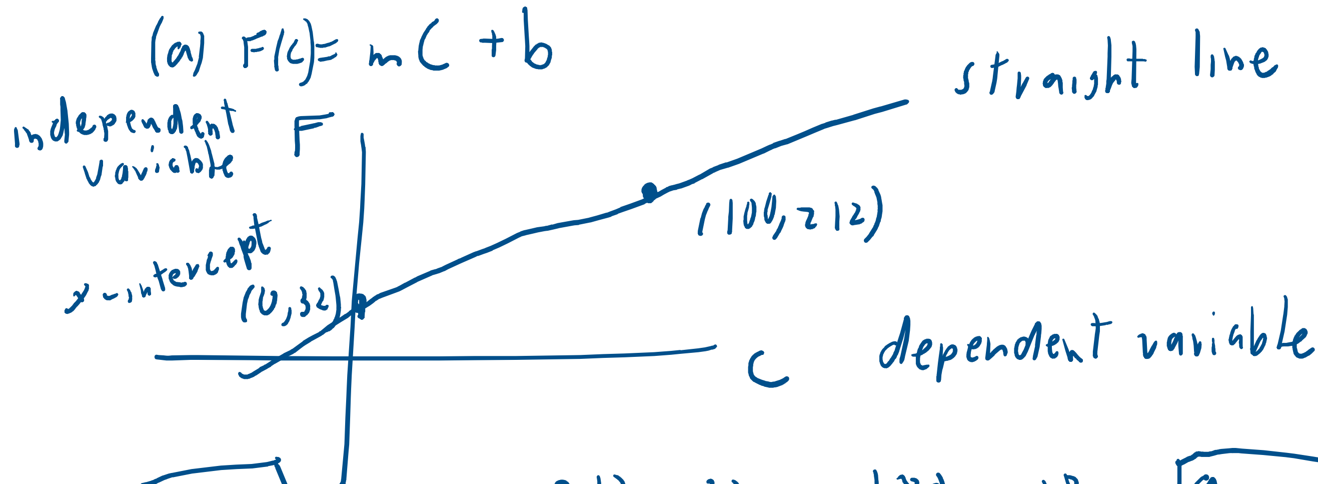
35. Water freezes at 0° Celsius and 32° Fahrenheit and it boils at 100°C and 212°F.

(a) Find a linear function F that expresses temperature in the Fahrenheit scale in terms of degrees Celsius. Use this function to convert 20°C into Fahrenheit.

(b) Find a linear function C that expresses temperature in the Celsius scale in terms of degrees Fahrenheit. Use this function to convert 110°F into Celsius.

(c) Is there a temperature n such that $F(n) = C(n)$?

(a) $F(C) = mC + b$



$\boxed{b = 32}$ $\quad m = \dfrac{212 - 32}{100 - 0} = \dfrac{180}{100} = \dfrac{18}{10} = \boxed{\dfrac{9}{5} = m}$

$$\boxed{F(C) = \frac{9C}{5} + 32}$$

Good notation

$$\frac{9C}{5} = \left(\frac{9}{5}\right)C$$

$9/5\,C$ bad notation

$$F(20) = \frac{(9)(20)}{5} + 32$$

$$= 36 + 32$$

$$\boxed{F(20°C) = 68°F}$$

... a linear function of F

b) Find a C as a linear function of F

$F = \frac{9C}{5} + 32$

solve for C

$5F = 9c + (5)(32)$

$9c = 5F - (5)(32)$

$\boxed{C = \frac{5(F-32)}{9}}$

(c) $F(n) = C(n)$ solve for n, if it exists

$F(n) = \frac{9n}{5} + 32$

$C(n) = \frac{5(n-32)}{9}$

$\frac{9n}{5} + 32 = \frac{5(n-32)}{9}$

solve for n

$\left(\frac{9n}{5}\right)(45) + (32)(45) = \frac{5}{9}(n-32)(45)$

32*45=1,440
25*32=800
2240/56=40

$81n + 1440 = 25(n-32)$

$(81-25)n = 25(-32) - 1440$

$56n = -800 - 1440$

$56n = -2240$

$$56n = -2240$$

$$n = \frac{-2240}{56}$$

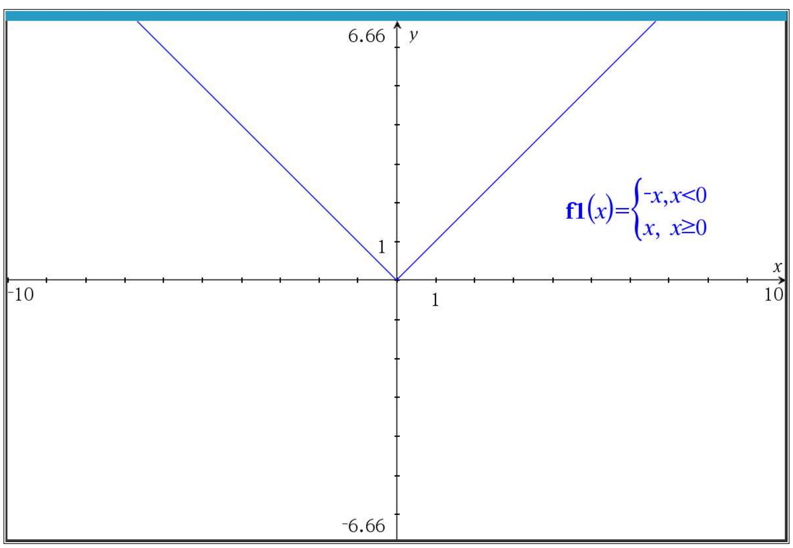
$$\boxed{n = -40}$$

35. (a) $F(C) = \frac{9}{5}C + 32$ (b) $C(F) = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$

(c) $F(-40) = -40 = C(-40)$.

TI-83, TI-84

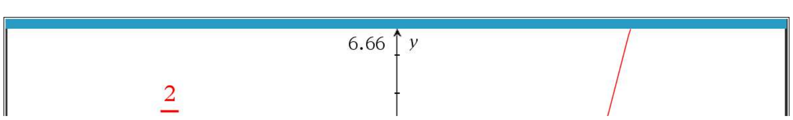
$$Y_1 = (-x)*(x<0) + (x)(x \geq 0)$$



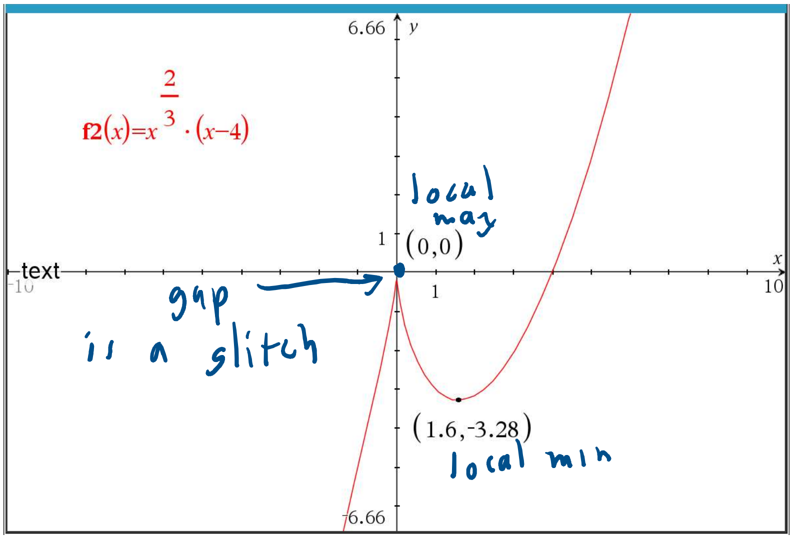
1.6: 75

In Exercises 74 - 77, use your graphing calculator to approximate the local and absolute extrema of the given function. Approximate the intervals on which the function is increasing and those on which it is decreasing. Round your answers to two decimal places.

75. $f(x) = x^{2/3}(x - 4)$



no global max



no global max
no global min

increasing $(-\infty, 0), (1.6, \infty)$

decreasing $(0, 1.6)$

1.4: 2

In Exercises 1 - 10, find an expression for $f(x)$ and state its domain.

2. f is a function that takes a real number x and performs the following three steps in the order given: (1) add 3; (2) multiply by 2; (3) divide by 4.

$$\frac{2(x+3)}{4} = \boxed{\frac{x+3}{2} = f(x)}$$

no division of 0
no square root of negative number
$\therefore$ domain of $f = (-\infty, \infty)$
$= \{x \mid x \in \mathbb{R}\}$
$= \{x \mid -\infty < x < \infty\}$

the set of all x (singular variable) such that x belongs to the set of all real numbers

such that x belongs to the
set of all real numbers

$$\{x\} \neq x$$

$$x \in \{x\}$$

$$f(x) = x$$

$$g(x) = x$$

$$h(x) = \frac{f(x)}{g(x)}$$

Find domain of h

$$h(x) = \frac{x}{x} = 1 \text{ if } x \neq 0$$

domain of $h = \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

When finding the domain of a compound or
composite function, look at the unsimplified form.

1.5

In Exercises 46 - 50, $C(x)$ denotes the cost to produce x items and $p(x)$ denotes the price-demand function in the given economic scenario. In each Exercise, do the following:

- Find and interpret $C(0)$.
- Find and interpret $p(5)$
- Find and simplify $P(x)$.
- Find and interpret $\overline{C}(10)$.
- Find and simplify $R(x)$.
- Solve $P(x) = 0$ and interpret.

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

47. The cost, in dollars, to produce x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is $C(x) = 10x + 100$, $x \geq 0$ and the price-demand function, in dollars per bottle, is $p(x) = 35 - x$, $0 \leq x \leq 35$.

$C(0) = 10(0) + 100 = 100$

The fixed cost is \$100

$\overline{C}(10) = \frac{C(10)}{10} = \frac{10(10)+100}{10} = \frac{200}{10} = \boxed{20}$

The average cost of producing 10 bottles is \$20

$p(5) = 35 - 5 = 30$

The cost per bottle is \$30 when producing 5 bottles

$R(x) = x\,p(x) = x(35 - x)$

$\boxed{R(x) = -x^2 + 35x}$

$P(x) = R(x) - C(x)$

$P(x) = R(x) - C(x)$

$= (-x^2 + 35x) - (10x + 100)$

$= -x^2 + 35x - 10x - 100$

$$\boxed{P(x) = -x^2 + 25x - 100}$$

$P(x) = 0$

$-x^2 + 25x - 100 = 0$

$x^2 - 25x + 100 = 0$

$(x - 20)(x - 5) = 0$

$$\boxed{x = 20, 5}$$

We break-even if we produce 5 or 20 bottles of tonic.