09-11-25 MTH 167-002N

1.6 Graphs of Functions
  1.6.2 Exercises
  page 107 (119): 1, 7, 9, 14, 21, 24, 32, 75

1.7 Transformations
  1.7.1 Exercises
  page 140 (152): 1,4, 19, 24, 48, 56

2 Linear and Quadratic Functions

2.1 Linear Functions
  2.1.1 Exercises
  page 163 (175): 5, 7, 15, 18, 26, 28, 32, 35, 45

6*1.6=9.6 we are behind 1 section

Exam 1, Thursday, 09/18/25, 1.1 - 1.7, 2.1

1.6:75

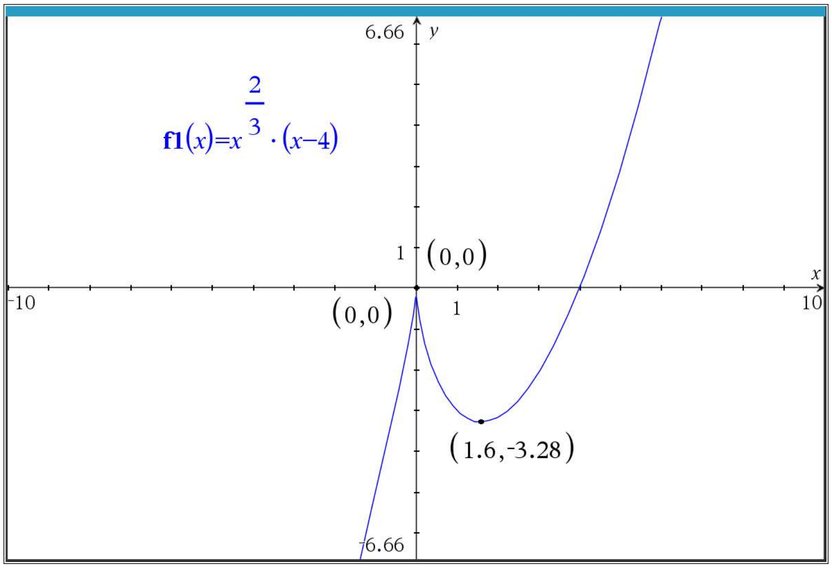
In Exercises 74 - 77, use your graphing calculator to approximate the local and absolute extrema of the given function. Approximate the intervals on which the function is increasing and those on which it is decreasing. Round your answers to two decimal places.

75. $f(x) = x^{2/3}(x-4)$

TI- 83

x ^ (2/3) x (x - 4)

TI-nspire



local max $(0,0)$

local min $(1.6, -3.28)$

TI-84 local max (9.0116E-7, - 3.732E-4)
local min (1.59999997, ?)
(1.6000011,-3.28154)

$9.001\ E{-7} = 9.001 \times 10^{-7}$

$= \frac{9}{10 \text{ million}} \approx 0$

$0.\underline{00}0000009 \approx 0.00$

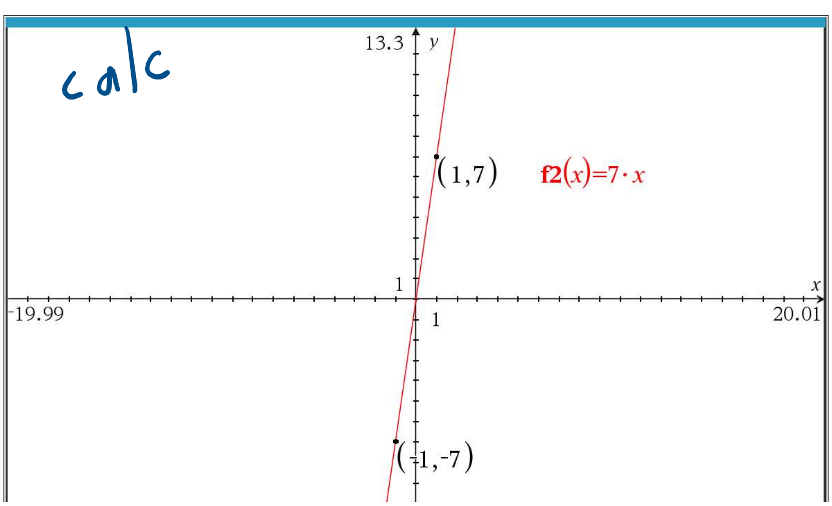
1.6: 21

In Exercises 21 - 41, determine analytically if the following functions are even, odd or neither.

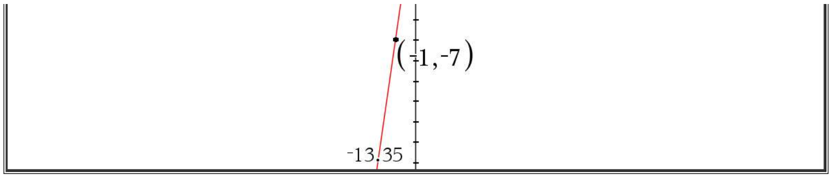
21. $f(x) = 7x$

$f(-x) = 7(-x) = -7x = -f(x)$

$\therefore$ f is odd



Graph supports, but does not prove our conclusion



1.7

Memorize

Theorem 1.2. Vertical Shifts. Suppose f is a function and k is a positive number.

- To graph $y = f(x) + k$, shift the graph of $y = f(x)$ up k units by adding k to the y-coordinates of the points on the graph of f.
- To graph $y = f(x) - k$, shift the graph of $y = f(x)$ down k units by subtracting k from the y-coordinates of the points on the graph of f.

Memorize

Theorem 1.3. Horizontal Shifts. Suppose f is a function and h is a positive number.

- To graph $y = f(x + h)$, shift the graph of $y = f(x)$ left h units by subtracting h from the x-coordinates of the points on the graph of f.
- To graph $y = f(x - h)$, shift the graph of $y = f(x)$ right h units by adding h to the x-coordinates of the points on the graph of f.

Memorize

Theorem 1.4. Reflections. Suppose f is a function.

- To graph $y = -f(x)$, reflect the graph of $y = f(x)$ across the x-axis by multiplying the y-coordinates of the points on the graph of f by -1.
- To graph $y = f(-x)$, reflect the graph of $y = f(x)$ across the y-axis by multiplying the x-coordinates of the points on the graph of f by -1.

Memorize

Theorem 1.5. Vertical Scalings. Suppose f is a function and $a > 0$. To graph $y = af(x)$, multiply all of the y-coordinates of the points on the graph of f by a. We say the graph of f has been vertically scaled by a factor of a.

- If $a > 1$, we say the graph of f has undergone a vertical stretching (expansion, dilation) by a factor of a.
- If $0 < a < 1$, we say the graph of f has undergone a vertical shrinking (compression, contraction) by a factor of $\frac{1}{a}$.

Memorize

Theorem 1.6. Horizontal Scalings. Suppose f is a function and $b > 0$. To graph $y = f(bx)$, divide all of the x-coordinates of the points on the graph of f by b. We say the graph of f has been horizontally scaled by a factor of $\frac{1}{b}$.

- If $0 < b < 1$, we say the graph of f has undergone a horizontal stretching (expansion, dilation) by a factor of $\frac{1}{b}$.
- If $b > 1$, we say the graph of f has undergone a horizontal shrinking (compression, contraction) by a factor of b.

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Supplied

Theorem 1.7. Transformations. Suppose f is a function. If $A \neq 0$ and $B \neq 0$, then to graph

$$g(x) = Af(Bx + H) + K$$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the left if $H > 0$ or right if $H < 0$.
2. Divide the x-coordinates of the points on the graph obtained in Step 1 by B. This results in a horizontal scaling, but may also include a reflection about the y-axis if $B < 0$.
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if $A < 0$.
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift up if $K > 0$ or down if $K < 0$.

$f(x)$

$g(x) = 3f(2x+1) - 4$

Assume the point $(5, 8)$ is on the graph $y = f(x)$

Find the point on the graph of $y = g(x)$

Use theorem 1.7

$A = 3$ $\quad$ $H = 1$

$B = 2$ $\quad$ $K = -4$

$(5, 8)$

③ $(2, (8)(3))$

① $(5-1, 8)$

$\boxed{(4, 8)}$ $\quad$ $\boxed{(2, 24)}$

① $(5-1, 8)$

$\boxed{(4, 8)}$

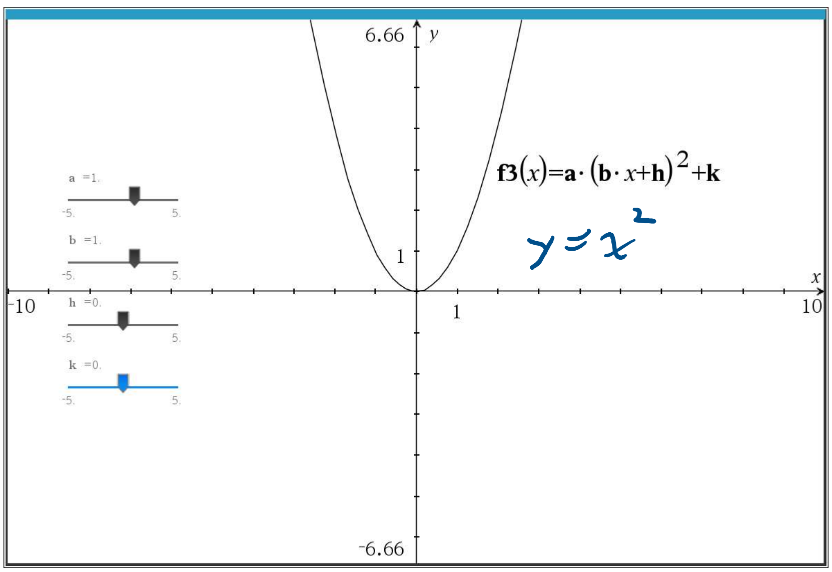
$\boxed{(2, 24)}$

② $\left(\frac{4}{2}, 8\right)$

$\boxed{(2, 8)}$

④ $(2, 24-4)$

$\boxed{(2, 20)}$ on $y = g(x)$



2.1
Memorize

Equation 2.1. The **slope** m of the line containing the points $P(x_0, y_0)$ and $Q(x_1, y_1)$ is:

$$m = \frac{y_1 - y_0}{x_1 - x_0},$$

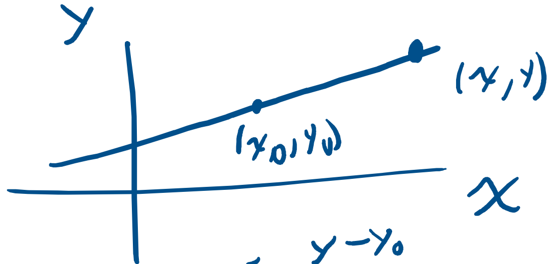
provided $x_1 \neq x_0$.

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

Memorize

Equation 2.2. The **point-slope form** of the line with slope m containing the point (x_0, y_0) is the equation $y - y_0 = m(x - x_0)$.

This is a direct consequence of Equation 2.1
Note: Here (x , y) is a variable point on the line.



$$m = \frac{y - y_0}{x - x_0}$$

$$\Leftrightarrow \quad y - y_0 = m(x - x_0)$$

Memorize

Equation 2.3. The **slope-intercept form** of the line with slope m and y-intercept $(0, b)$ is the equation $y = mx + b$.

Rewrite the point-slope equation as a slope-intercept equation. What are m and b now?

$$y - y_0 = m(x - x_0)$$

$$y = mx - mx_0 + y_0$$

$$y = mx + (-mx_0 + y_0)$$

slope $= m$

y-intercept $= b = -mx_0 + y_0$

Find the point-slope equation of the line with slope $= 3$ and passing through the point $(1, 5)$.

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 5 = 3(x - 1)}$$ solve for y

change to slope-intercept equation

$$y = 5 + 3x - 3 = (5 - 3) + 3x$$
$$= 2 + 3x$$

$y = 5 + 3x - 3$

$= 2 + 3x$

$= 3x + 2$

$\boxed{y = 3x + 2}$

$m = 3, \quad b = 2$

Memorize

Definition 2.1. A **linear function** is a function of the form

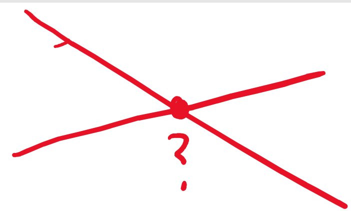
$$f(x) = mx + b,$$

where m and b are real numbers with $m \neq 0$. The domain of a linear function is $(-\infty, \infty)$.

$5x - y = 3$

$2x + y = 10$

intersection ?



Memorize

Definition 2.2. A **constant function** is a function of the form

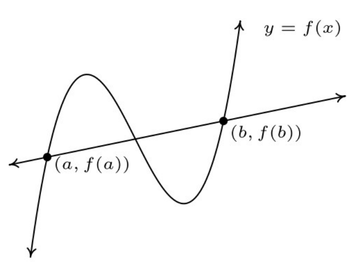
$$f(x) = b,$$

where b is real number. The domain of a constant function is $(-\infty, \infty)$.

Memorize

Definition 2.3. Let f be a function defined on the interval $[a, b]$. The **average rate of change** of f over $[a, b]$ is defined as:

$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$



The graph of $y = f(x)$ and its secant line through $(a, f(a))$ and $(b, f(b))$

2.1

In Exercises 44 - 49, compute the average rate of change of the function over the specified interval.

44. $f(x) = x^3$, $[-1, 2]$

average rate of change on $[-1, 2]$

is $\frac{\Delta f}{\Delta x} = \frac{f(2) - f(-1)}{2-(-1)}$

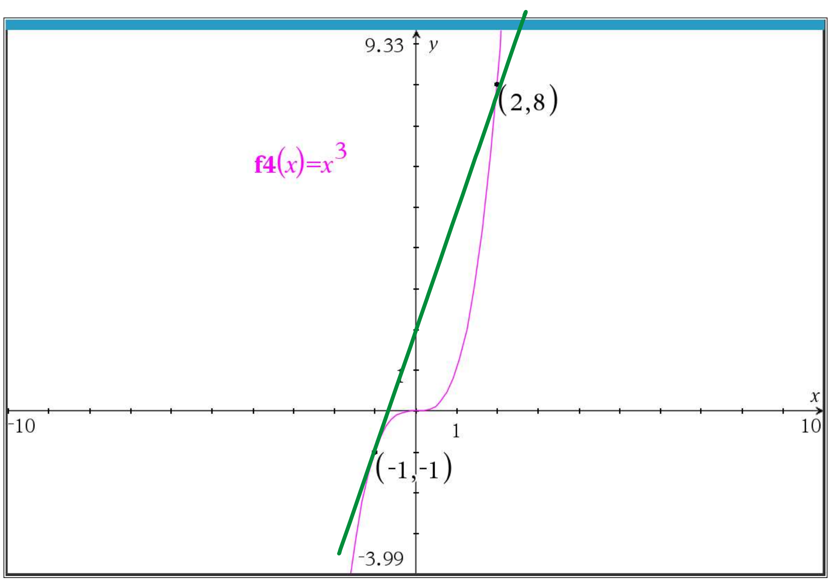
$= \frac{2^3 - (-1)^3}{2+1}$

$= \frac{8 - (-1)}{3}$

$= \frac{8+1}{3}$

$= \frac{9}{3}$

$\boxed{\frac{\Delta f}{\Delta x} = 3}$



Find slope-intercept equation of this secant line

$y = mx + b$

$m = 8-(-1) \quad = 9 \quad = 3$

$y = mx + b$

$m = \frac{8-(-1)}{2-(-1)} = \frac{9}{3} = 3$

$y = 3x + b$

$-1 = 3(-1) + b$

$-1 = -3 + b$

$b = 3 - 1$

$\boxed{b = 2}$

$\boxed{y = 3x + 2}$

point-slope equation

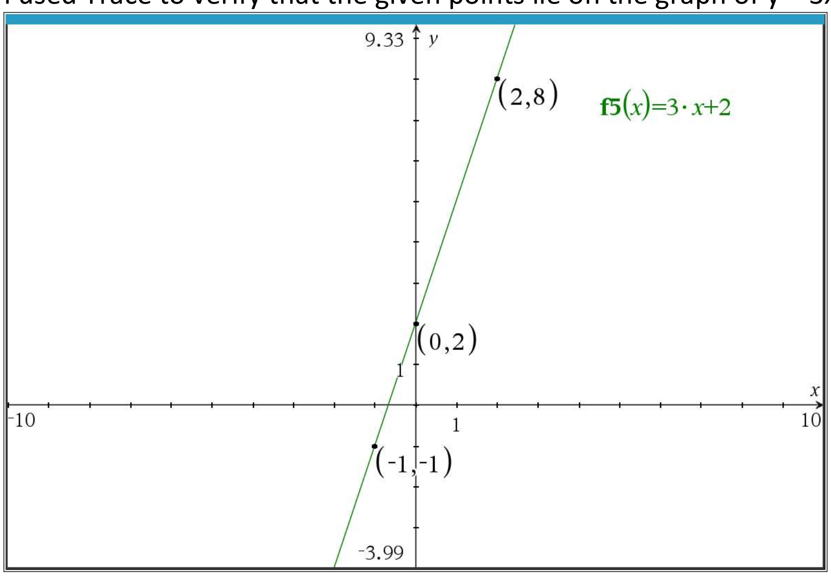
$y - (-1) = 3(x - (-1))$

$y + 1 = 3(x + 1)$

$y + 1 = 3x + 3$

$\boxed{y = 3x + 2}$

Use calculator to check my linear equation.
I graphed my linear equation.
I used Trace to verify that the given points lie on the graph of y = 3x+2.



$\{x \mid x \geq -5 \text{ and } x \neq 2\}$

$= [-5, 2) \cup (2, \infty)$

for a precalculus class Can you give me an example of a function? For which we have to find the domain. But don't give me the domain. Just give me one hint.

For a precalculus class, here is an example of a function where you would need to find the domain:

$$f(x) = \frac{\sqrt{x+5}}{x-2}$$

Hint: The domain of a function is the set of all possible input values (x) for which the function is defined.

To help you find the domain, what part of the function do you think you should focus on first?

the denominator

For the function $f(x) = \frac{\sqrt{x+5}}{x-2}$, the denominator is $x - 2$.

What do you think is a key rule about the denominator of a fraction that helps determine the domain?

It cannot be zero

That's exactly right. The denominator of a fraction can never be equal to zero. Therefore, for the function $f(x) = \frac{\sqrt{x+5}}{x-2}$, we know that $x - 2 \neq 0$, which means $x \neq 2$.

Now that you've considered the denominator, what part of the function do you think you should analyze next to find the rest of the domain?

the numerator.

That's correct. The numerator is $\sqrt{x+5}$.

Now, thinking about the rules of real numbers, what can you tell me about the expression under a square root sign? What kind of values must it be?

non-negative

That's correct. The expression under a square root must be non-negative, which means it must be greater than or equal to zero.

For our function, this gives us the inequality $x + 5 \geq 0$.

Now you have two conditions for the domain:

1. From the denominator: $x \neq 2$
2. From the numerator: $x + 5 \geq 0$

What is the next step to find the complete domain of the function?

solve the inequality, giving x greater than or equal to -5, and say x not equal to 2

You have correctly solved the inequality for the numerator, finding that $x \geq -5$, and you have correctly identified that $x \neq 2$ from the denominator.

Now, to find the complete domain of the function, you need to combine these two conditions. How would you write the final domain using interval notation?

$[-5,2) \cup (2,\infty)$ I pasted a screen shot of this into Gemini.

$[-5,2) \cup (2,\infty)$

Analysis

That is the correct domain for the function $f(x) = \frac{\sqrt{x+5}}{x-2}$.

You have successfully combined the two conditions:

- The first interval, $[-5, 2)$, represents all numbers greater than or equal to -5, up to (but not including) 2.
- The second interval, $(2, \infty)$, represents all numbers greater than 2.
- The union symbol, $\cup$, correctly joins these two parts to show the complete set of valid inputs.

Would you like to try another example with a different type of function to practice finding the domain?