09-09-25 MTH 167-002N

1.5 Function Arithmetic

1.5.1 Exercises

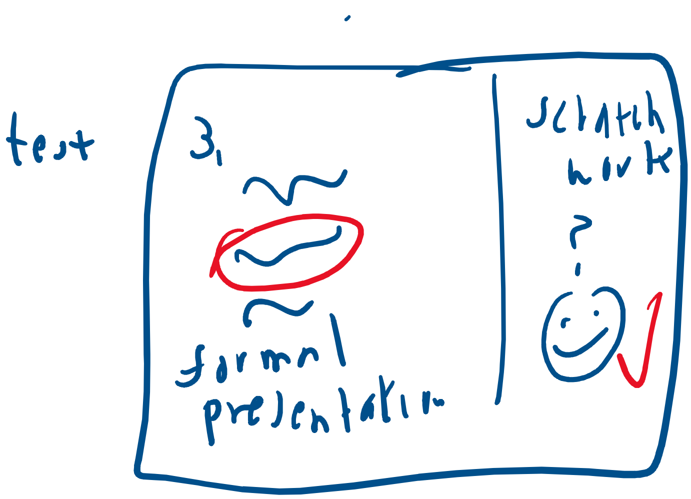
page 84 (96): 1, 11, 17, 21, 23, 25, 46, 57

5*1.6=8 sections by today to be on track

1.6 Graphs of Functions

1.6.2 Exercises

page 107 (119): 1, 7, 9, 14, 21, 24, 32, 75



1.5:46

In Exercises 46 - 50, $C(x)$ denotes the cost to produce x items and $p(x)$ denotes the price-demand function in the given economic scenario. In each Exercise, do the following:

- Find and interpret $C(0)$.
- Find and interpret $\overline{C}(10)$.
- Find and interpret $p(5)$
- Find and simplify $R(x)$.
- Find and simplify $P(x)$.
- Solve $P(x) = 0$ and interpret.

46. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, $0 \leq x \leq 15$.

$$c(0) = 2(0) + 26 = 26$$

\$26 is the start-up cost

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

$$\overline{C}(10) = \frac{C(10)}{10} = \frac{2(10)+26}{10} = \frac{20+26}{10} = \frac{46}{10}$$

$$\boxed{\overline{C}(10) = 4.6}$$

The average cost for producing 10 T-shirts is \$4.60

$$p(5) = 30 - 2(5) = 30 - 10 = 20$$

If 5 T-shirts are produced, the price per T-shirt is \$20.

$$R(x) = x\,p(x) = x(30 - 2x) = 30x - 2x^2$$

$$P(x) = R(x) - C(x)$$

$$= 30x - 2x^2 - (2x + 26)$$

$$= 30x - 2x^2 - 2x - 26$$

$$\boxed{P(x) = -2x^2 + 28x - 26}$$

Solve $P(x) = 0$ and interpret.

$$-2x^2 + 28x - 26 = 0$$

$$\frac{-2x^2}{-2} + \underline{28x} - \underline{26} = \frac{0}{-2}$$

$$\frac{-2x^2}{-2} + \frac{28x}{-2} - \frac{26}{-2} = \frac{0}{-2}$$

$$x^2 - 14x + 13 = 0$$

$$(x-1)(x-13) = 0$$

$$\boxed{x = 1, 13}$$

If 1 or 13 T-shirts are produced, then the profit is zero. That is, we break even.

1.5

In Exercises 21 - 45, find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the given function.

25. $f(x) = -x^2 + 2x - 1$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[-(x+h)^2 + 2(x+h) - 1\right] - \left[-x^2 + 2x - 1\right]}{h}$$

$$= \frac{-(x^2 + 2xh + h^2) + 2x + 2h - 1 + x^2 - 2x + 1}{h}$$

$$= \frac{-\cancel{x^2} - 2xh - h^2 + \cancel{2x} + 2h \cancel{-1} + \cancel{x^2} \cancel{-2x} \cancel{+1}}{h}$$

$$= \frac{-2xh - h^2 + 2h}{h}$$

$$= \frac{\cancel{h}(-2x - h + 2)}{\cancel{h}}$$

$= \cancel{h}(-2x - h + 2)$

$\overline{\qquad\cancel{h}\qquad}$

$$\boxed{\frac{\Delta f}{\Delta x} = -2x - h + 2}$$

1.6

Memorize

The Fundamental Graphing Principle for Functions

The graph of a function f is the set of points which satisfy the equation $y = f(x)$. That is, the point (x, y) is on the graph of f if and only if $y = f(x)$.

Is the point $(3, 7)$ on the graph

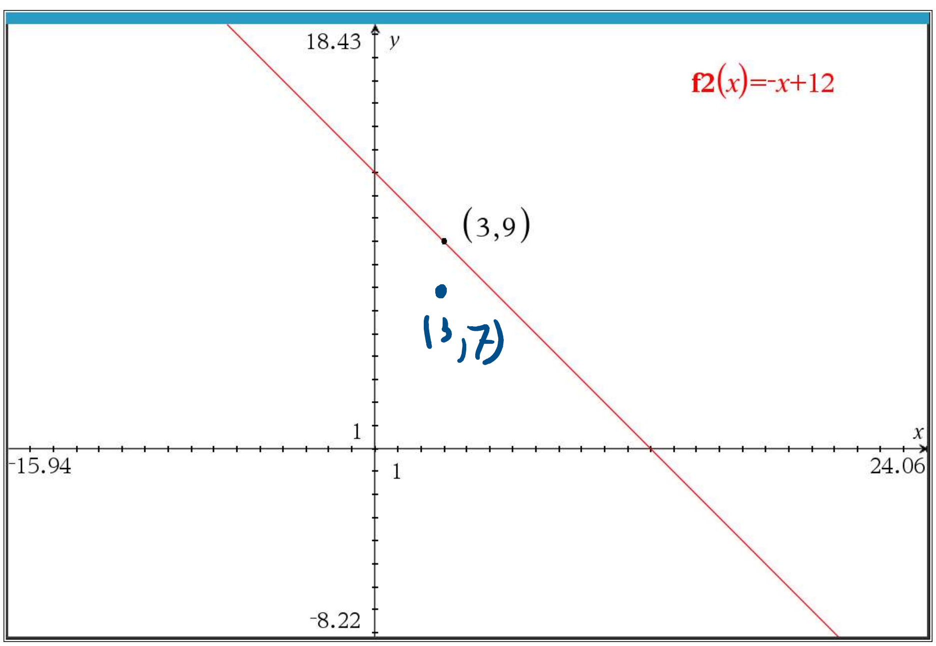
$y = -x + 12$

? yes

maybe NO ✓

$7 \overset{?}{=} -3 + 12$

$7 \neq 9$



Memorize

Definition 1.9. The **zeros** of a function f are the solutions to the equation $f(x) = 0$. In other words, x is a zero of f if and only if $(x, 0)$ is an x-intercept of the graph of $y = f(x)$.

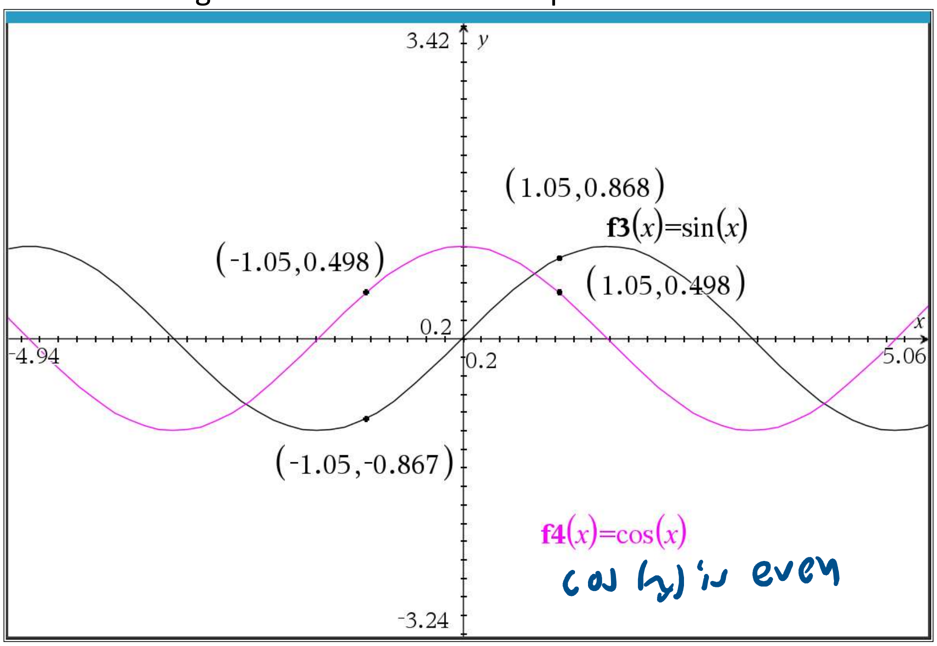
Memorize

Testing the Graph of a Function for Symmetry

The graph of a function f is symmetric

- about the y-axis if and only if $f(-x) = f(x)$ for all x in the domain of f. even
- about the origin if and only if $f(-x) = -f(x)$ for all x in the domain of f. odd

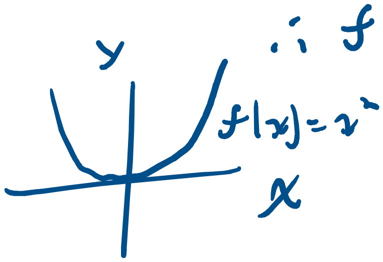
We know that the sine function is odd, so the graph is misleading because -0.868 is not equal to -0.867.



$f(x) = x^2$ is f even or odd

$f(-x) = (-x)^2 = x^2 = f(x)$

$\therefore f$ is even

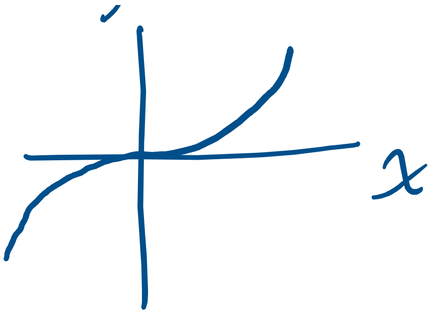


even

$f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

odd



$f(-x) = (-x)^3 = -x^3 = -f(x)$

$\therefore$ f is odd

Memorize

> **Definition 1.10.** Suppose f is a function defined on an interval I. We say f is:
>
> - **increasing** on I if and only if $f(a) < f(b)$ for all real numbers a, b in I with $a < b$.
> - **decreasing** on I if and only if $f(a) > f(b)$ for all real numbers a, b in I with $a < b$.
> - **constant** on I if and only if $f(a) = f(b)$ for all real numbers a, b in I.

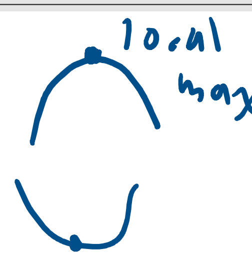
Memorize

> **Definition 1.11.** Suppose f is a function with $f(a) = b$.
>
> - We say f has a **local maximum** at the point (a, b) if and only if there is an open interval I containing a for which $f(a) \geq f(x)$ for all x in I. The value $f(a) = b$ is called 'a local maximum value of f' in this case.
> - We say f has a **local minimum** at the point (a, b) if and only if there is an open interval I containing a for which $f(a) \leq f(x)$ for all x in I. The value $f(a) = b$ is called 'a local minimum value of f' in this case.
> - The value b is called the **maximum** of f if $b \geq f(x)$ for all x in the domain of f.
> - The value b is called the **minimum** of f if $b \leq f(x)$ for all x in the domain of f.

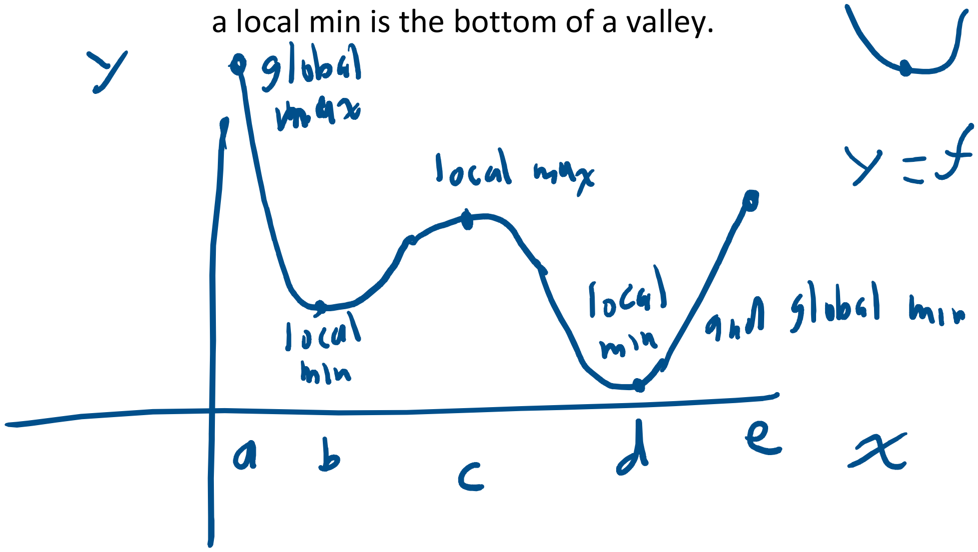
relative

global
absolute

Informal definition: a local max is the top of a small hill,

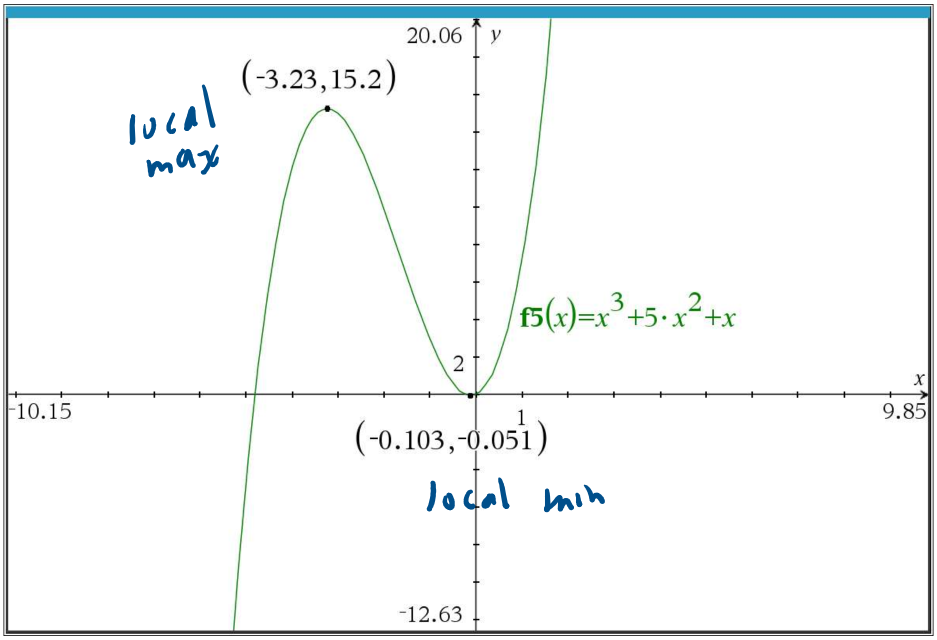


a local min is the bottom of a valley.



$y = f(x)$

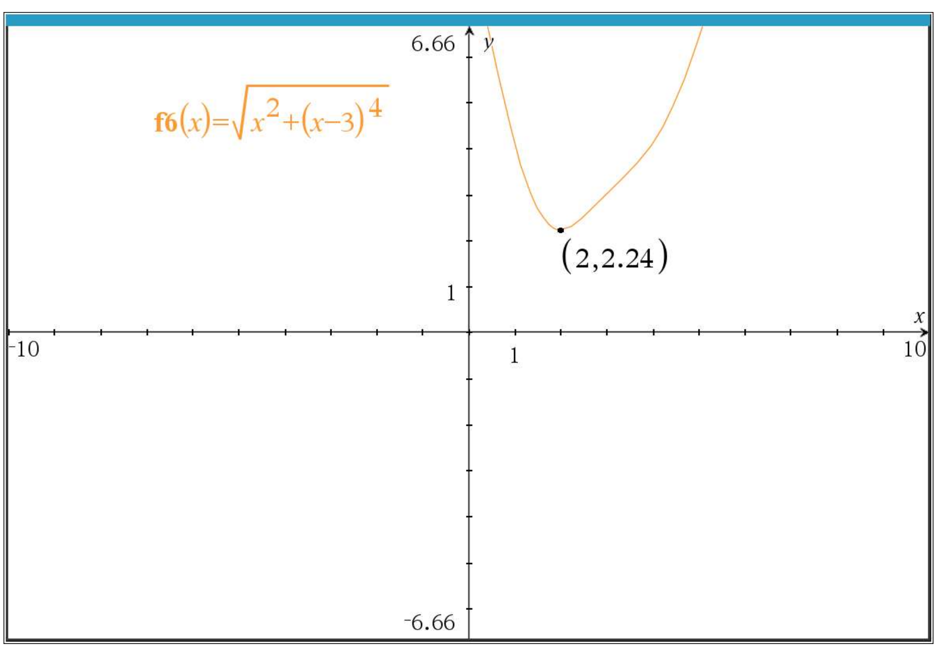
calc



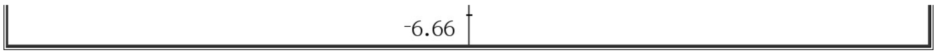
Example 1.6.6. Find the points on the graph of $y = (x-3)^2$ which are closest to the origin. Round your answers to two decimal places.

Solution. Suppose a point (x, y) is on the graph of $y = (x-3)^2$. Its distance to the origin $(0,0)$ is given by

$$\begin{aligned} d &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \\ &= \sqrt{x^2 + [(x-3)^2]^2} \qquad \text{Since } y = (x-3)^2 \\ &= \sqrt{x^2 + (x-3)^4} \end{aligned}$$



x = 2 gives min d

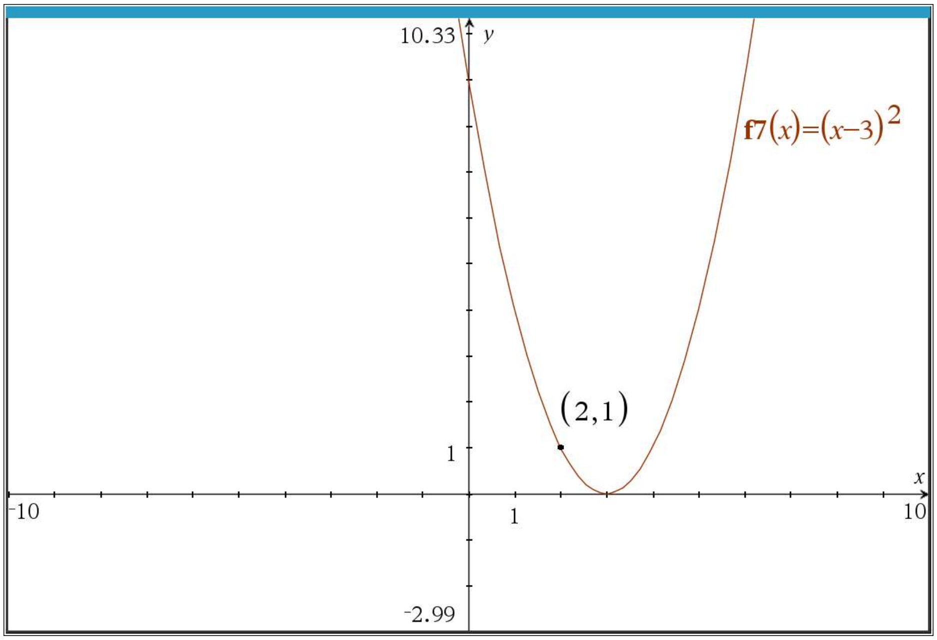


$x = 2$ gives min d

but $x = 2$ on original graph

$\Rightarrow y = (2-3)^2 = (-1)^2 = 1$

$\therefore (2,1)$ is the desired point



Quiz 2 Your Name MTH 167-002N calculator OK

1. Find and simplify the difference quotient for $f(x) = x^2 - 1$.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 - 1] - [x^2 - 1]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 \cancel{-1} - \cancel{x^2} \cancel{+1}}{h} = \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{\therefore \frac{\Delta f}{\Delta x} = 2x + h}$$

2. Find the domain of $\frac{f(x)}{g(x)}$ for $f(x) = x$ and $g(x) = \sqrt{x}$.
   Show reasoning and calculations.

$$\frac{f(x)}{g(x)} = \frac{x}{\sqrt{x}}$$

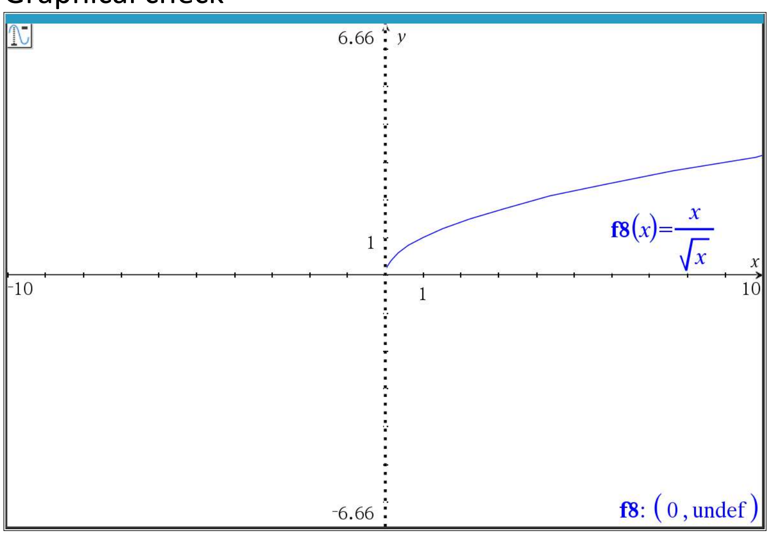
To avoid square root of negative number

$$\boxed{x \geq 0}$$

To avoid division by 0, $x \neq 0$

$$\boxed{\therefore \text{domain} = \{x \mid x > 0\} = (0, \infty)}$$

Graphical check



Note: if $x \neq 0$,

$$\frac{x}{\sqrt{x}} = \sqrt{x}$$

This simplified function has domain $[0, \infty)$
$= \{x \mid x \geq 0\}$