09-04-24 MTH 167-002N

1.3 Introduction to Functions
1.3.1 Exercises
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1.4 Function Notation
1.4.2 Exercises
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1.5 Function Arithmetic
1.5.1 Exercises
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1.3: 46

In Exercises 33 - 47, determine whether or not the equation represents y as a function of x.

46. $2xy = 4$

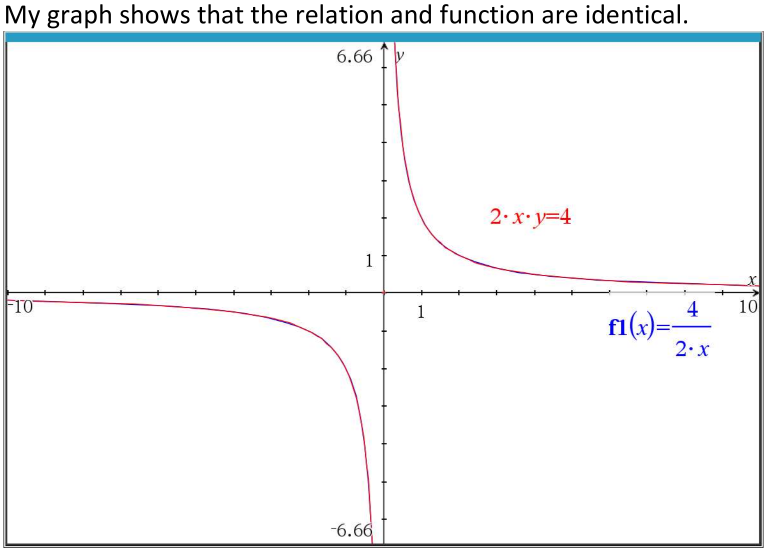
solve for y

$$\boxed{y = \frac{4}{2x}}$$

For each x not equal to 0, there is exactly one corresponding y-value, so y is a function of x,

Note domain of function $= \{x \mid x \neq 0\}$

My graph shows that the relation and function are identical.



1.4:15

In Exercises 11 - 18, use the given function f to find and simplify the following:

- $f(3)$
- $f(-1)$
- $f\left(\frac{3}{2}\right)$
- $f(4x)$
- $4f(x)$
- $f(-x)$
- $f(x-4)$
- $f(x)-4$
- $f\left(x^2\right)$

15. $f(x) = \dfrac{x}{x-1}$

$$f(\text{input}) = \frac{\text{input}}{\text{input}-1}$$

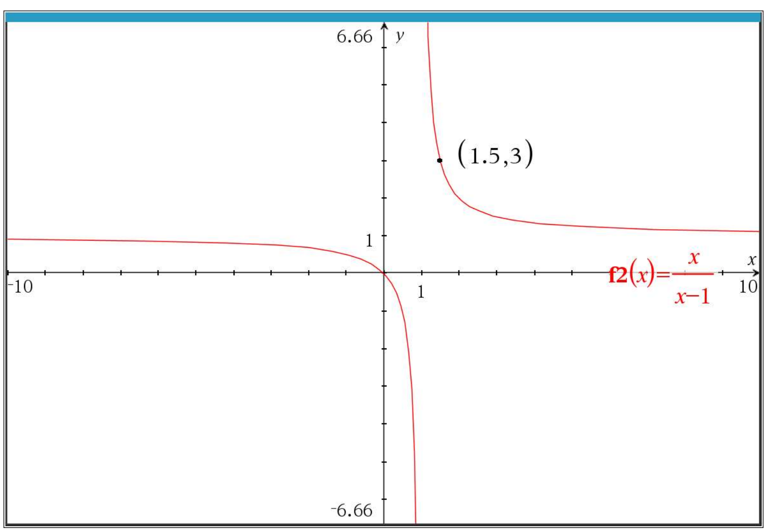
$$f\left(\frac{3}{2}\right) = \frac{\frac{3}{2}}{\frac{3}{2}-1} \quad \frac{\frac{3}{2}}{\frac{3}{2}-1\left(\frac{2}{2}\right)} = \frac{\frac{3}{2}}{\frac{3}{2}-\frac{2}{2}}$$

$$= \frac{\frac{3}{2}}{\frac{3-2}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = \left(\frac{3}{2}\right)\left(\frac{2}{1}\right)$$

$$= \left(\frac{2}{2}\right)\left(\frac{3}{1}\right)$$

$$= (1)(3) = \boxed{3}$$

Graphical confirmation



1.5
Memorize

Function Arithmetic

Suppose f and g are functions and x is in both the domain of f and the domain of g.[a]

Function Arithmetic

Suppose f and g are functions and x is in both the domain of f and the domain of g.[a]

- The **sum** of f and g, denoted $f+g$, is the function defined by the formula

$$(f+g)(x) = f(x) + g(x)$$

- The **difference** of f and g, denoted $f-g$, is the function defined by the formula

$$(f-g)(x) = f(x) - g(x)$$

- The **product** of f and g, denoted fg, is the function defined by the formula

$$(fg)(x) = f(x)g(x)$$

- The **quotient** of f and g, denoted $\dfrac{f}{g}$, is the function defined by the formula

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},$$

provided $g(x) \neq 0$.

[a] Thus x is an element of the intersection of the two domains.

Let $h = f+g$ def

$h(x) = (f+g)(x) = f(x) + g(x)$

Let $f(x) = 3x$

$g(x) = 5$

$\Rightarrow (f+g)(x) = f(x) + g(x) = 3x+5$

implies

$f(x) = x$ domain of $f = \mathbb{R} = (-\infty, \infty)$

$g(x) = x$ domain of $g = \mathbb{R} = (-\infty, \infty)$

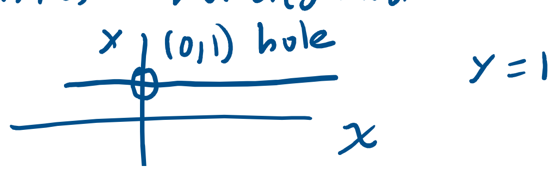
Let $h(x) = \dfrac{f(x)}{g(x)}$

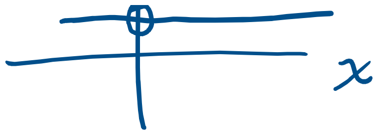
Find the domain of $h(x)$

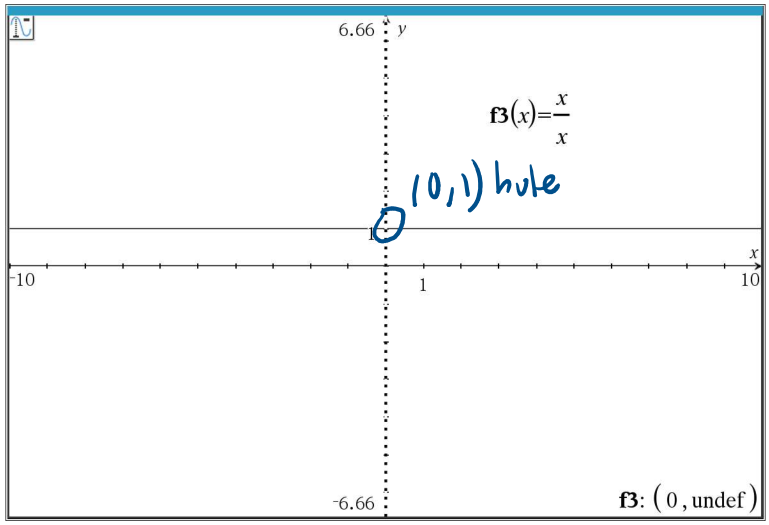
Find the range of $h(x)$

$h(x) = \dfrac{x}{x} = 1$ if $x \neq 0$

$h(0) =$ not defined







domain of $h = \{x \mid x \neq 0\}$
$= (-\infty, 0) \cup (0, \infty)$

Memorize

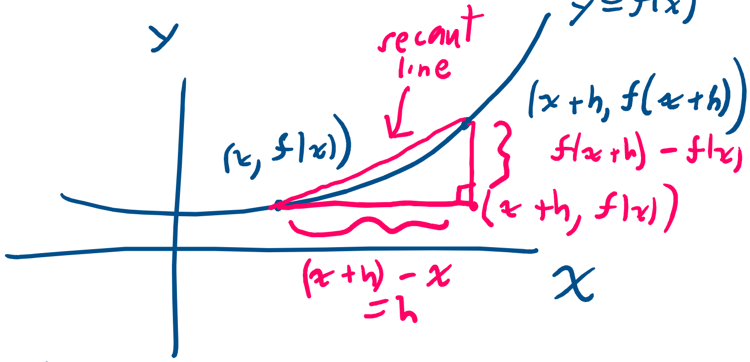
Definition 1.8. Given a function f, the **difference quotient** of f is the expression

$$\frac{f(x+h) - f(x)}{h}, \; h \neq 0$$

$$(x+h) - x = x + h - x$$
$$= x - x + h$$
$$= (0) + h$$
$$= h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\Delta f}{\Delta x}$$

Δ = change



slope of secant line $= \dfrac{\Delta y}{\Delta x}$

$= \dfrac{f(x+h) - f(x)}{h}$ = difference quotient

$$- \quad \frac{}{h} \quad \text{quotient}$$

Supplied

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

1.5

In Exercises 51 - 62, let f be the function defined by

$$f = \{(-3,4), (-2,2), (-1,0), (0,1), (1,3), (2,4), (3,-1)\}$$

and let g be the function defined

$$g = \{(-3,-2), (-2,0), (-1,-4), (0,0), (1,-3), (2,1), (3,2)\}$$

. Compute the indicated value if it exists.

$$(f+g)(5) = f(5) + g(5)$$
$$= \text{not defined} \quad 5 \notin \text{domain of } f \text{ or } g$$

$$(f+g)(-2) \overset{\text{def}}{=} f(-2) + g(-2) = 2 + 0 = \boxed{2}$$

the quantity "f plus g" of -2

$f(\text{input}) = 5(\text{input}) + 3$

$f(w) = 5w + 3$

Let $f(x) = 5x + 3$

Find and simplify the difference quotient

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

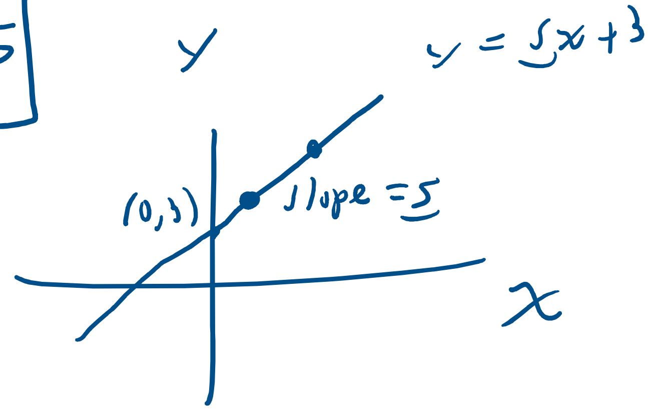
$\Delta x \qquad h$

$$= \frac{[5(x+h)+3] - [5x+3]}{h}$$

$$= \frac{\cancel{5x} + 5h \cancel{+3} \cancel{-5x} \cancel{-3}}{h}$$

$$= \frac{5\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 5}$$



$f(x) = x^2$

Find and simplify $\frac{\Delta f}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \cancel{h}(2x+h)$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2x + h}$$