09-02-25 MTH 167-002N

1 Relations and Functions
1.1 Sets of Real Numbers and the Cartesian Coordinate Plane
1.1.4 Exercises
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1.2 Relations
1.2.2 Exercises
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1.3 Introduction to Functions
1.3.1 Exercises
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1.4 Function Notation
1.4.2 Exercises
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1.2:18

1.2.2 Exercises

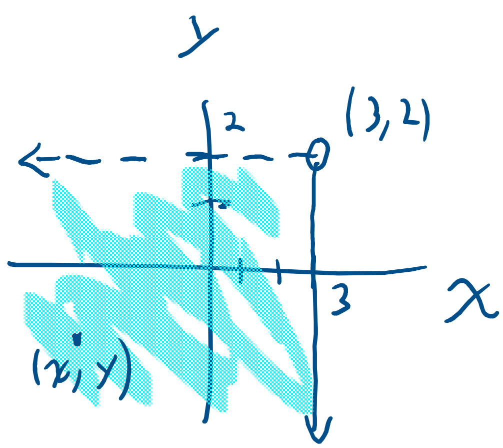
In Exercises 1 - 20, graph the given relation.

18. $\{(x, y) \mid x \leq 3,\ y < 2\}$ $= R$

$(1, 0) \in R$

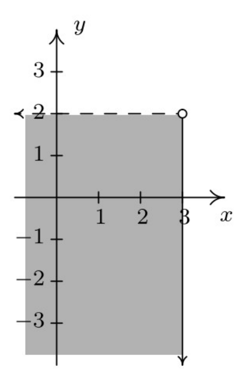
$1 \leq 3,\ 0 < 2$

$(4, 1) \notin R$



Textbook answer

18.



1.2: 50

For each equation given in Exercises 41 - 52:

- Find the x- and y-intercept(s) of the graph, if any exist.
- Follow the procedure in Example 1.2.3 to create a table of sample points on the graph of the equation.
- Plot the sample points and create a rough sketch of the graph of the equation.
- Test for symmetry. If the equation appears to fail any of the symmetry tests, find a point on the graph of the equation whose reflection fails to be on the graph as was done at the end of Example 1.2.4

50. $x^2 - y^2 = 1$

x-intercept

Let $y=0$

$x^2 - 0^2 = 1$

solve for x

$x^2 = 1$

$\boxed{x = \pm 1}$

points $(-1,0), (1,0)$

y-intercept

Let $x=0$

solve for y

$0^2 - y^2 = 1$

$y^2 = -1$

$y = \pm\sqrt{-1} \notin \mathbb{R}$

$\therefore$ no y-intercept

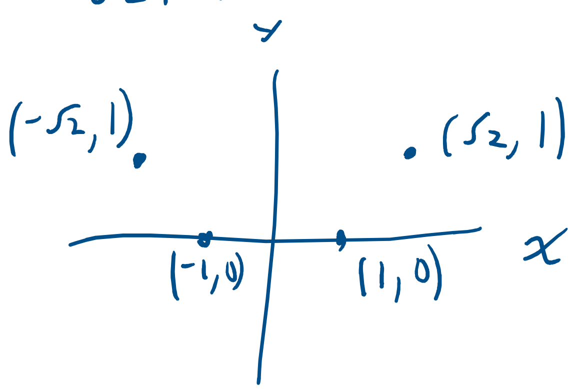
x	y
-1	0
1	0
$-\sqrt{2}$	1
$\sqrt{2}$	1

$x^2 - 1^2 = 1$

$x^2 - 1 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$



$x^2 - y^2 = 1$

Graph on TI-84

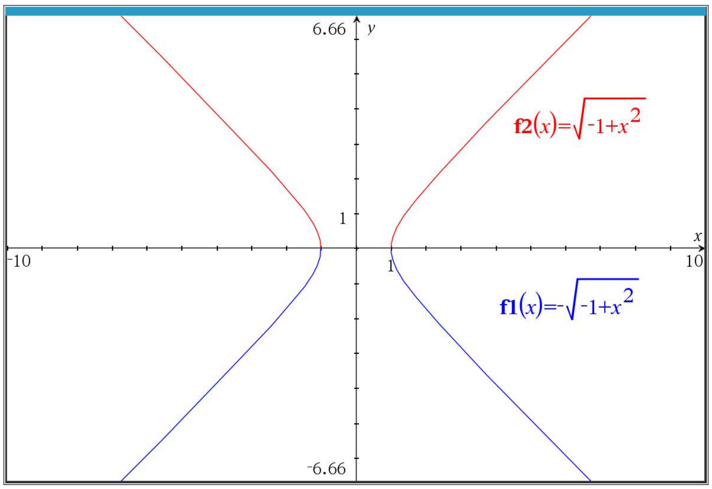
solve for y

solve for y

$$-y^2 = 1 - x^2$$

$$y^2 = -1 + x^2$$

$$\boxed{y = \pm\sqrt{-1+x^2}}$$



hyperbola

test for symmetry

$$\boxed{x^2 - y^2 = 1}$$

x-axis symmetry

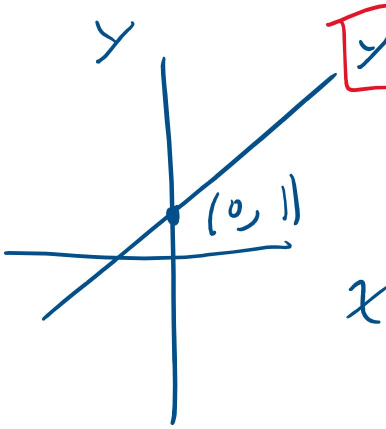
same

$$x^2 - (-y)^2 = 1$$

$$\boxed{x^2 - y^2 = 1}$$

$$(-y)^2 = (-y)(-y)$$
$$= (-1)(-1)(y)(y)$$
$$= (1)\,y^2$$
$$= y^2$$

∴ x-axis symmetry



$$\boxed{y = x+1}$$

not the same

x-axis symmetry

∴ No x-axis symmetry

x-axis symmetry

$-y = x + 1$

$\boxed{y = -x - 1}$

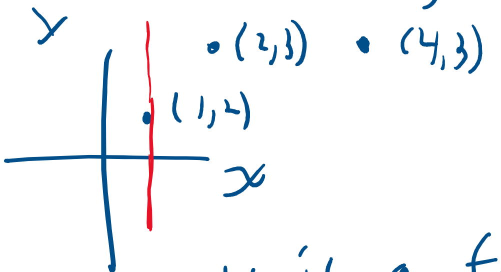
$\therefore$ No x-axis symmetry

1.3

Definition 1.6. A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a **function** of x.

memorize

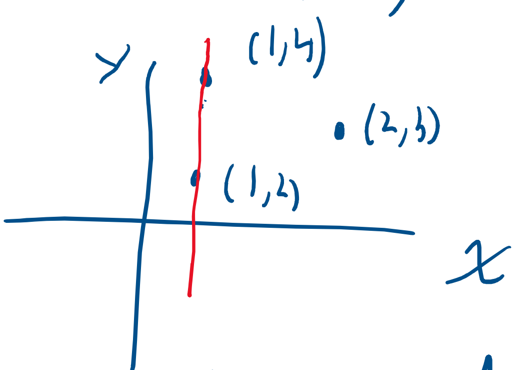
$R_1 = \{(1,2), (2,3), (4,3)\}$



graph passes the vertical line test

y is a function of x

$R_2 = \{(1,2), (2,3), (1,4)\}$



graph fails the vertical line test

The x-coordinate 1 has two y-coordinates, namely 2 and 4

$\therefore$ y is not a function of x

$R_3 = \{(1,2), (2,3), (1,2)\} = \{(1,2), (2,3)\}$

memorize

Theorem 1.1. The Vertical Line Test: A set of points in the plane represents y as a function

Memorize

Theorem 1.1. The Vertical Line Test: A set of points in the plane represents y as a function of x if and only if no two points lie on the same vertical line.

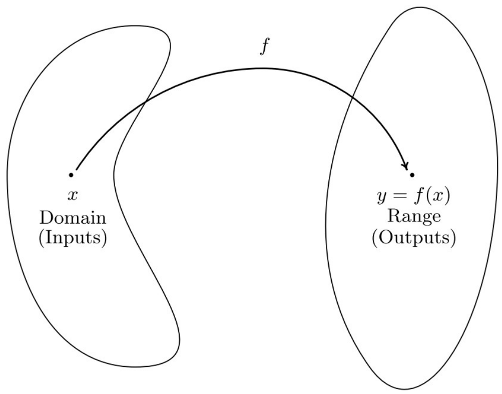
Memorize

Definition 1.7. Suppose F is a relation which describes y as a function of x.

- The set of the x-coordinates of the points in F is called the **domain** of F.
- The set of the y-coordinates of the points in F is called the **range** of F.

We can generalize this to any relation.

1.4



General definition: A <u>function</u> is a rule that associates to each element in one set, called the domain, a unique corresponding element in another set, called the range.

Definition: the implied domain of a function $f(x)$ is the largest set of real numbers such that the function is well-defined, that is, so that $f(x)$ can be calculated.

Let $f(x) = \frac{1}{x}$

Find the domain of f.

$f(1) = \frac{1}{1} = 1$

$1 \in$ domain

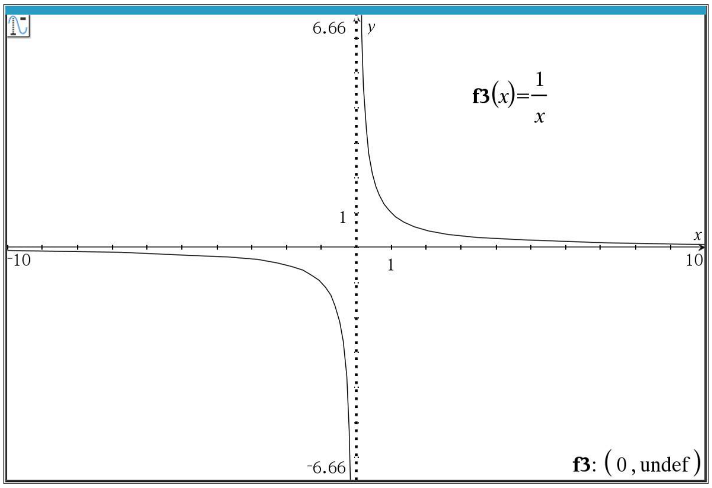
$f(100) = \frac{1}{100} \Rightarrow 100 \in$ domain

$f(0) = \frac{1}{0}$ not defined

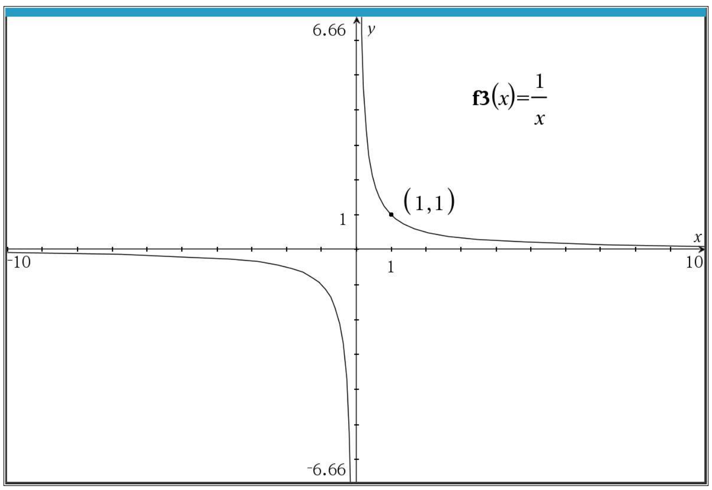
$\therefore 0 \notin$ domain

$\therefore 0 \notin$ domain

domain of $f = \{x \mid x \neq 0\}$



$0 \notin$ domain of $f(x) = \frac{1}{x}$



$1 \in$ domain of $f(x) = \frac{1}{x}$

Definition: a mathematical model is a set of equations and formulas that describe a real-world system.

Co-pilot.

I'm in my pre-calculus class and I've got a question.

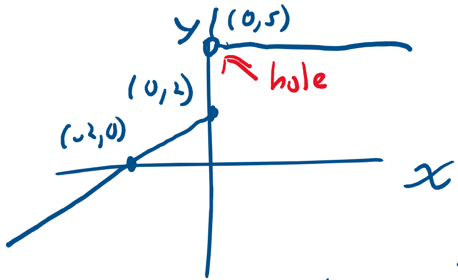
A mathematical model is basically using math to represent, describe, and analyze real-world situations or systems. So, imagine you're using equations, functions, graphs, or even algorithms to predict or explain behaviors and outcomes. It's kind of like a blueprint, but instead of bricks and mortar, you're building with numbers and symbols. Does that help?—

Piecewise-defined function

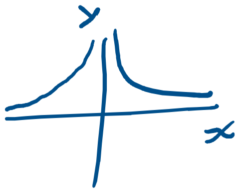
$$f(x) = \begin{cases} x+2 & \text{for } x \le 0 \\ 5 & \text{for } x > 0 \end{cases}$$

(0,5)

$f(x) = \{ 5$ for $x > 0$



domain $= (-\infty, \infty) = \mathbb{R}$



$f(x) = \frac{1}{x^2}$

domain $= \{x \mid x \neq 0\}$
$= (-\infty, 0) \cup (0, \infty)$

$1 + 1 = 2$

1

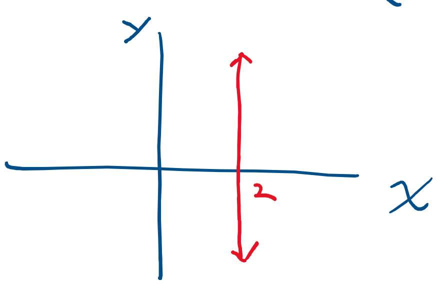
$1 + 1 = 3$

0

TI-84 $Y_1 = (x+2)*(x \le 0)$
$+ 5*(x > 0)$

Your name MTH 167-002N Quiz1

① Graph the relation $R = \{(x, y) \mid x = 2\}$



② what is the domain of R ?

$\boxed{\text{domain of } R = \{2\}}$

③ what is the range of R ?

$= (-\infty, \infty) = \mathbb{R}$ = set of

③ what is the range of R ?

$\boxed{\text{range of } R = (-\infty, \infty) = \mathbb{R} = \text{set of all real numbers}}$

④ Let $g(x) = \dfrac{1}{\sqrt{4-x}}$

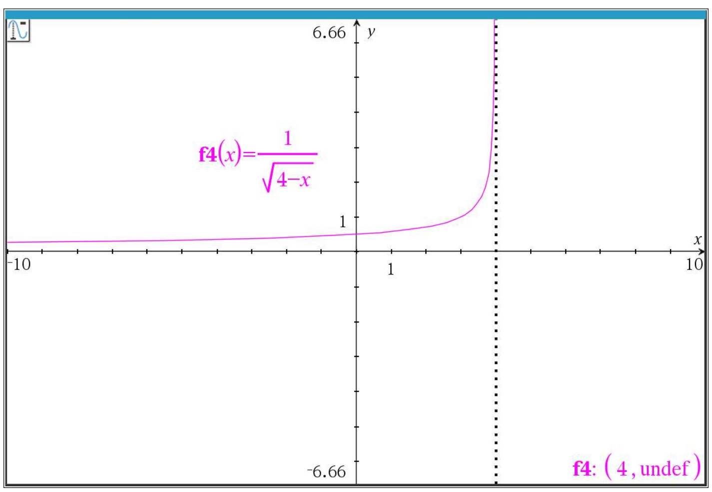
what is the domain of g ?

To avoid division by 0, $\boxed{x \neq 4}$

To avoid taking the square root of a negative number $4-x \geq 0$

$\Rightarrow \boxed{x \leq 4}$

$\Rightarrow$ domain $= \{x \mid x \neq 4 \text{ and } x \leq 4\} = \boxed{\{x \mid x < 4\} = (-\infty, 4)}$



⑤ Does R in #1 represent y as a function of x?

why or why not?

No. For x = 2, there are an infinite number of possible y values. The graph fails the vertical line test.