08-28-25 MTH 167-002N

1 Relations and Functions
1.1 Sets of Real Numbers and the Cartesian Coordinate Plane
1.1.4 Exercises
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1.2 Relations
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1.1
Memorize

Definition 1.1. A **set** is a well-defined collection of objects which are called the 'elements' of the set. Here, 'well-defined' means that it is possible to determine if something belongs to the collection or not, without prejudice.

Memorize

Ways to Describe Sets

1. **The Verbal Method:** Use a sentence to define a set.
2. **The Roster Method:** Begin with a left brace '{', list each element of the set *only once* and then end with a right brace '}'.
3. **The Set-Builder Method:** A combination of the verbal and roster methods using a "dummy variable" such as x.

$A = \{2, 4, 6\}$ list (roster) notation
$= \{6, 4, 2\} = \{2, 2, 4, 6\}$

Verbal method: A is the set of even integers greater than 1 and less than 7

Set-builder notation $\{A = \{x | x \in \mathbb{Z}, x \text{ is even}, 1 < x < 7\}\}$

Memorize

Sets of Numbers

1. The **Empty Set**: $\emptyset = \{\} = \{x \,|\, x \neq x\}$. This is the set with no elements. Like the number '0,' it plays a vital role in mathematics.[a]
2. The **Natural Numbers**: $\mathbb{N} = \{1, 2, 3, \ldots\}$ The periods of ellipsis here indicate that the natural numbers contain 1, 2, 3, 'and so forth'.
3. The **Whole Numbers**: $\mathbb{W} = \{0, 1, 2, \ldots\}$
4. The **Integers**: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
5. The **Rational Numbers**: $\mathbb{Q} = \left\{\frac{a}{b} \,|\, a \in \mathbb{Z} \text{ and } b \in \mathbb{Z}\right\}$. Rational numbers are the ratios of integers (provided the denominator is not zero!) It turns out that another way to describe the rational numbers[b] is:

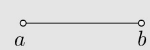
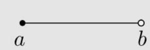
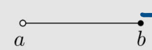
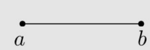
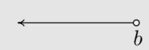
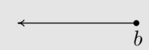
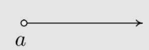
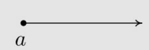
$$\mathbb{Q} = \{x \,|\, x \text{ possesses a repeating or terminating decimal representation.}\}$$

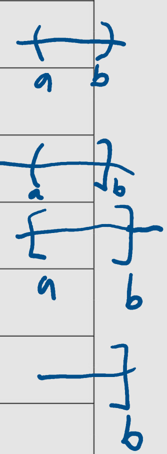
6. The **Real Numbers**: $\mathbb{R} = \{x \,|\, x \text{ possesses a decimal representation.}\}$
7. The **Irrational Numbers**: $\mathbb{P} = \{x \,|\, x \text{ is a non-rational real number.}\}$ Said another way, an irrational number is a decimal which neither repeats nor terminates.[c]
8. The **Complex Numbers**: $\mathbb{C} = \{a + bi \,|\, a, b \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$ Despite their importance, the complex numbers play only a minor role in the text.[d]

[a] ... which, sadly, we will not explore in this text.
[b] See Section 9.2.
[c] The classic example is the number π (See Section 10.1), but numbers like $\sqrt{2}$ and $0.101001000100001\ldots$ are other fine representatives.
[d] They first appear in Section 3.4 and return in Section 11.7.

Interval Notation

Let a and b be real numbers with $a < b$.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \,|\, a < x < b\}$	(a, b)	
$\{x \,|\, a \leq x < b\}$	$[a, b)$	
$\{x \,|\, a < x \leq b\}$	$(a, b]$	
$\{x \,|\, a \leq x \leq b\}$	$[a, b]$	
$\{x \,|\, x < b\}$	$(-\infty, b)$	
$\{x \,|\, x \leq b\}$	$(-\infty, b]$	
$\{x \,|\, x > a\}$	(a, ∞)	
$\{x \,|\, x \geq a\}$	$[a, \infty)$	
$\mathbb{R}$	$(-\infty, \infty)$	



set notation $x \in A$ x is an element (member) of set A

$x \notin A$

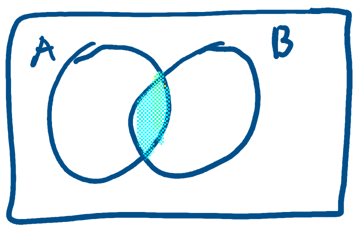
x is not an element of A

Memorize

> **Definition 1.2.** Suppose A and B are two sets.
>
> - The **intersection** of A and B: $A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}$
> - The **union** of A and B: $A \cup B = \{x \,|\, x \in A \text{ or } x \in B \text{ (or both)}\}$

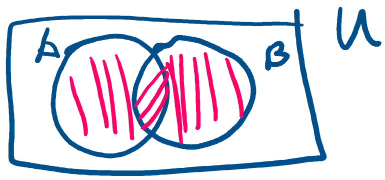
↑ inclusive or

Venn diagram



$\mathcal{U}$ = universal set
= set of discourse

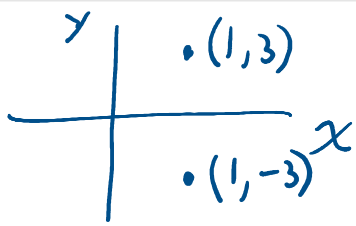
$A \cap B$



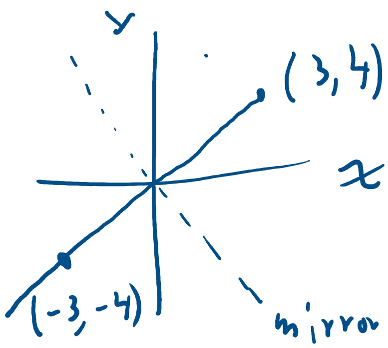
$A \cup B$

Memorize

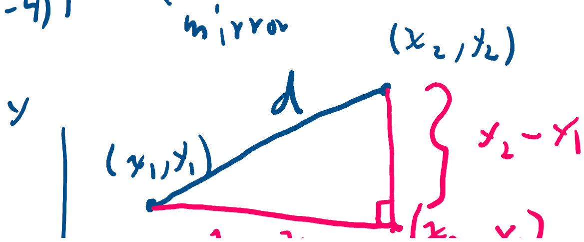
> **Definition 1.3.** Two points (a, b) and (c, d) in the plane are said to be
>
> - **symmetric about the x-axis** if $a = c$ and $b = -d$
> - **symmetric about the y-axis** if $a = -c$ and $b = d$
> - **symmetric about the origin** if $a = -c$ and $b = -d$

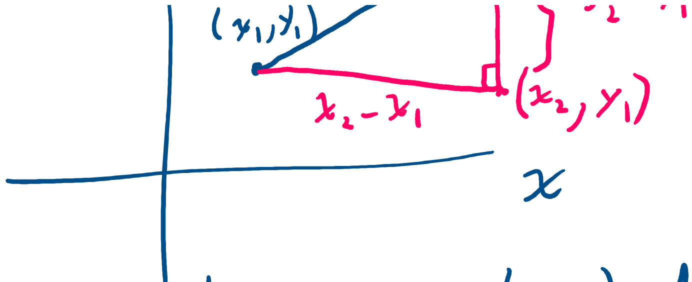


(1,3) and (1,-3)
are symmetric
about the x-axis



origin symmetry





d = distance between (x_1, y_1) and (x_2, y_2)

Pythagorean Theorem

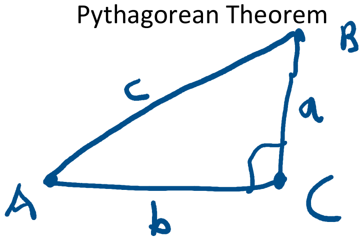
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

distance formula

Memorize

Pythagorean Theorem



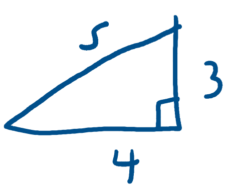
memorize

If $a^2 + b^2 = c^2$, then $\angle C = 90^0$ = right angle

$\Rightarrow \triangle ABC$ is a right triangle

If $\triangle ABC$ is a right triangle with hypotenuse c

Then $a^2 + b^2 = c^2$



is this a right triangle

$3^2 + 4^2 = 9 + 16 = 25$

$5^2 = 25$

$25 = 25$

$\Rightarrow 3^2 + 4^2 = 5^2$

$\therefore$ right $\triangle$

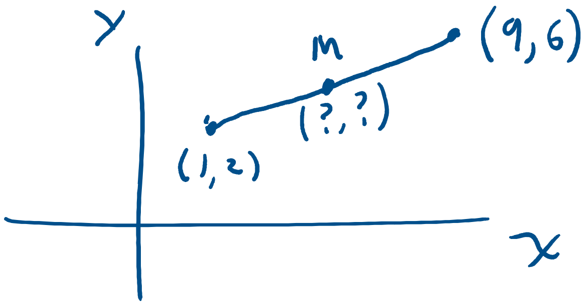
$\Rightarrow 3^2+4^2=5^2$

$\therefore$ right $\triangle$

Memorize

Equation 1.2. The Midpoint Formula: The midpoint M of the line segment connecting $P(x_0, y_0)$ and $Q(x_1, y_1)$ is:

$$M = \left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}\right)$$



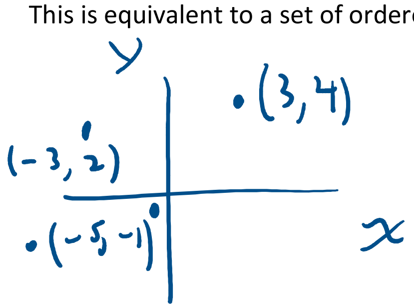
$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{1+9}{2}, \frac{2+6}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{8}{2}\right)$$
$$= \boxed{(5,4)}$$

1.2

Memorize

Definition 1.4. A **relation** is a set of points in the plane.

This is equivalent to a set of ordered pairs of real numbers.



$R = \{(3,4), (-3,2), (-5,-1)\}$

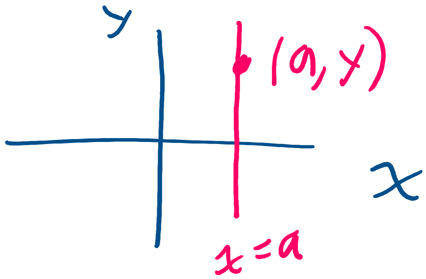
R

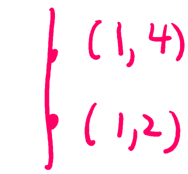
Memorize

Equations of Vertical and Horizontal Lines

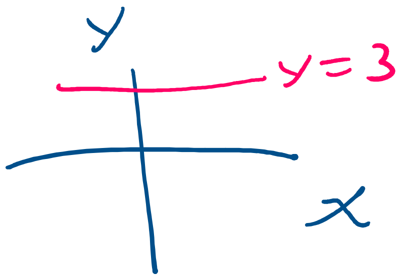
- The graph of the equation $x = a$ is a **vertical line** through $(a, 0)$.
- The graph of the equation $y = b$ is a **horizontal line** through $(0, b)$.

- The graph of the equation $y = b$ is a **horizontal line** through $(0, b)$.





$$\text{slope} = \frac{2-4}{1-1} = \frac{-2}{0} = \text{not defined}$$



$$y = 0 \cdot x + 3 = 3$$

Memorize

> **The Fundamental Graphing Principle**
> The graph of an equation is the set of points which satisfy the equation. That is, a point (x, y) is on the graph of an equation if and only if x and y satisfy the equation.

Memorize

> **Definition 1.5.** Suppose the graph of an equation is given.
>
> - A point on a graph which is also on the x-axis is called an ***x*-intercept** of the graph.
> - A point on a graph which is also on the y-axis is called an ***y*-intercept** of the graph.

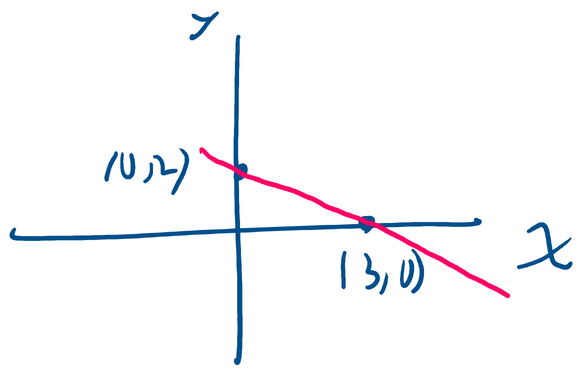
Graph the line $2x + 3y = 6$ by finding and plotting the x-intercept ane y-intercept.

x-intercept
set $y = 0$
solve for x

$2x + 3(0) = 6$
$2x = 6$
$\boxed{x = 3}$
or the point $(3, 0)$

y-intercept
Set $x = 0$
solve for y

$2(0) + 3y = 6$
$3y = 6$
$\boxed{y = 2}$
or the point $(0, 2)$



Memorize

Testing the Graph of an Equation for Symmetry

To test the graph of an equation for symmetry

- about the y-axis – substitute $(-x, y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the y-axis.
- about the x-axis – substitute $(x, -y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the x-axis.
- about the origin - substitute $(-x, -y)$ into the equation and simplify. If the result is equivalent to the original equation, the graph is symmetric about the origin.

$y = x^2$ test for symmetry

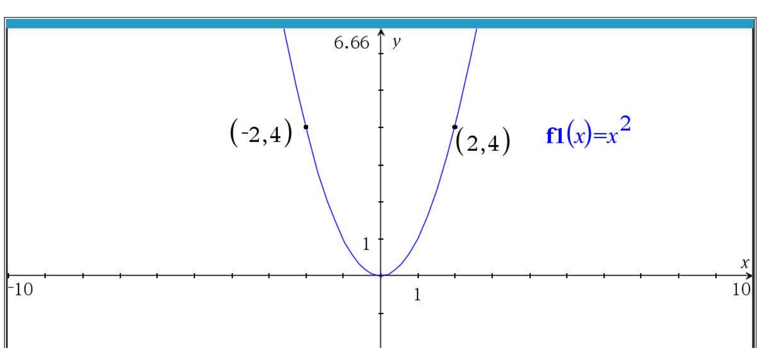
$y = (-x)^2 = x^2 \Rightarrow$ y-axis symmetry

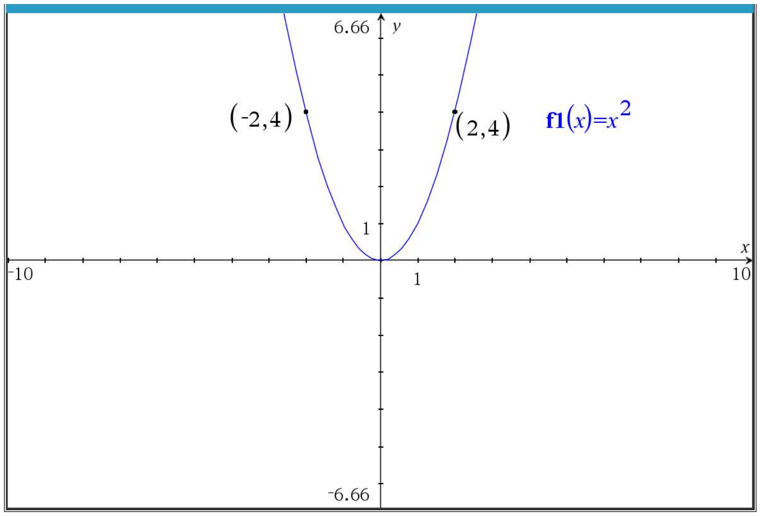
$-y = x^2 \Rightarrow y = -x^2 \Rightarrow$ no x-axis symmetry

$-y = (-x)^2 \Rightarrow -y = x^2$

$\Rightarrow (-y)(-1) = (-1)x^2$ no origin symmetry

$\Rightarrow y = -x^2$





From the graph, it appears that the relation has only
y-axis symmetry