04-02-25 161-C06N

3.3 Real Zeros of Polynomials

3.3.3: Exercises

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3.3: 48

In Exercises 45 - 54, solve the polynomial inequality and state your answer using interval notation.

48. $4x^3 \geq 3x + 1$

$4x^3 - 3x - 1$

≥ 0 Let $f(x) = 4x^3 - 3x - 1$

solve $4x^3 - 3x - 1 = 0$

textbook strategy: try small integer values of x

$f(0) = 4(0^3) - 3(0) - 1 = -1 \neq 0$

$f(1) = 4(1^3) - 3(1) - 1 = 4 - 4 = 0$ ✓

$\therefore x = 1$ is a zero of $f(x)$

$\Rightarrow x - 1$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & (0) \end{array}$$

remainder

$4x^2 + 4x + 1$

$f(x) = (x-1)(4x^2 + 4x + 1)$

$4x^2 + 4x + 1 = 0$

bad choice $(4x + 1)(x + 1) = 0$

bad choice

$(4x+1)(x+1)=0$

$4x^2+5x+1$

$(2x+1)(2x+1)=0$

$2x+1=0$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$ mult. 2

$4x^2+4x+1=0$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a=4$
$b=4$
$c=1$

$$x=\frac{-4\pm\sqrt{4^2-(4)(4)(1)}}{2(4)}$$

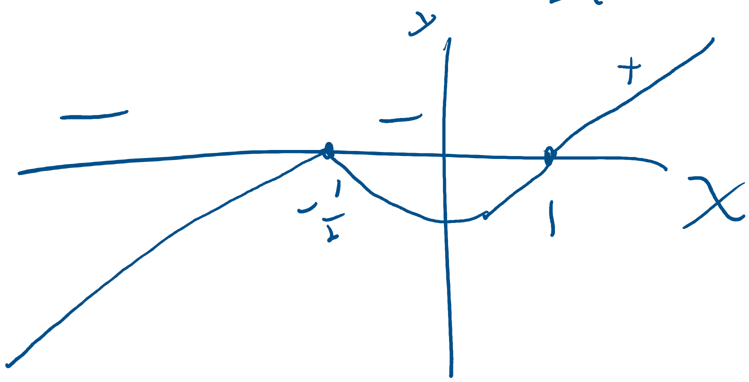
$$x=\frac{-4\pm\sqrt{16-16}}{8}$$

$$x=\frac{-4\pm0}{8}=\frac{-4}{8}$$

$\boxed{x=-\frac{1}{2}}$

zeros of $f(x)$ are $x=1$ (mult. 1)
$x=-\frac{1}{2}$ (mult 2)

$x = -\frac{1}{2}$ (mul t 2)



solution $[1, \infty)$

test values $f(x) = (2x+1)^2(x-1)$

$f(-1) = (2(-1)+1)^2(-1-1)$

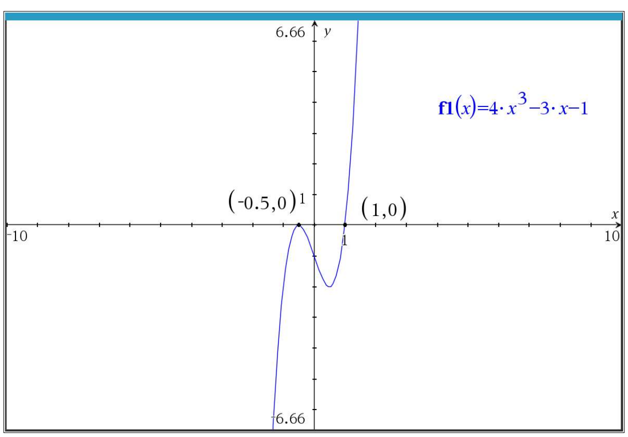
$= (+)(-) < 0$

$f(0) = (2\cdot 0+1)^2(0-1)$

$= (+)(-) < 0$

$f(2) = (2\cdot 2+1)(2-1)$

$= (+)(+) > 0$



54. $x^6 + x^3 \geq 6$ Solve the inequality

$x^6 + x^3 - 6 \geq 0$

$$x^6 + x^3 - 6 \geq 0$$

Let $f(x) = x^6 + x^3 - 6$

Let $y = x^3$

$$\Rightarrow y^2 = (x^3)^2 = x^6$$

$$f(x) = y^2 + y - 6$$

solve $y^2 + y - 6 = 0$

$$(y+3)(y-2) = 0$$

$$\boxed{y = -3, 2}$$

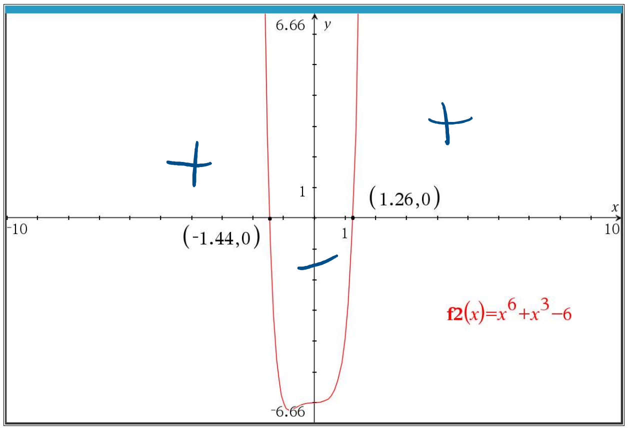
$$x = \sqrt[3]{y}$$

$$x = \sqrt[3]{-3}, \sqrt[3]{2}$$

$$\boxed{x = -\sqrt[3]{3}, \sqrt[3]{2}}$$

-(3^(1/3))=-1.4422

2^(1/3)=1.2599



solution

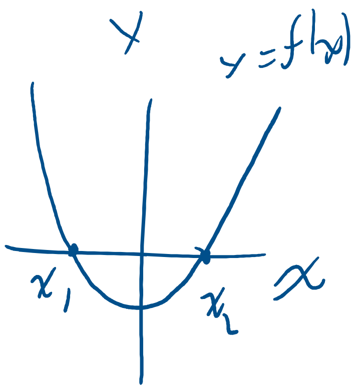
$$\left(-\infty, -\sqrt[3]{3}\right] \cup \left[\sqrt[3]{2}, \infty\right)$$

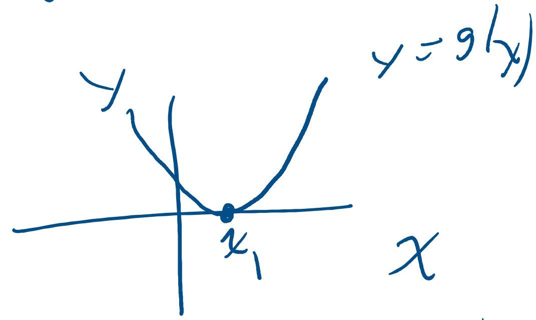
$-\sqrt[3]{3}$	$-3^{\frac{1}{3}}$
$\sqrt[3]{-3}$	$-3^{\frac{1}{3}}$

Brief intro to complex numbers

define i by $i^2 = -1$

$i = \sqrt{-1}$



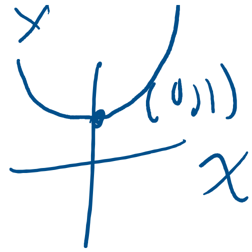


2 real distinct zeros

1 real solution, mult 2

no real solution

$y = x^2 + 1$



$y = x^2 + 1$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

2 complex solutions