03-31-25 MTH 161-C06N

3.2 The Factor Theorem and the Remainder Theorem
   3.2.1 Exercises
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3.3 Real Zeros of Polynomials
   3.3.3: Exercises
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3.2:35

In Exercises 31 - 40, you are given a polynomial and one of its zeros. Use the techniques in this section to find the rest of the real zeros and factor the polynomial.

35. $x^3 + 2x^2 - 3x - 6, \quad c = -2$

Let $p(x) = x^3 + 2x^2 - 3x - 6$

$c = -2$ is a zero of $p(x)$

$\Rightarrow (x - (-2)) = (x + 2)$ is a factor of $p(x)$

$\exists$ a poly $q(x)$ such that $p(x) = (x+2)\,q(x)$

there exists

To find $q(x)$, note that

$q(x) = p(x) \div (x + 2)$

$$\begin{array}{r|l} & x^2 \qquad\quad -3 \\ \hline x + 2 & x^3 + 2x^2 - 3x - 6 \\ & x^3 + 2x^2 \\ \hline \end{array}$$

$$x^3 + 2x^2$$

$$\begin{array}{r} -3x-6 \\ -3x-6 \\ \hline 0 \end{array}$$

$q(x) = x^2 - 3$

$p(x) = (x+2)(x^2-3)$

$p(x) = (x+2)(x+\sqrt{3})(x-\sqrt{3})$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

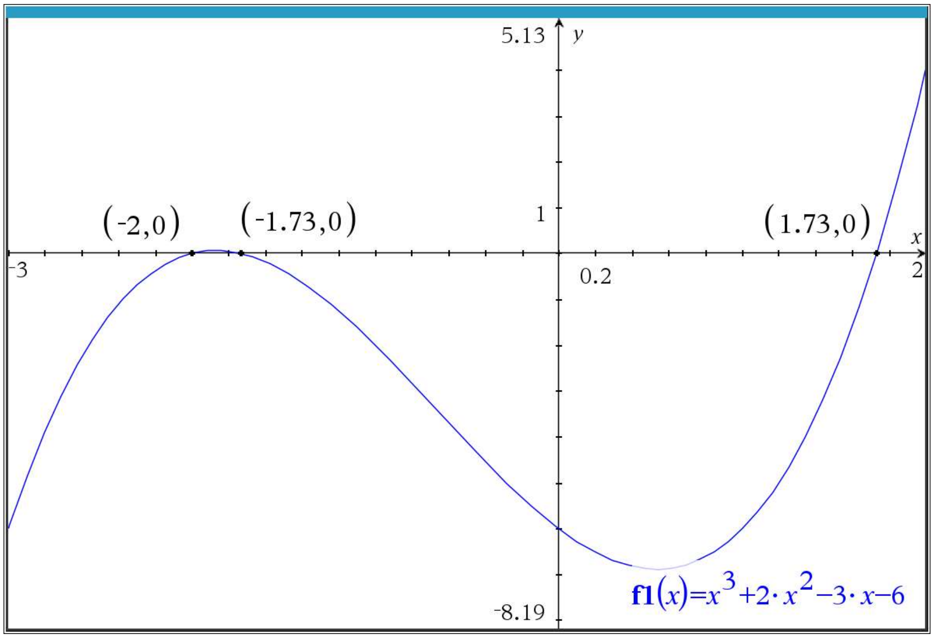
$\boxed{\text{Zeros of } p(x) \text{ are } -2, \pm\sqrt{3}}$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & & -2 & 0 & 6 \\ \hline & 1 & 0 & -3 & (0) \end{array}$$

remainder

$q(x) = x^2 - 3$

Graphical confirmation of our algebra



Sqrt(3)=1.732050807568877

3.3
Supplied

Theorem 3.8. Cauchy's Bound: Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$. Let M be the largest of the numbers: $\frac{|a_0|}{|a_n|}, \frac{|a_1|}{|a_n|}, \ldots, \frac{|a_{n-1}|}{|a_n|}$. Then all the real zeros of f lie in in the interval $[-(M+1), M+1]$.

$f(x) = x^3 + 2x^2 - 3x - 6$

$a_3 = 1, \quad a_2 = 2, \quad a_3 = -3, \quad a_0 = -6$

$M = \max\left\{ \frac{|-6|}{|1|}, \frac{|-3|}{|-1|}, \frac{|2|}{|-1|} \right\}$

$M = \max\{6, 3, 2\} = 6$

$\boxed{M+1 = 7}$

$\therefore$ $[-7, 7]$ includes all real zeros of $f(x)$

Supplied

Theorem 3.9. Rational Zeros Theorem: Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$, and a_0, a_1, $\ldots a_n$ are integers. If r is a rational zero of f, then r is of the form $\pm\frac{p}{q}$, where p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.

$a_n = a_3 = 1$

$a_0 = -6$

$p|q$ means "p divides into q"

or "p is a factor of q"

$p|a_0 \Rightarrow p|6 \Rightarrow p = \pm 1, \pm 2, \pm 3, \pm 6$

$q|a_3 \Rightarrow q|1 \Rightarrow q = \pm 1$

$$\boxed{\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6}$$ candidates for rational zeros

$x = -2$ is a zero of $f(x)$ from our graph

$x = \pm 1.73$ are <u>not</u> in our list

$\therefore x \approx \pm 1.73$ are irrational

Supplied

Theorem 3.10. Descartes' Rule of Signs: Suppose $f(x)$ is the formula for a polynomial function written with descending powers of x.

- If P denotes the number of variations of sign in the formula for $f(x)$, then the number of positive real zeros (counting multiplicity) is one of the numbers $\{P, P-2, P-4, \ldots\}$.
- If N denotes the number of variations of sign in the formula for $f(-x)$, then the number of negative real zeros (counting multiplicity) is one of the numbers $\{N, N-2, N-4, \ldots\}$.

$f(x) = x^3 + 2x^2 - 3x - 6$

0 1 0

There is 1 sign change, so the number of positive real zeros is 1.
This prediction agrees with our graphical result.

$$f(-x) = (-x)^3 + 2(-x)^2 - 3(-x) - 6$$

$$f(-x) = -x^3 + 2x^2 + 3x - 6$$

1 0 1

There are 2 sign changes, so there are 2 or 2-2=0 negative real zeros.

In fact, our graph shows 2 negative real zeros, agreeing with the prediction.

Omit upper and lower bound theorem