03-26-25 MTH 161-C06N

3.1 Graphs of Polynomials
  3.1.1 Exercises
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3.2 The Factor Theorem and the Remainder Theorem
  3.2.1 Exercises
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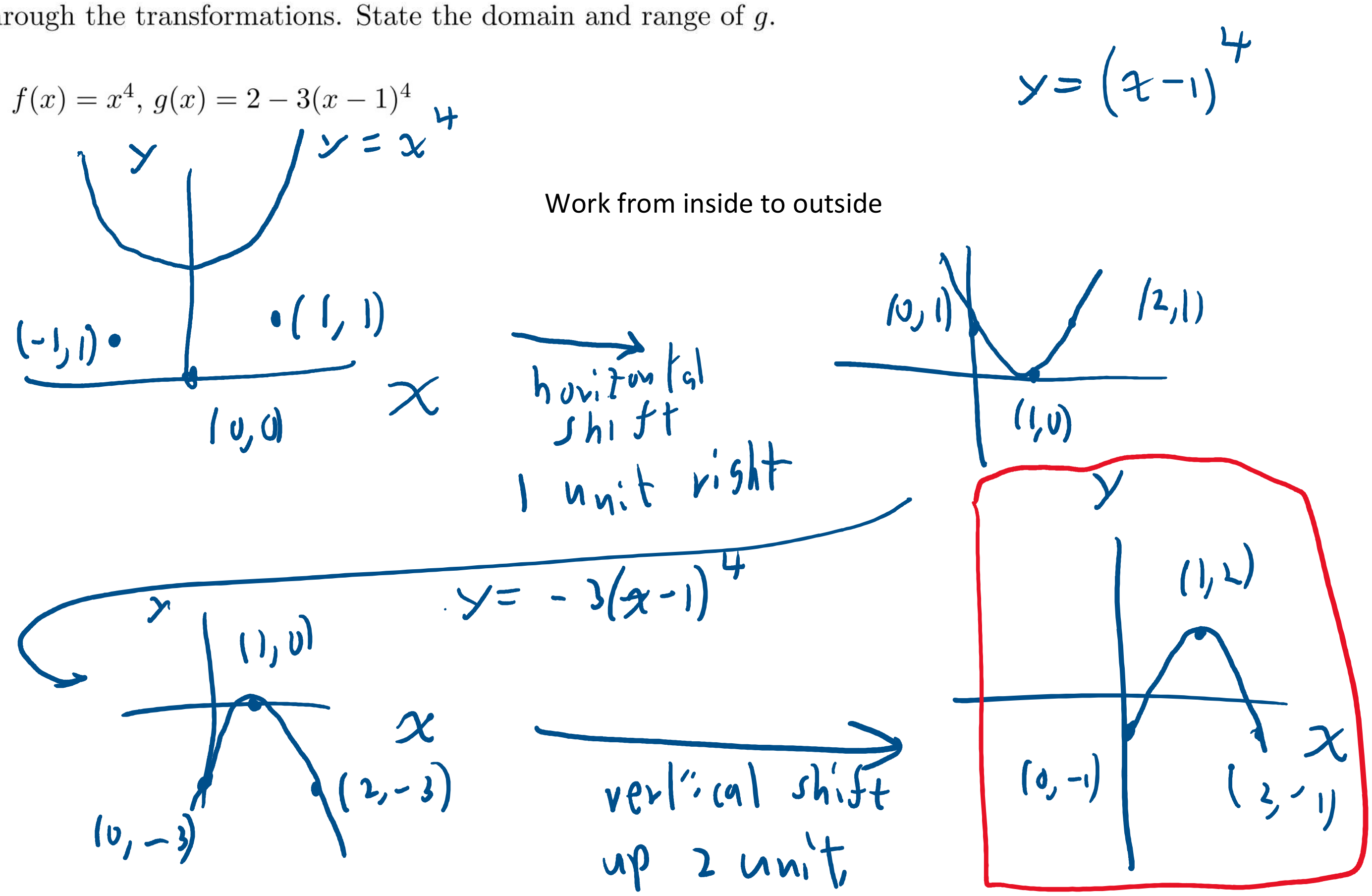
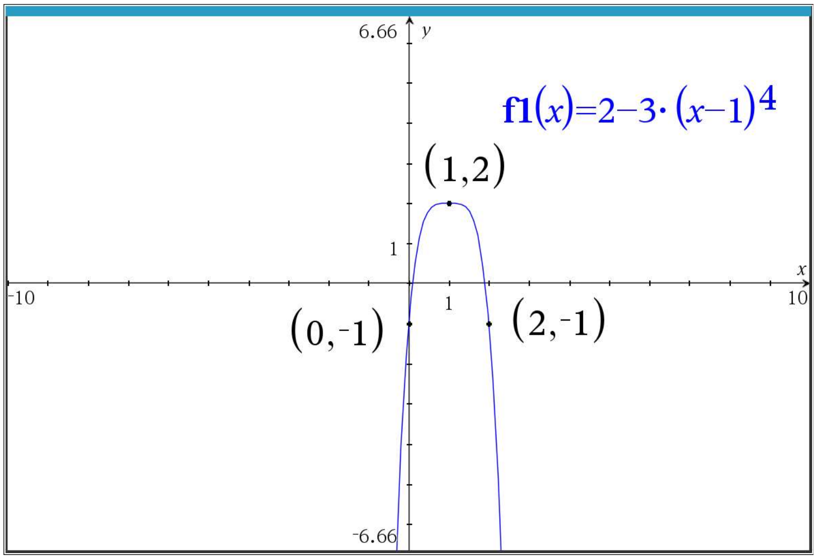
3.1:

In Exercises 21 - 26, given the pair of functions f and g, sketch the graph of $y = g(x)$ by starting with the graph of $y = f(x)$ and using transformations. Track at least three points of your choice through the transformations. State the domain and range of g.

23. $f(x) = x^4$, $g(x) = 2 - 3(x-1)^4$

Work from inside to outside





3.1

In Exercises 1 - 10, find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the given polynomial.

6. $s(t) = -4.9t^2 + v_0 t + s_0$

Degree is the highest exponent in the polynomial
Thus, the degree of s(t) is 2.

The leading term is the term (power of variable multiplied by a constant coefficient) with the highest power
Leading term is $-4.9t^2$.

The leading coefficient is -4.9
The constant term is s_0.

end behavior

$t \to -\infty$, then $s(t) \to -\infty$

$t \to \infty$, then $s(t) \to -\infty$

3.2
Long division of numbers

$$\begin{array}{r} 1 \\ 3\overline{)782} \\ 3 \\ \hline (4) \end{array}$$

$\frac{3}{4}$

$4 > 3$

$\therefore$ 1 is not correct

$$
\begin{array}{r|l}
 & 260\tfrac{2}{3} \\
3 & 782 \\
 & \underline{6} \\
 & 18 \\
 & \underline{18} \\
 & \quad 2 \\
 & \quad \underline{0} \\
 & \quad 2
\end{array}
$$

= remainder

$$\boxed{782 \div 3 = 260\tfrac{2}{3}}$$

check $3\left(260\tfrac{2}{3}\right) = 3(260) + 3\left(\tfrac{2}{3}\right)$

$= 780 + 2$

$= 782$ ✓

$$
\begin{array}{r|l}
 & 2x^2 - 7x + 8 - \frac{12}{x+1} \\
x+1 & 2x^3 - 5x^2 + x - 4 \\
 & \underline{2x^3 + 2x^2} \\
 & \quad -7x^2 + x \\
 & \quad \underline{-7x^2 - 7x} \\
 & \qquad 8x - 4 \\
 & \qquad \underline{8x + 8} \\
 & \qquad\quad -12
\end{array}
$$

x

$\underline{-(-7x)}$

$\underline{x + 7x}$

$8x$

$0x + 0 \quad | \quad 8x$

-12

Check:

$$(x+1)\left(2x^2 - 7x + 8 - \frac{12}{x+1}\right)$$

$$\begin{aligned} = \; & 2x^3 - 7x^2 + 8x \\ & \phantom{2x^3 -{}} 2x^2 - 7x + 8 - 12 \\ \hline & 2x^3 - 5x^2 + x - 4 \end{aligned}$$

$\checkmark$

$$(x+1)\left(\frac{-12}{x+1}\right)$$

Supplied

Theorem 3.4. Polynomial Division: Suppose $d(x)$ and $p(x)$ are nonzero polynomials where the degree of p is greater than or equal to the degree of d. There exist two unique polynomials, $q(x)$ and $r(x)$, such that $p(x) = d(x)\,q(x) + r(x)$, where either $r(x) = 0$ or the degree of r is strictly less than the degree of d.

Given polynomials $d(x)$ and $p(x)$ there exist (but we don't yet know what they are) polynomials $q(x)$ and $r(x)$

$$\underbrace{p(x)}_{\text{polynomial}} = \underbrace{d(x)}_{\text{divisor}}\, \underbrace{q(x)}_{\text{quotient}} + \underbrace{r(x)}_{\text{remainder}}$$

Memorize the theorem, understand the proof

Theorem 3.5. The Remainder Theorem: Suppose p is a polynomial of degree at least 1 and c is a real number. When $p(x)$ is divided by $x - c$ the remainder is $p(c)$.

$$p(x) = d(x)\, q(x) + r(x)$$

given $p(x)$, $d(x) = x - c$

Then Thm 3.4 $\Rightarrow \exists$ $q(x)$ $r(x)$

$p(x)$, $d(x) = x - c$

Then Thm 3.4 $\Rightarrow \exists$ $q(x), r(x)$

there exists

$$p(x) = (x - c)\, q(x) + r(x)$$

prove $r(x) = p(c)$

$$p(c) = (c - c)\, q(c) + r(c)$$

$$p(c) = (0)\, q(c) + r(c)$$

$$\boxed{p(c) = r(c)}$$

Thm 3.4 $\Rightarrow$ $r(x) = 0$ all x

or $\deg r < \deg d(x)$

$= \deg (x - c)$

$= 1$

either $r(x) = 0$ all x

or $\deg r = 0 \Rightarrow r(x) = \text{constant} \neq 0$

If $r(x) = \text{constant} \neq 0$

Then $r(x) = r(c)$

$\therefore$ $p(x) = r(x)$

If $r(x) = 0$ all x

Again $r(c) = 0$

$\therefore$ $p(x) = r(x)$
