03-24-25 MTH 161-C06n

3.1 Graphs of Polynomials
   3.1.1 Exercises
   page 246: 3, 7, 13, 21, 27

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16 textbook sections remaining
13 MTH 161 class meetings before final exam
16/13=1.2308
1 or 2 sections each class meeting

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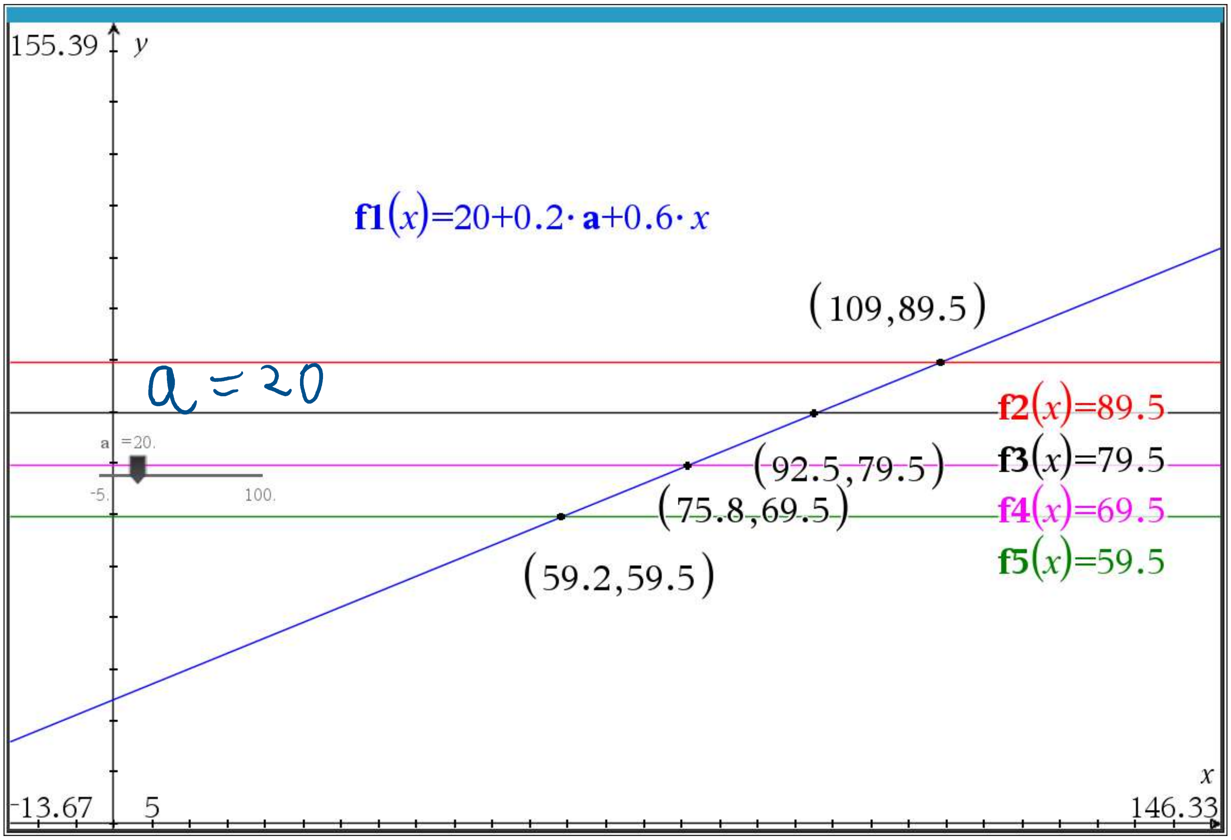
Exam 2		stem & leaf		
34.76923	mean			A-0
28	median	8	03	B-2
23.85905	st. dev	7		C-0
5	min	6		D-0
83	max	5	9	F- 11
13	count	4	3	
		3	1	
		2	01689	
		1	25	
		0	5	

Exam 1		stem & leaf		
39.25	mean			A-0
28.5	median			B-0
19.66013	st. dev	7	26	C-2
15	min	6	6	D-1
76	max	5		F- 9
12	count	4	4	
		3	4	
		2	577889	
		1	5	



3.1
Memorize

Definition 3.1. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0,$$

where a_0, a_1, $\ldots$, a_n are real numbers and $n \geq 1$ is a natural number. The domain of a polynomial function is $(-\infty, \infty)$.

memorize

Definition 3.2. Suppose f is a polynomial function.

- Given $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$, we say
  - The natural number n is called the **degree** of the polynomial f.
  - The term $a_n x^n$ is called the **leading term** of the polynomial f.
  - The real number a_n is called the **leading coefficient** of the polynomial f.
  - The real number a_0 is called the **constant term** of the polynomial f.
- If $f(x) = a_0$, and $a_0 \neq 0$, we say f has degree 0.
- If $f(x) = 0$, we say f has no degree.[a]

[a]Some authors say $f(x) = 0$ has degree $-\infty$ for reasons not even we will go into.

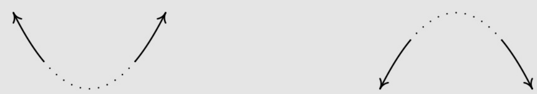
Memorize

End Behavior of functions $f(x) = ax^n$, n even.

Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and n is an even natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:



$a > 0$ $\qquad$ $a < 0$

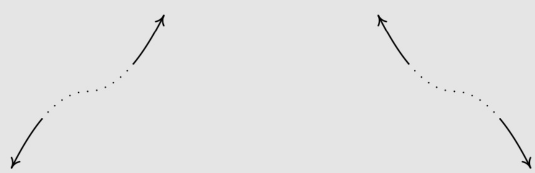
memorize

End Behavior of functions $f(x) = ax^n$, n odd.

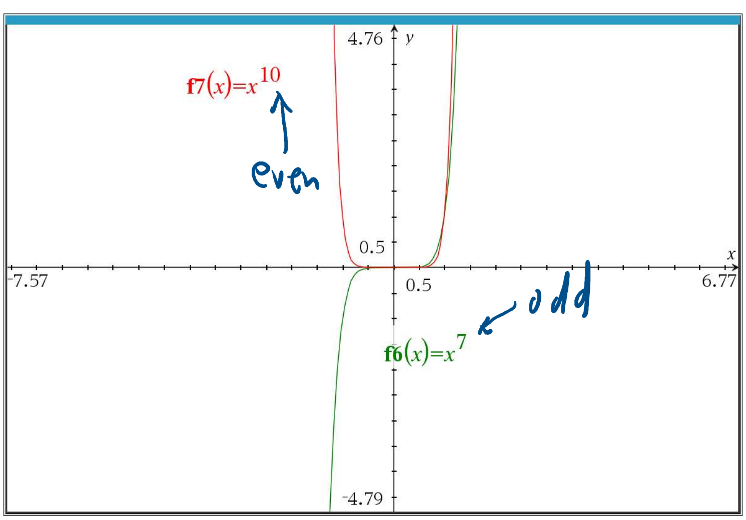
Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and $n \geq 3$ is an odd natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:

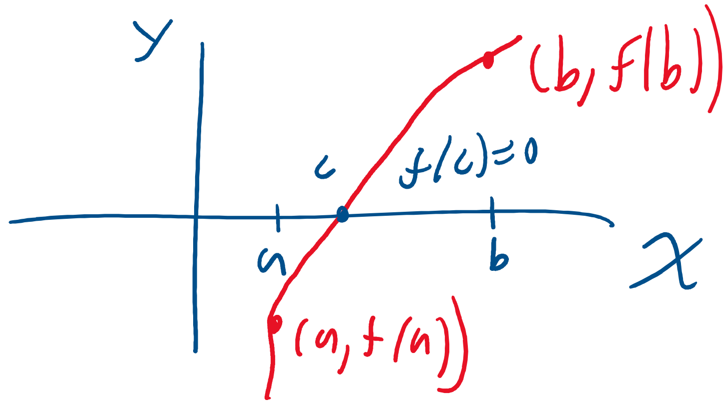


$a > 0$ $\qquad$ $a < 0$



Supplied

Theorem 3.1. The Intermediate Value Theorem (Zero Version): Suppose f is a continuous function on an interval containing $x = a$ and $x = b$ with $a < b$. If $f(a)$ and $f(b)$ have different signs, then f has at least one zero between $x = a$ and $x = b$; that is, for at least one real number c such that $a < c < b$, we have $f(c) = 0$.



$f(b) > 0$

$f(a) < 0$

Know this process

Steps for Constructing a Sign Diagram for a Polynomial Function

Suppose f is a polynomial function.

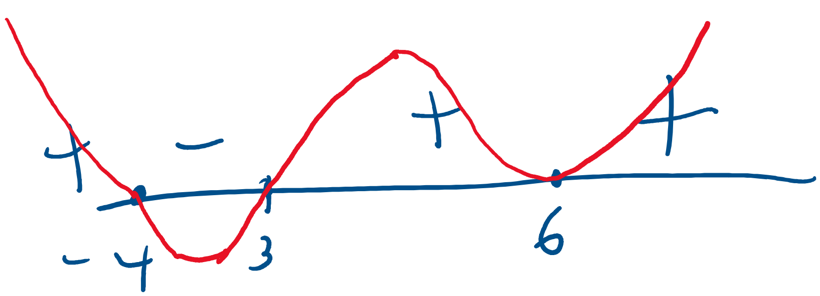
1. Find the zeros of f and place them on the number line with the number 0 above them.
2. Choose a real number, called a **test value**, in each of the intervals determined in step 1.
3. Determine the sign of $f(x)$ for each test value in step 2, and write that sign above the corresponding interval.

$f(x) = (x-3)(x+4)(x-6)^2$

$\deg f = 4$

leading term $= x^4$

leading coef $= 1$



$f(-5) = (-5-3)(-5+4)(-5-6)^2$

$f(-5) = (-5-3)(-5+4)(-5-6)^2$

$f(-5) = (-)(-)(+) > 0$

5

$f(5) = (5-3)(5+4)(5-6)^2$

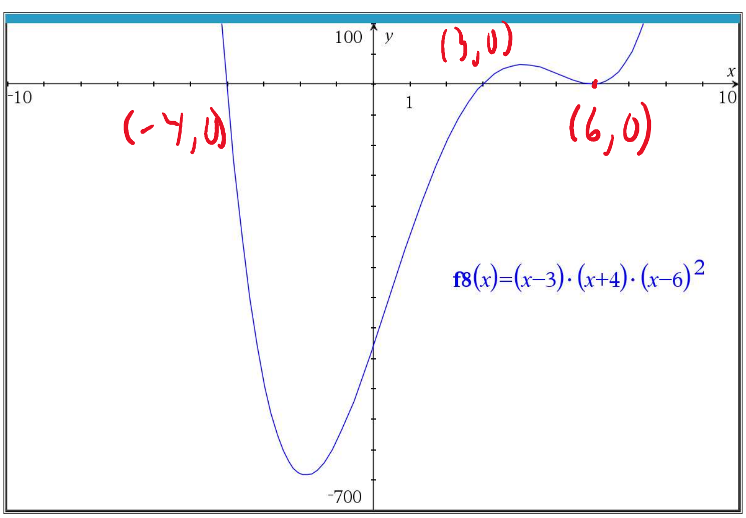
$f(5) = (+)(+)(+) > 0$

$f(0) = (0-3)(0+4)(0-6)^2$

$f(0) = (-)(+)(+) < 0$

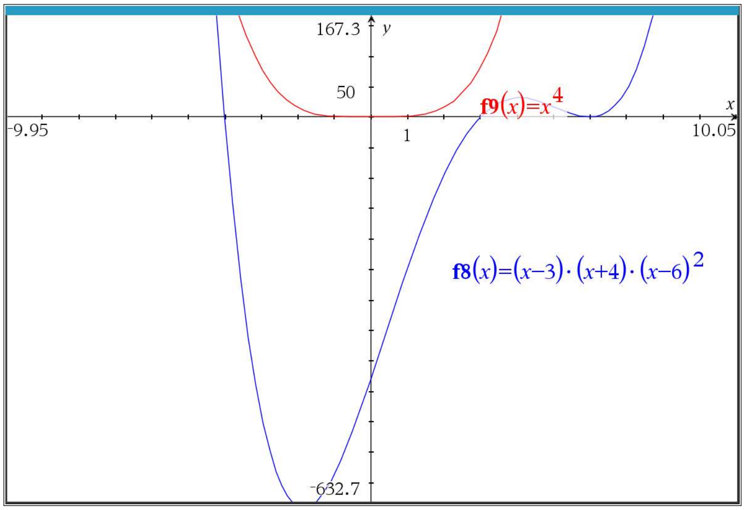
$f(7) = (7-3)(7+4)(7-6)^2$

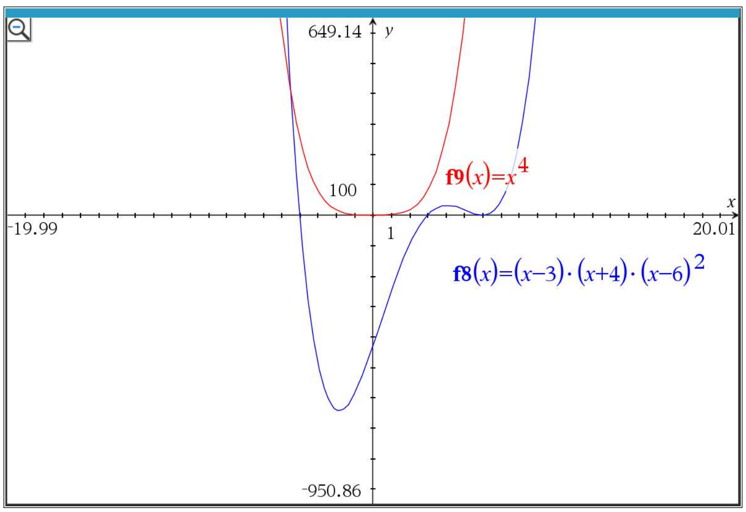
$= (+)(+)(+) > 0$

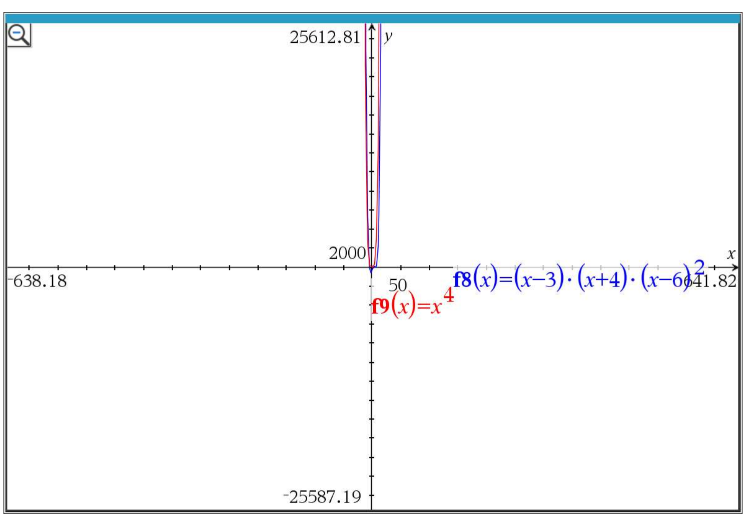


Memorize

Theorem 3.2. End Behavior for Polynomial Functions: The end behavior of a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ with $a_n \neq 0$ matches the end behavior of $y = a_n x^n$.







Memorize

Definition 3.3. Suppose f is a polynomial function and m is a natural number. If $(x-c)^m$ is a factor of $f(x)$ but $(x-c)^{m+1}$ is not, then we say $x = c$ is a zero of **multiplicity** m.

Memorize

Theorem 3.3. The Role of Multiplicity: Suppose f is a polynomial function and $x = c$ is a zero of multiplicity m.

- If m is even, the graph of $y = f(x)$ touches and rebounds from the x-axis at $(c, 0)$.
- If m is odd, the graph of $y = f(x)$ crosses through the x-axis at $(c, 0)$.