03-03-25 MTH 161-C06N

2.2 Absolute Value Functions

2.2.1 Exercises

page 183: 1, 2, 15, 17, 22, 29

2.3 Quadratic Functions

2.3.1 Exercises

page 200: 1, 4, 11, 21, 26

2.2:17

Prove that if $|f(x)| = |g(x)|$ then either $f(x) = g(x)$ or $f(x) = -g(x)$. Use that result to solve the equations in Exercises 16 - 21.

17. $|3x + 1| = |4x|$

$f(x) = 3x+1$

$g(x) = 4x$

given $|f(x)| = |g(x)|$ for #17

$\Rightarrow f(x) = g(x)$ or $f(x) = -g(x)$

$\Rightarrow 3x+1 = 4x$ or $3x+1 = -(4x)$

$\boxed{1 = x}$

$7x = -1$

$\boxed{x = -\frac{1}{7}}$

check

$|3(1)+1| \stackrel{?}{=} |4(1)|$

$|4| \stackrel{?}{=} |4|$

$4 = 4$ ✓

$\left|3\left(-\frac{1}{7}\right)+1\right| \stackrel{?}{=} \left|4\left(-\frac{1}{7}\right)\right|$

$\left|-\frac{3}{7}+1\right| \stackrel{?}{=} \left|-\frac{4}{7}\right|$

$\left|-\frac{3}{7}+\frac{7}{7}\right| \stackrel{?}{=} \frac{4}{7}$

$$\left|\frac{4}{7}\right| \stackrel{?}{=} \frac{4}{7}$$

$$\frac{4}{7} = 4/7 \checkmark$$

Given $|f(x)| = |g(x)|$,
Prove $f(x) = \pm g(x)$.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

Strategy: consider all the cases

Case 1) $f(x) \geq 0$ and $g(x) \geq 0$ for some x

$\Rightarrow |f(x)| = f(x)$

$|g(x)| = g(x)$

$|f(x)| = |g(x)| \Rightarrow f(x) = g(x)$

case 2) $f(x) \geq 0$, $g(x) < 0$

$|f(x)| = f(x)$

$|g(x)| = -g(x)$

$|f(x)| = |g(x)| \Rightarrow f(x) = -g(x)$

Finish case 3: $f(x) < 0$, $g(x) \geq 0$

case 4: $f(x) < 0$, $g(x) < 0$

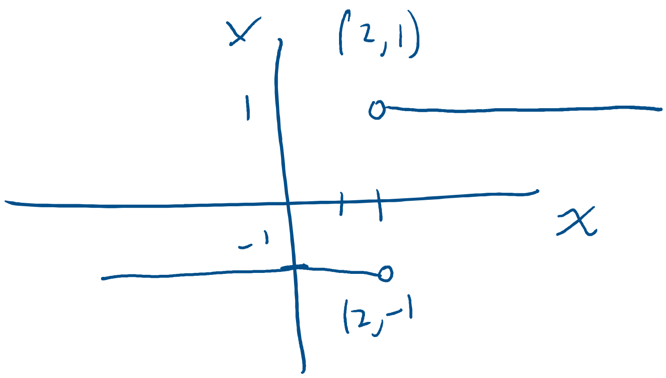
2.2: 29

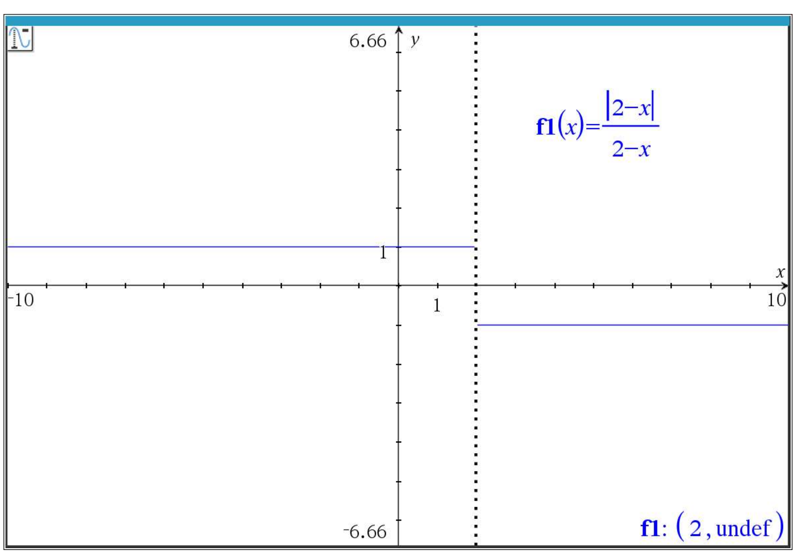
In Exercises 22 - 33, graph the function. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

29. $f(x) = \dfrac{|2-x|}{2-x}$

$$f(x) = \begin{cases} \dfrac{2-x}{2-x} & \text{if } 2-x \geq 0 \text{ and } \cancel{x \neq 2} \iff x < 2 \\ \dfrac{-(2-x)}{2-x} & \text{if } 2-x < 0 \iff x > 2 \end{cases}$$

$$f(x) = \begin{cases} 1 & \text{if } x \leq 2 \\ -1 & \text{if } x > 2 \end{cases}$$





2.3 memorize

Definition 2.5. A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c,$$

where a, b and c are real numbers with $a \neq 0$. The domain of a quadratic function is $(-\infty, \infty)$.

Memorize

Definition 2.6. Standard and General Form of Quadratic Functions: Suppose f is a quadratic function.

- The **general form** of the quadratic function f is $f(x) = ax^2 + bx + c$, where a, b and c are real numbers with $a \neq 0$.
- The **standard form** of the quadratic function f is $f(x) = a(x-h)^2 + k$, where a, h and k are real numbers with $a \neq 0$. Vertex form

Show that the vertex form actually gives a quadratic function.

$f(x) = a(x-h)^2 + k$

Find b, c in terms of a, h, k

$f(x) = a(x^2 - 2hx + h^2) + k$

$(p-q)^2 = (p-q)(p-q)$

$= p^2 - pq - pq + q^2$

$= p^2 - 2pq + q^2$

$$f(x) = ax^2 \underbrace{-2ah}_{b}x + \underbrace{ah^2 + k}_{c}$$

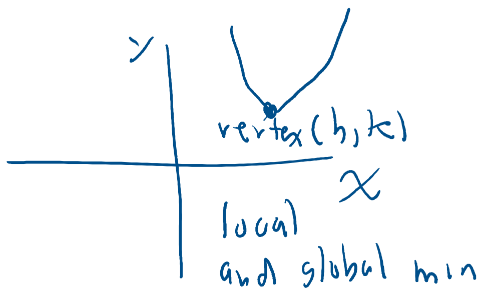
$f(x) = ax^2 + bx + c$

$\therefore$ vertex form is equivalent to the general form

$f(x) = a(x-h)^2 + k$, assume $a > 0$

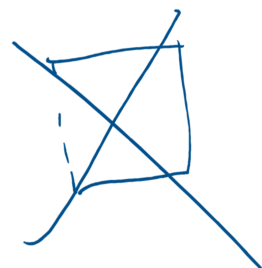
y

$f(h) = a(h-h)^2 + k$



$$f(h) = a(h-h)^2 + k$$
$$= a(0)^2 + k$$
$$= 0 + k$$
$$= k$$

convert from general form to vertex form
by completing the square



Let $f(x) = 2x^2 + 6x - 5$

① group x terms together

$$f(x) = (2x^2 + 6x) - 5$$

② factor out the coefficial of x^2

$$f(x) = 2(x^2 + 3x) - 5$$

③ Take $\left(\frac{1}{2}\right)$ coefficient of x
square it
add and subtract the result
inside the parentheses

$$\left(\frac{1}{2}\right)(3) = \frac{3}{2}$$
$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$f(x) = 2\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4}\right) - 5$$

④ Take the negative constant
outside the parentheses

$$f(x) = 2\left(x^2 + 3x + \frac{9}{4}\right) - (2)\left(\frac{9}{4}\right) - 5$$

$$f(x) = 2\left(x^2 + 3x + \frac{9}{4}\right) - (2)\left(\frac{9}{4}\right) - 5$$

(5) inside the parentheses
we have a perfect square

$$f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} - 5$$

$$= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} - \frac{10}{2}$$

$$\boxed{f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{19}{2}}$$