1.7 Transformations

1.7.1 Exercises

page 140: 1,4, 19, 24, 48, 56

---

2 Linear and Quadratic Functions

2.1 Linear Functions

2.1.1 Exercises

page 163: 5, 7, 15, 18, 26, 28, 32, 35, 45

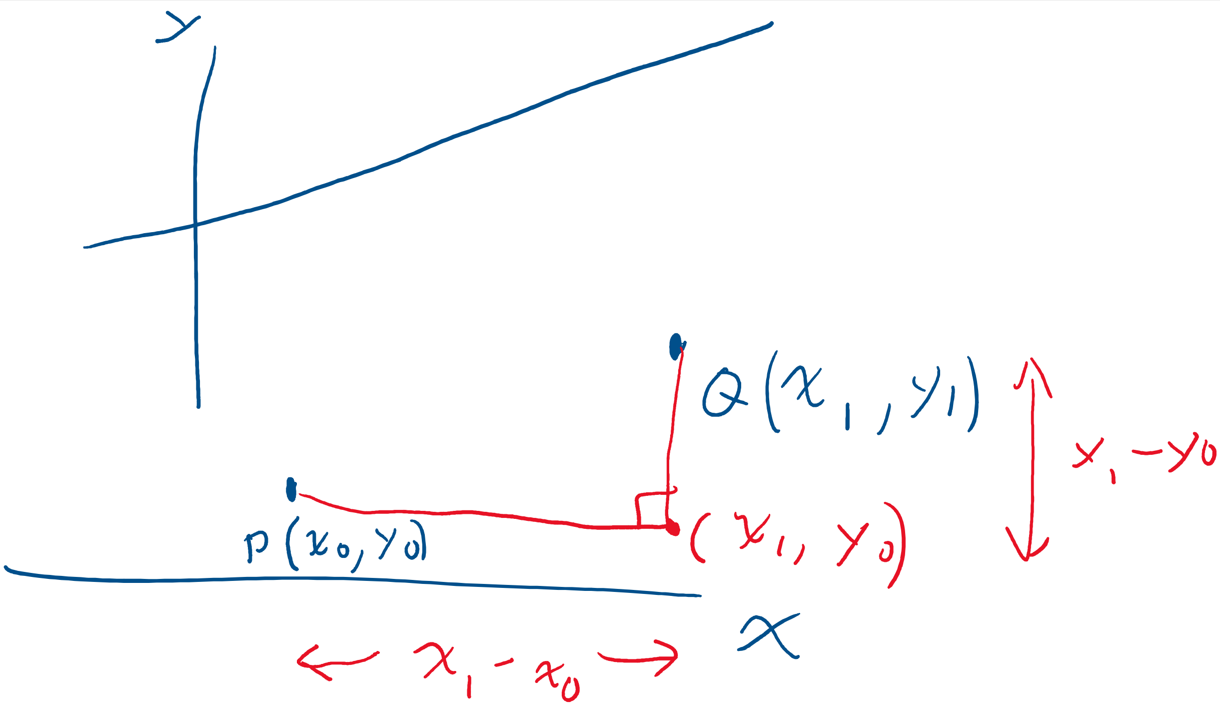
2.1

Memorize

Equation 2.1. The **slope** m of the line containing the points $P\,(x_0, y_0)$ and $Q\,(x_1, y_1)$ is:

$$m = \frac{y_1 - y_0}{x_1 - x_0},$$

provided $x_1 \neq x_0$.



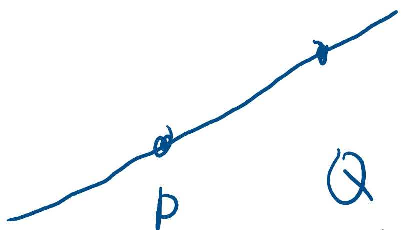
$m = \dfrac{\Delta y}{} = \dfrac{\text{rise}}{} =$ rate of change

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \text{rate of change of } y \text{ with respect to } x$$

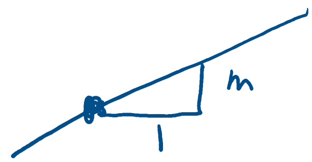
Δ = change

memorize

Equation 2.2. The **point-slope form** of the line with slope m containing the point (x_0, y_0) is the equation $y - y_0 = m(x - x_0)$.

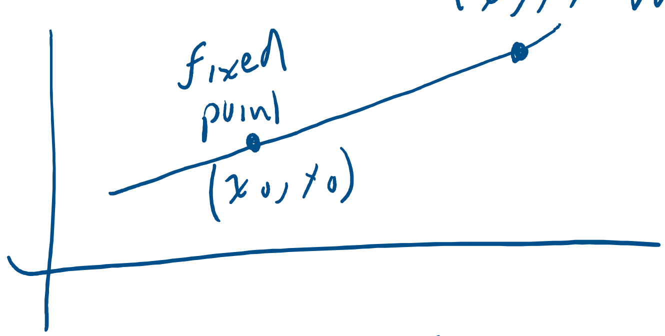


Two points determine a line



One point and a slope determine a line

(x, y) variable point



$$m = \frac{y - y_0}{x - x_0}$$

multiply both sides by $x - x_0$

$$m(x - x_0) = \left(\frac{y - y_0}{\cancel{x - x_0}}\right)(\cancel{x - x_0})$$

$$\boxed{y - y_0 = m(x - x_0)}$$

Memorize

Equation 2.3. The **slope-intercept form** of the line with slope m and y-intercept $(0, b)$ is the equation $y = mx + b$.

Derive the slope-intercept form from the point-slope form.

$$y - y_0 = m(x - x_0)$$

solve for y and match corresponding item

$$y - y_0 = mx - mx_0$$

$$y = mx - mx_0 + y_0$$

Let $b = -mx_0 + y_0$ = constant

Then $y = mx + b$

Memorize

Definition 2.1. A **linear function** is a function of the form

$$f(x) = mx + b,$$

where m and b are real numbers with $m \neq 0$. The domain of a linear function is $(-\infty, \infty)$.

Definition 2.2. A **constant function** is a function of the form

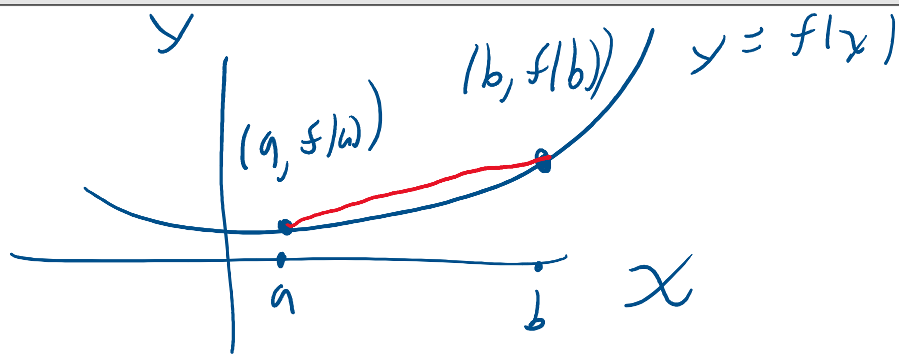
$$f(x) = b,$$

where b is real number. The domain of a constant function is $(-\infty, \infty)$.

Memorize

Definition 2.3. Let f be a function defined on the interval $[a, b]$. The **average rate of change** of f over $[a, b]$ is defined as:

$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$



2.1

2.1.1 Exercises

In Exercises 1 - 10, find both the point-slope form and the slope-intercept form of the line with the given slope which passes through the given point.

2. $m = -2, \quad P(-5, 8)$

Equation 2.2. The **point-slope form** of the line with slope m containing the point (x_0, y_0) is the equation $y - y_0 = m(x - x_0)$.

Equation 2.3. The **slope-intercept form** of the line with slope m and y-intercept $(0, b)$ is the equation $y = mx + b$.

$y - y_0 = m(x - x_0)$

$m = -2$

$x_0 = -5, \quad y_0 = 8$

$y - 8 = -2(x - (-5))$

$\boxed{y - 8 = -2(x + 5)}$ point-slope

$y - 8 = -2x - 10$

$y - 8 + 8 = -2x - 10 + 8$

enough

$y = -2x - 10 + 8$

$$y - 8 + 8 = -2x - 10 + 8$$
$$y + (-8 + 8) = -2x + (-10 + 8)$$
$$y + 0 = -2x + (-2)$$
$$\boxed{y = -2x - 2}$$

$$y = -2x - 10 + 8$$
$$\boxed{y = -2x - 2}$$

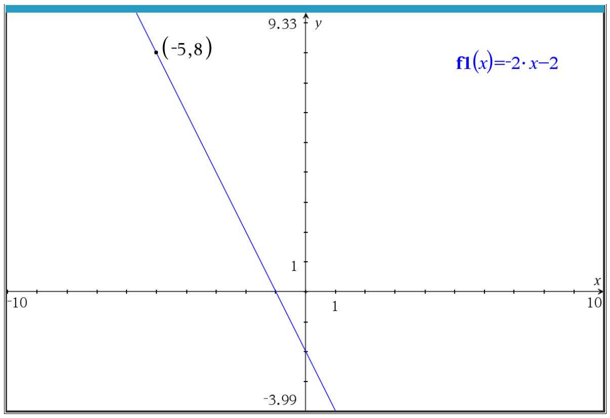
Derive the slope-intercept form directly from the fixed point and the slope

$$m = -2$$
$$(x_0, y_0) = (-5, 8)$$
$$y = mx + b$$
$$y = -2x + b$$

This equation is true for all points on the line.
In particular, it is true for (-5,8).

$$8 = (-2)(-5) + b$$
$$8 = 10 + b$$
$$b = 8 - 10$$
$$\boxed{b = -2}$$
$$\boxed{y = -2x - 2}$$

Graph equation on calculator
Use trace with x = -5 to verify that
the point (-5,8) is on the line.



2.1

In Exercises 11 - 20, find the slope-intercept form of the line which passes through the given points.

12. $P(-1,-2),\ Q(3,-2)$

$y - y_0 = m(x - x_0)$

$y = mx + b$

$m = \dfrac{y_1 - y_0}{x_1 - x_0}$

$m = \dfrac{-2-(-2)}{3-(-1)}$

$= \dfrac{-2+2}{3+1} = \dfrac{0}{4} = 0$

$\boxed{m = 0}$

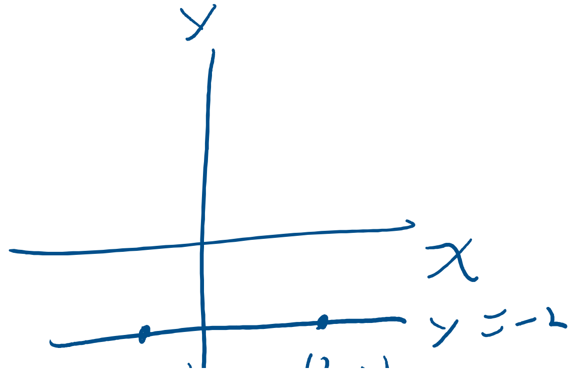
$y = mx + b$

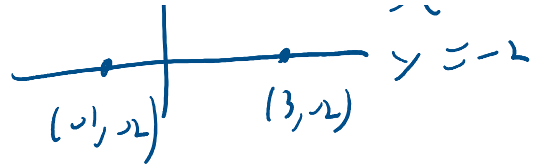
$y = 0 \cdot x + b$

$\boxed{y = b}$

$\boxed{b = -2}$

$\boxed{y = -2}$

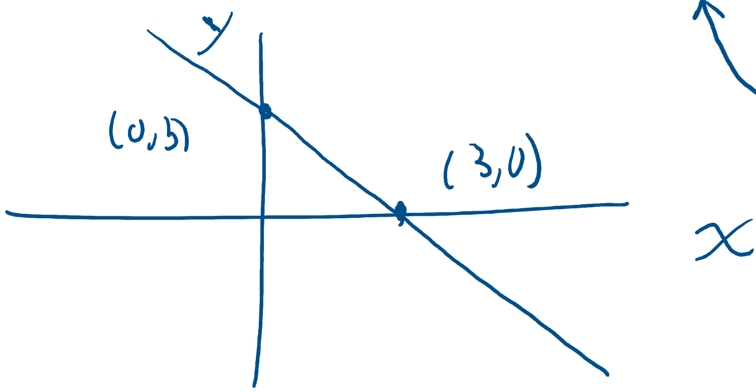




2.1

In Exercises 21 - 26, graph the function. Find the slope, y-intercept and x-intercept, if any exist.

22. $f(x) = 3 - x \iff y = -x + 3$



$\boxed{m = -1}$

x-intercept
set $y = 0$
solve for x

$3 - x = 0$

$-x = -3$

$\boxed{x = 3}$

y-intercept
set $x = 0$
solve for y

$3 - 0 = y$

$\boxed{y = 3}$