04-01-25 MTH 161-004N

3.2 The Factor Theorem and the Remainder Theorem

3.2.1 Exercises

page 265: 1, 3, 9, 21, 35, 42

3.3 Real Zeros of Polynomials

3.3.3: Exercises

page 280: 1, 31, 37, 48

Quiz 5, closed notes, closed book

3.2:35

In Exercises 31 - 40, you are given a polynomial and one of its zeros. Use the techniques in this section to find the rest of the real zeros and factor the polynomial.

35. $x^3 + 2x^2 - 3x - 6, \quad c = -2$

After your manual calculations, find the zeros graphically on your calculator and compare the results.

Let $p(x) = x^3 + 2x^2 - 3x - 6$

Factor Thm. $\Rightarrow$ $x-(-2) = x+2$ is a factor of $p(x)$

$\Rightarrow p(x) = (x+2)\,q(x)$

$q(x$ = unknown quadratic polynomial

long division

$$
\begin{array}{r|l}
 & x^2 \qquad\quad -3 \\
\hline
x+2 & x^3 + 2x^2 - 3x - 6 \\
 & x^3 + 2x^2 \\
\hline
 & \qquad\quad -3x - 6 \\
 & \qquad\quad -3x - 6 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-2 & 1 & 2 & -3 & -6 \\
 & & -2 & 0 & 6 \\
\hline
 & 1 & 0 & -3 & (0)
\end{array}
$$

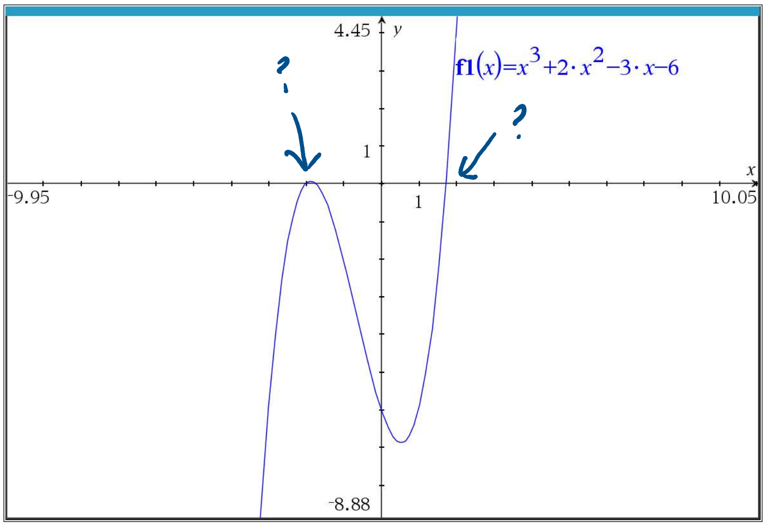
$\boxed{q(x) = x^2 - 3}$

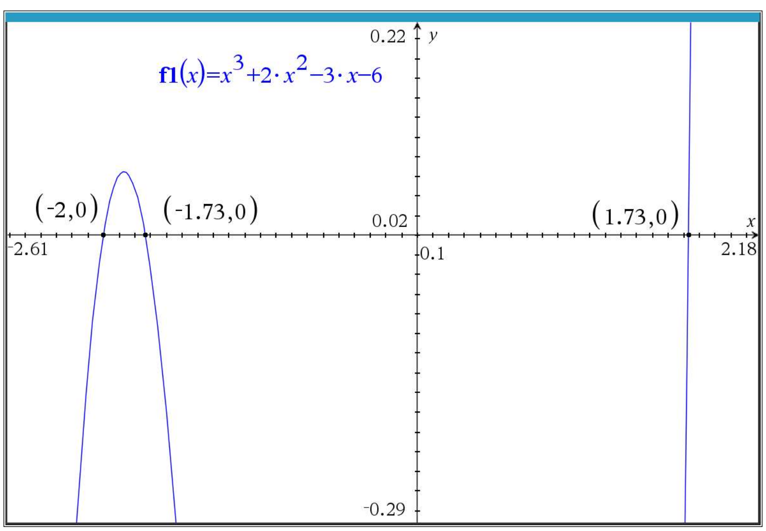
$$\boxed{q(x) = x^2 - 3}$$

$$p(x) = (x+2)(x^2-3)$$

$$\boxed{p(x) = (x+2)(x+\sqrt{3})(x-\sqrt{3})}$$

The zeros are $\boxed{-2, \pm\sqrt{3}}$





From graph

$$\boxed{x \approx -2, \pm 1.73}$$

Sqrt(3)=1.732050807568877

3.3

Supplied

Theorem 3.8. Cauchy's Bound: Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$. Let M be the largest of the numbers: $\frac{|a_0|}{|a_n|}, \frac{|a_1|}{|a_n|}, \ldots, \frac{|a_{n-1}|}{|a_n|}$. Then all the real zeros of f lie in in the interval $[-(M+1), M+1]$.

$f(x) = x^3 + 2x^2 - 3x - 6$

$a_0 = -6$

$a_1 = -3$

$a_2 = 2$

$a_3 = 1$

$M = \max\left\{ \frac{|-6|}{|1|}, \frac{|-3|}{|1|}, \frac{|2|}{|1|} \right\}$

$M = \max\{6, 3, 2\} = 6$

$M + 1 = 7$

$\therefore [-7, 7]$ includes all real zeros.

Our examples supports the theorem.

supplied

> **Theorem 3.9. Rational Zeros Theorem:** Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$, and $a_0, a_1, \ldots a_n$ are integers. If r is a rational zero of f, then r is of the form $\pm\frac{p}{q}$, where p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.

$f(x) = x^3 + 2x^2 - 3x - 6$

$a \mid b$ means "a divides into b evenly"

or "a is a factor of b"

$a_n = a_3 = 1$

$a_0 = -6$

$p \mid -6 \Rightarrow p = \pm 1, \pm 2, \pm 3, \pm 6$

$q \mid 1 \Rightarrow q = \pm 1$

$$\boxed{\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6}$$

$x = -2$ is in our list

... to verify that $f(-2) = 0$

$x = -2$ is in our ...

check to verify that $f(-2) = 0$

Is $x = \pm 1.73$ in our list? No

$\therefore x \approx \pm 1.73$ are irrational zeros.

Supplied

Theorem 3.10. Descartes' Rule of Signs: Suppose $f(x)$ is the formula for a polynomial function written with descending powers of x.

- If P denotes the number of variations of sign in the formula for $f(x)$, then the number of positive real zeros (counting multiplicity) is one of the numbers $\{P, P-2, P-4, \ldots\}$.
- If N denotes the number of variations of sign in the formula for $f(-x)$, then the number of negative real zeros (counting multiplicity) is one of the numbers $\{N, N-2, N-4, \ldots\}$.

$f(x) = x^3 + 2x^2 - 3x - 6$

$0 \quad 1 \quad 0$

$P = 1, \; 1, \cancel{1-2}, \cancel{1-4}, \ldots$

$P = 1 \quad \cancel{-1}, \cancel{-3}, \ldots$

$\therefore$ there is exactly 1 positive real zero

This agrees with the prediction of Descartes' rule of signs.

$f(-x) = (-x)^3 + 2(-x)^2 - 3(-x) - 6$

$f(-x) = -x^3 + 2x^2 + 3x - 6$

$1 \quad 0 \quad 1$

$N = 2$

$\therefore$ there are 2 or $2-2=0$ negative real zeros

In fact, there are 2 neg, real zeros

This agrees with Descartes' rule of signs

In fact, there are 2 neg, real zeros
This agrees with Descartes' rule of signs

Omit theorem 3.11, upper and lower bound theorem