03-25-25 MTH 161-004N

3.1 Graphs of Polynomials

3.1.1 Exercises

page 246: 3, 7, 13, 21, 27

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16 textbook sections remaining

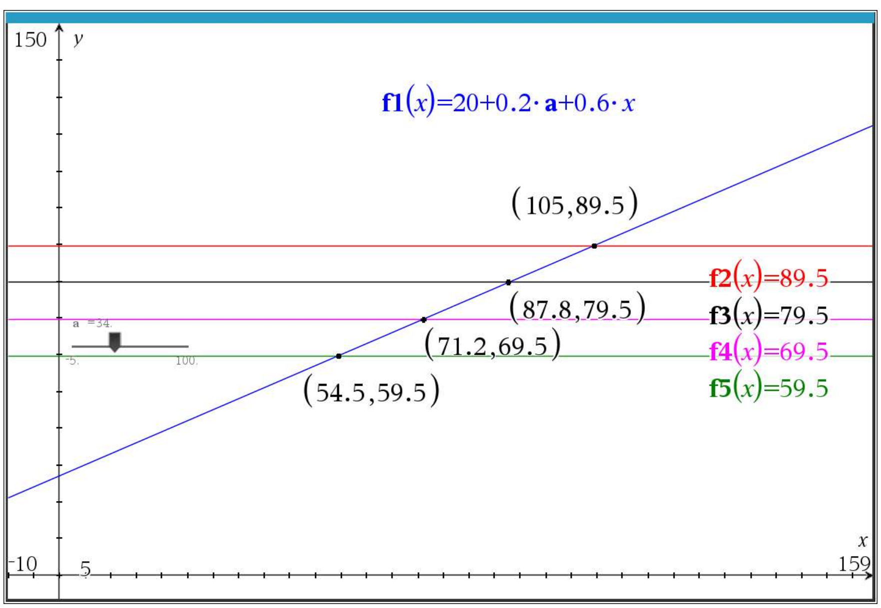
12 class meetings before final exam

16/12=1.3333

1 or 2 sections each class meeting

Exam 2		stem & leaf		
35	mean	9	1	A-1
27	median	8		B-0
20.97618	st. dev	7		C-0
18	min	6		D-0
91	max	5		F-9
10	count	4	49	
		3		
		2	34778	
		1	89	

Exam 1		stem & leaf		
36.8	mean	9	6	A-1
34	median	8		B-0
21.10039	st. dev	7		C-0
15	min	6	9	D-1
96	max	5		F-13
15	count	4	078	
		3	447	
		2	0349	
		1	579	



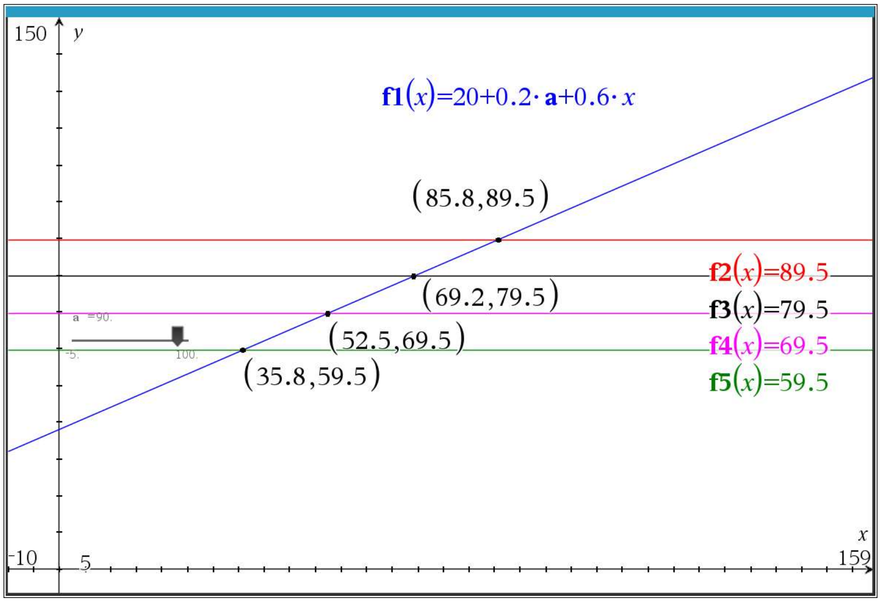
To obtain the following course grades, the median student needs the following final exam scores.

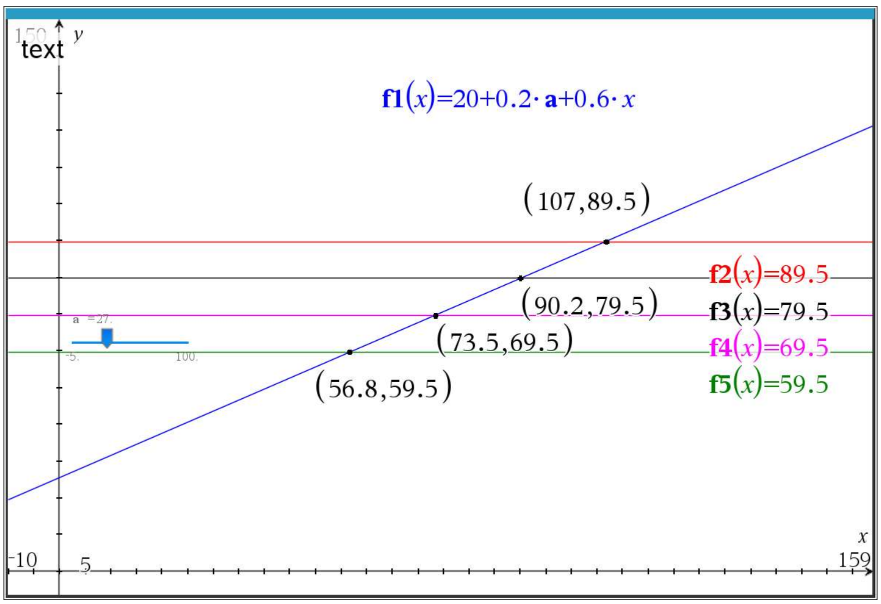
105% - A

88% - B

72% - C

55% - D





3.1
Memorize

Definition 3.1. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0,$$

where a_0, a_1, $\ldots$, a_n are real numbers and $n \geq 1$ is a natural number. The domain of a polynomial function is $(-\infty, \infty)$.

Memorize

Definition 3.2. Suppose f is a polynomial function.

- Given $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ with $a_n \neq 0$, we say
  - The natural number n is called the **degree** of the polynomial f.
  - The term $a_n x^n$ is called the **leading term** of the polynomial f.
  - The real number a_n is called the **leading coefficient** of the polynomial f.

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  - The real number a_n is called the **leading coefficient** of the polynomial f.
  - The real number a_0 is called the **constant term** of the polynomial f.
- If $f(x) = a_0$, and $a_0 \neq 0$, we say f has degree 0.
- If $f(x) = 0$, we say f has no degree.[a]

[a] Some authors say $f(x) = 0$ has degree $-\infty$ for reasons not even we will go into.

Copilot

What is the degree of the polynomial function f(x) = 0?

The degree of a polynomial is determined by the highest power of the variable in its terms. In the case of (f(x) = 0), there are no terms involving a variable, so it's essentially a constant function.

By convention, the degree of the zero polynomial (where all coefficients are zero, resulting in (f(x) = 0)) is **undefined**. Some contexts may assign it a degree of (-\infty) for theoretical purposes, but the generally accepted answer is **undefined**.

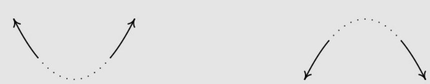
memorize

End Behavior of functions $f(x) = ax^n$, n even.

Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and n is an even natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$

Graphically:



$a > 0$ $\qquad$ $a < 0$

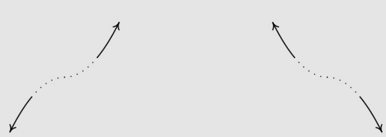
Memorize

End Behavior of functions $f(x) = ax^n$, n odd.

Suppose $f(x) = ax^n$ where $a \neq 0$ is a real number and $n \geq 3$ is an odd natural number. The end behavior of the graph of $y = f(x)$ matches one of the following:

- for $a > 0$, as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$
- for $a < 0$, as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$

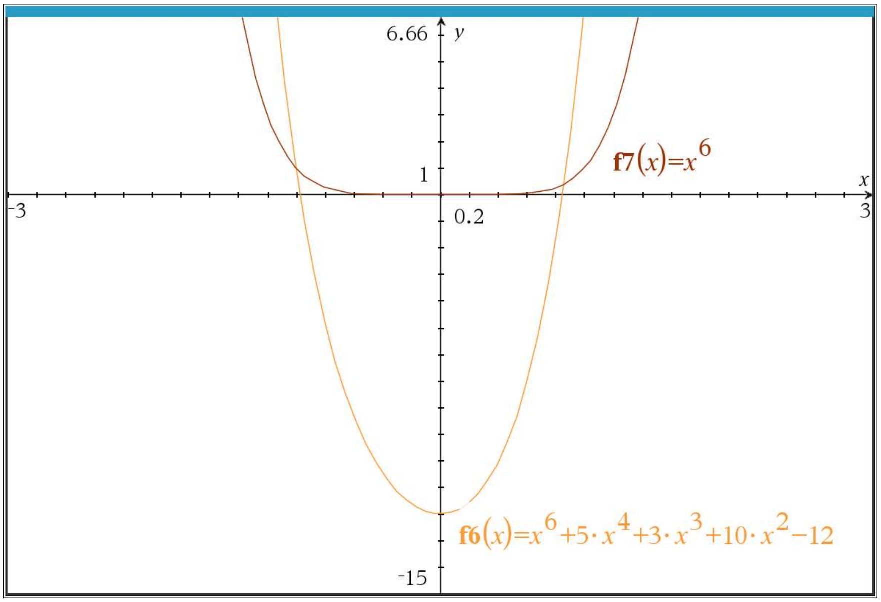
Graphically:

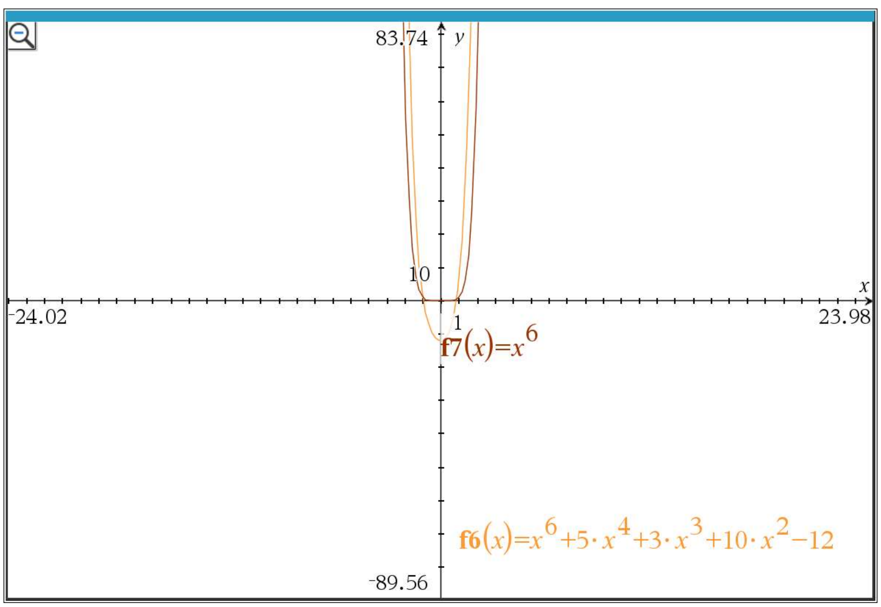


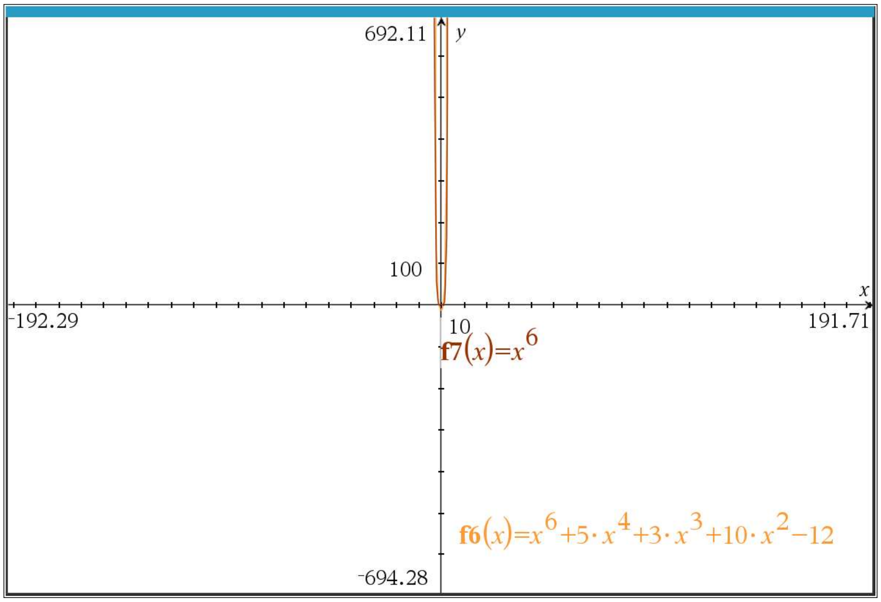
$a > 0$ $\qquad$ $a < 0$

Memorize

Theorem 3.2. End Behavior for Polynomial Functions: The end behavior of a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with $a_n \neq 0$ matches the end behavior of $y = a_n x^n$.

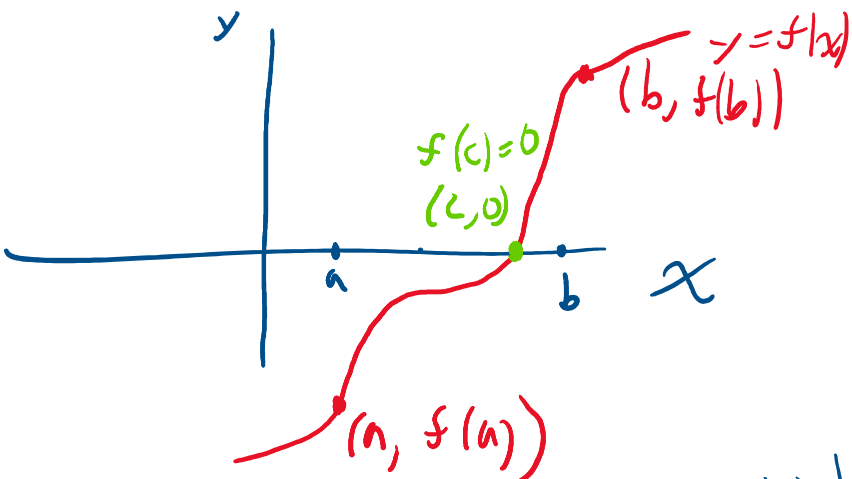


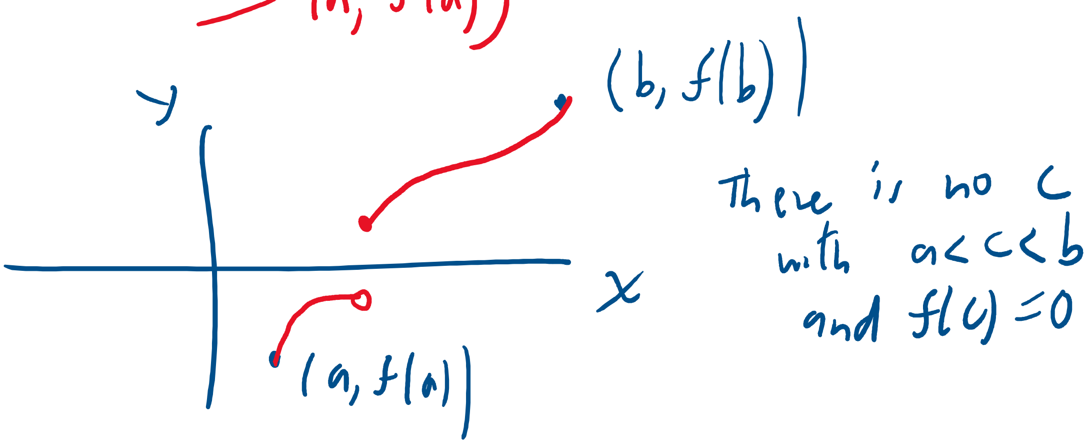




Supplied

> **Theorem 3.1. The Intermediate Value Theorem (Zero Version):** Suppose f is a continuous function on an interval containing $x = a$ and $x = b$ with $a < b$. If $f(a)$ and $f(b)$ have different signs, then f has at least one zero between $x = a$ and $x = b$; that is, for at least one real number c such that $a < c < b$, we have $f(c) = 0$.

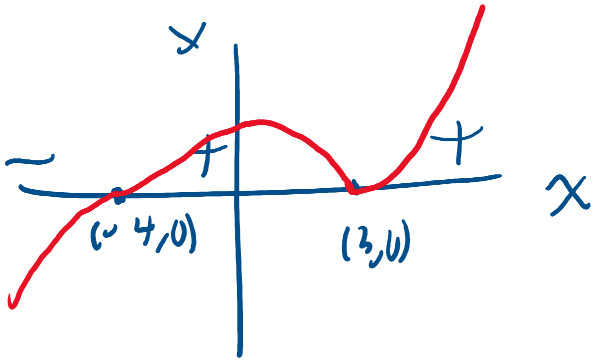




This example does not violate the intermediate value theorem, because the function is not continuous.

Use a sign diagram to make a rough sketch of a polynomial.

$$f(x) = (x+4)(x-3)^2$$



$$f(-5) = (-5+4)(-5-3)^2$$

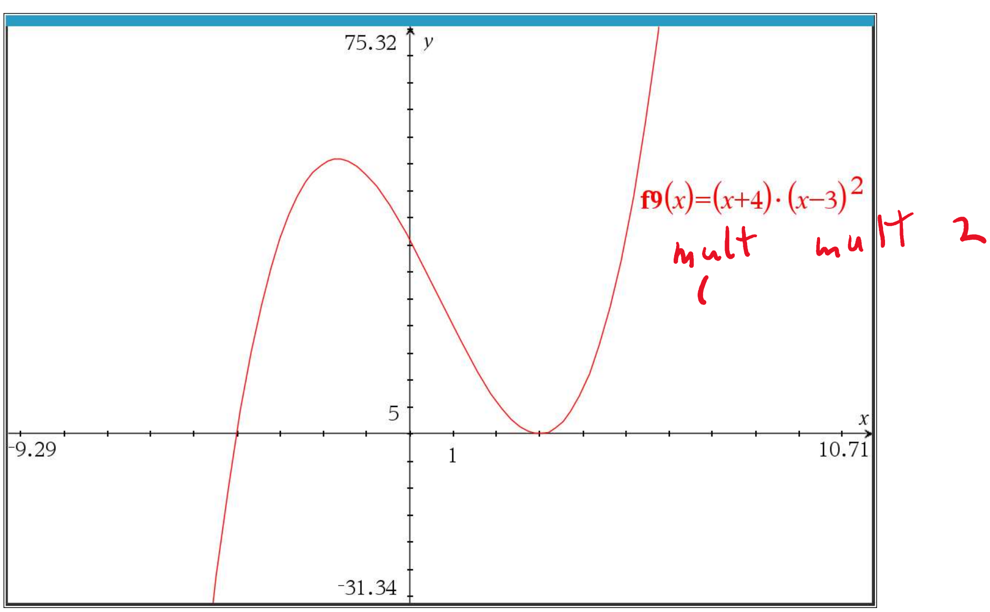
$$= (-)(+) < 0$$

$$f(0) = (0+4)(0-3)^2$$

$$= (+)(+) > 0$$

$$f(4) = (4+4)(4-3)^2$$

$$= (+)(+) > 0$$



memorize

Definition 3.3. Suppose f is a polynomial function and m is a natural number. If $(x-c)^m$ is a factor of $f(x)$ but $(x-c)^{m+1}$ is not, then we say $x=c$ is a zero of **multiplicity** m.

Memorize

Theorem 3.3. The Role of Multiplicity: Suppose f is a polynomial function and $x=c$ is a zero of multiplicity m.

- If m is even, the graph of $y=f(x)$ touches and rebounds from the x-axis at $(c,0)$.
- If m is odd, the graph of $y=f(x)$ crosses through the x-axis at $(c,0)$.