03-11-25 MTH 161-004N

2.4 Inequalities with Absolute Value and Quadratic Functions
  2.4.1 Exercises
  page 220: 1, 8, 17, 34, 36

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Exam 2, Thursday, 03/13/25
1.6-1.7, 2.1-2.4

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2.4:36

36. The height h in feet of a model rocket above the ground t seconds after lift-off is given by $h(t) = -5t^2 + 100t$, for $0 \le t \le 20$. When is the rocket at least 250 feet off the ground? Round your answer to two decimal places.

Find t such that h(t) is at least 250 feet off the ground.

solve $h(t) \geq 250$

$-5t^2 + 100t \geq 250$

$-5t^2 + 100t - 250 \geq 0$

$\Leftrightarrow \quad \frac{-5t^2}{-5} + \frac{100t}{-5} - \frac{250}{-5} \leq 0$

Let $f(t) = t^2 - 20t + 50 \leq 0$

solve $f(t) = 0$

$t = \frac{20 \pm \sqrt{400 - (4)(50)}}{2}$

$t = \frac{20 \pm \sqrt{400 - 200}}{2}$

$t = \frac{20 \pm \sqrt{200}}{2}$

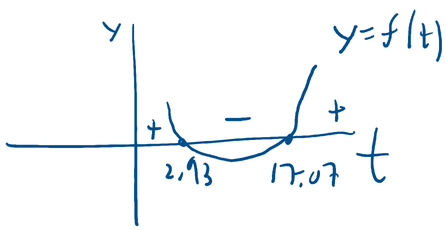
$t = \frac{20 \pm \sqrt{100 \cdot 2}}{2}$

$t = \frac{20 \pm 10\sqrt{2}}{2}$

$\boxed{t = 10 \pm 5\sqrt{2}}$

10-5*sqrt(2)=2.928932188134524 ≈ 2.93
10+5*sqrt(2)=17.07106781186548 ≈ 17.07

$f(t)$ is an upward opening parabola

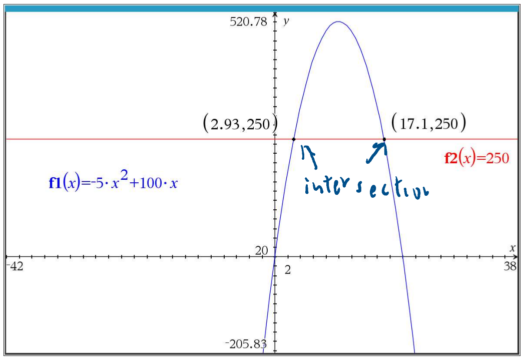


$f(t) \leq 0$ on $[2.93, 17.07]$

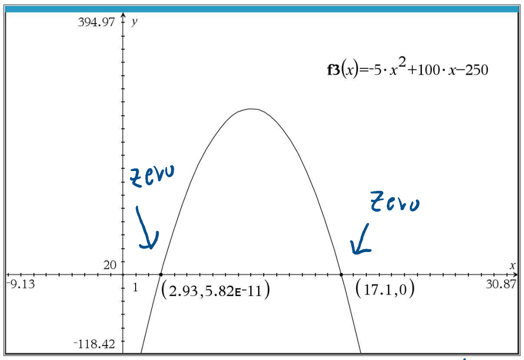
$\Leftrightarrow h(t) \geq 250$ on $[2.93, 17.07]$

The height of the rocket is at least 250 feet off the ground between 2.93 seconds and 17.07 seconds (including the end points) after lift-off.

Shorter method



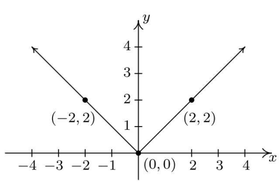
$2.93 \le t \le 17.1$



$5.82E^{-11} = 5.82 \times 10^{-11} \approx 0.0$

1.7

The complete graph of $y = f(x)$ is given below. In Exercises 19 - 27, use it and Theorem 1.7 to graph the given transformed function.



The graph for Ex. 19 - 27

24. $y = f(2x)$ $= g(x)$

Theorem 1.7. Transformations. Suppose f is a function. If $A \neq 0$ and $B \neq 0$, then to graph

$$g(x) = Af(Bx + H) + K$$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the left if $H > 0$ or right if $H < 0$.
2. Divide the x-coordinates of the points on the graph obtained in Step 1 by B. This results in a horizontal scaling, but may also include a reflection about the y-axis if $B < 0$.
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if $A < 0$.
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift up if $K > 0$ or down if $K < 0$.

$A = 1$ $H = 0$

$B = 2$ $k = 0$

$(-2, 2)$ $(0, 0)$ $(2, 2)$

(1) $(-2-0, 2)$ $(0-0, 0)$ $(2-0, 2)$

$(-2, 2)$ $(0, 0)$ $(2, 2)$

(1) $(-2-0, 2)$ $(0-0, 0)$ $(2-0, 2)$

$(-2, 2)$ $(0, 0)$ $(2, 2)$

(2) $(\frac{-2}{2}, 2)$ $(\frac{0}{2}, 0)$ $(\frac{2}{2}, 2)$

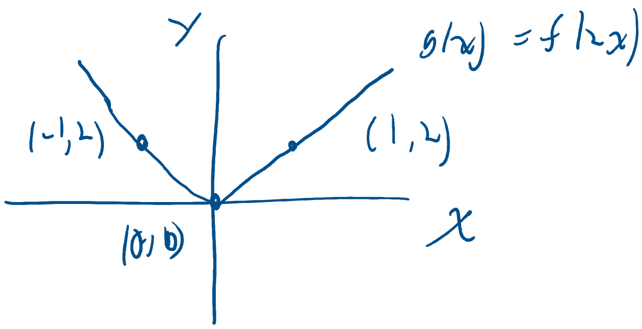
$(-1, 2)$ $(0, 0)$ $(1, 2)$

(3) $(-1, (2)(1))$ $(0, (0)(1))$ $(1, (2)(1))$

$(-1, 2)$ $(0, 0)$ $(1, 2)$

(4) $(-1, 2+0)$ $(0, 0+0)$ $(1, 2+0)$

$\boxed{(-1, 2) \quad (0, 0) \quad (1, 2)}$



After class notes

$\sqrt{4} = 2$

solve $x^2 = 4$

$x = \pm\sqrt{4}$

$\boxed{x = \pm 2}$

check $2^2 = 4$

$(-2)^2 = (-1)^2(2^2) = (1)(4) = 4$

2.3:

2.3.1 Exercises

In Exercises 1 - 9, graph the quadratic function. Find the x- and y-intercepts of each graph, if any exist. If it is given in general form, convert it into standard form; if it is given in standard form, convert it into general form. Find the domain and range of the function and list the intervals on which the function is increasing or decreasing. Identify the vertex and the axis of symmetry and determine whether the vertex yields a relative and absolute maximum or minimum.

4. $f(x) = -2(x+1)^2 + 4$ vertex form

$f(x) = -2(x-(-1))^2 + 4$

$f(x) = a(x-h)^2 + k$

$h = -1, \ k = 4$

convert to general form $f(x) = ax^2 + bx + c$

$f(x) = -2(x^2 + 2x + 1) + 4$

$f(x) = -2x^2 - 4x - 2 + 4$

$\boxed{f(x) = -2x^2 - 4x + 2}$ general form

$$\begin{array}{r} x+1 \\ \times \quad x+1 \\ \hline x+1 \\ x^2 + x \quad \\ \hline x^2 + 2x + 1 \end{array}$$

$(x+1)(x+1)$

$x^2 + 2x + 1$

y-intercept
set $x=0$, solve for y

$f(0) = -2(0^2) - 4(0) + 2 = 2$ or $(0, 2)$

x-intercept
set $y=0$, solve for x

$-2(x+1)^2 + 4 = 0$

$\frac{-2}{-2}(x+1)^2 + \frac{4}{-2} = \frac{0}{-2}$

$(x+1)^2 - 2 = 0$

$(x+1)^2 = 2$

$\sqrt{(x+1)^2} = \pm\sqrt{2}$

$x + 1 = \pm\sqrt{2}$

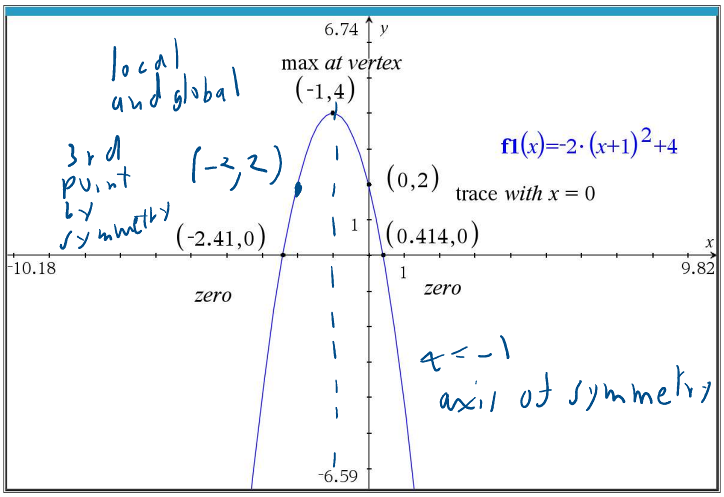
$\boxed{x = -1 \pm \sqrt{2}}$ exact answer

$x+1 = \pm\sqrt{2}$

$\boxed{x = -1 \pm \sqrt{2}}$ exact answer

-1-sqrt(2)=-2.414213562373095

-1+sqrt(2)=0.414213562373095



increasing on $(-\infty,-1)$

decreasing on $(-1,\infty)$

never constant

domain $=(-\infty,\infty)$

range $=(-\infty,4]$

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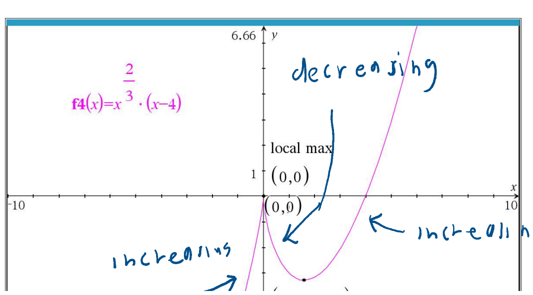
after class notes

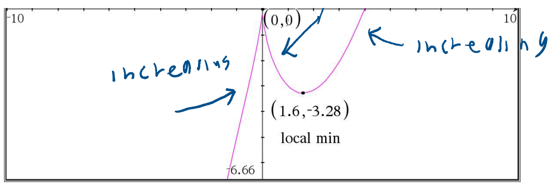
1.6: 75

In Exercises 74 - 77, use your graphing calculator to approximate the local and absolute extrema of the given function. Approximate the intervals on which the function is increasing and those on which it is decreasing. Round your answers to two decimal places.

75. $f(x) = x^{2/3}(x-4)$



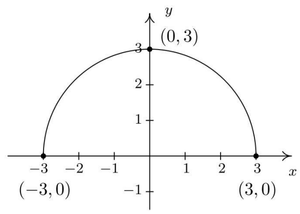
no global max or global min



increasing $(-\infty, 0)$, $(1.6, \infty)$
decreasing $(0, 1.6)$

1.7: 48

The complete graph of $y = f(x)$ is given below. In Exercises 38 - 49, use it and Theorem 1.7 to graph the given transformed function.



The graph for Ex. 38 - 49

48. $p(x) = 4 + f(1 - 2x)$

$A = 1$ $\quad H = 1$
$B = -2$ $\quad K = 4$

Theorem 1.7. Transformations. Suppose f is a function. If $A \neq 0$ and $B \neq 0$, then to graph

$$g(x) = Af(Bx + H) + K$$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the left if $H > 0$ or right if $H < 0$.
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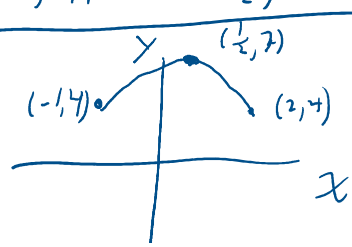
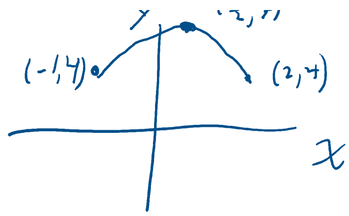
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if $A < 0$.
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift up if $K > 0$ or down if $K < 0$.

	$(-3, 0)$	$(0, 3)$	$(3, 0)$
1	$(-3-1, 0)$	$(0-1, 3)$	$(3-1, 0)$
	$(-4, 0)$	$(-1, 3)$	$(2, 0)$
2	$(\frac{-4}{-2}, 0)$	$(\frac{-1}{-2}, 3)$	$(\frac{2}{-2}, 0)$
	$(2, 0)$	$(\frac{1}{2}, 3)$	$(-1, 0)$
3	$(2, (0)(1))$	$(\frac{1}{2}, (3)(1))$	$(-1, (0)(1))$
	$(2, 0)$	$(\frac{1}{2}, 3)$	$(-1, 0)$
4	$(2, 0+4)$	$(\frac{1}{2}, 3+4)$	$(-1, 0+4)$
	$\boxed{(2, 4)}$	$\boxed{(\frac{1}{2}, 7)}$	$\boxed{(-1, 4)}$

transformed points





Possible essay question:

What is the quadratic formula used for?
We use the quadratic formula to solve quadratic equations.

I ask: solve $3x^2 - 10x + 24 = 0$

you think: Aha! I'll use the quadratic formula,

quadratic equation: $ax^2 + bx + c = 0$

quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2.1: 31

31. A plumber charges \$50 for a service call plus \$80 per hour. If she spends no longer than 8 hours a day at any one site, find a linear function that represents her total daily charges C (in dollars) as a function of time t (in hours) spent at any one given location.

linear function $f(x) = mx + b$, m = constant, b = constant

$$\boxed{c(t) = mt + b}$$

Find m, b

$$\boxed{c(t) = 80t + 50}$$

$0 \le t \le 8$

example: let $t = 3$ hr

$$c(3) = \frac{\$80}{\cancel{hr}}(3\cancel{hr}) + \$50$$

$$c(3) = \$240 + \$50$$

$$\boxed{c(3) = \$290}$$

For a 3 hour service call, the

plumber will charge $290.

2.2

In Exercises 22 - 33, graph the function. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

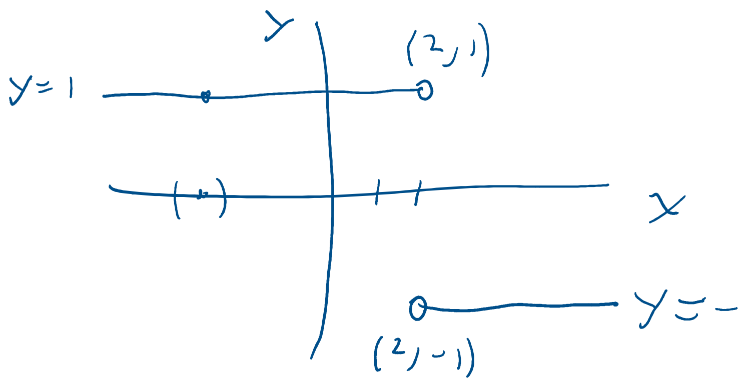
29. $f(x) = \dfrac{|2-x|}{2-x}$

$|2-x| = 2-x$ if $2-x \geq 0 \iff 2 \geq x$

$|2-x| = -2+x$ if $2-x < 0 \iff x > 2$

$$f(x) = \begin{cases} \dfrac{2-x}{2-x} & \text{if } x < 2 \quad (x \neq 2) \\ \dfrac{-(2-x)}{2-x} & \text{if } x > 2 \end{cases}$$

$$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ -1 & \text{if } x > 2 \\ \text{not defined} & \text{if } x = 2 \end{cases}$$



domain $= \{x \mid x \neq 2\} = (-\infty, 2) \cup (2, \infty)$

range $= \{-1, 1\}$

constant on $(-\infty, 2)$ and on $(2, \infty)$

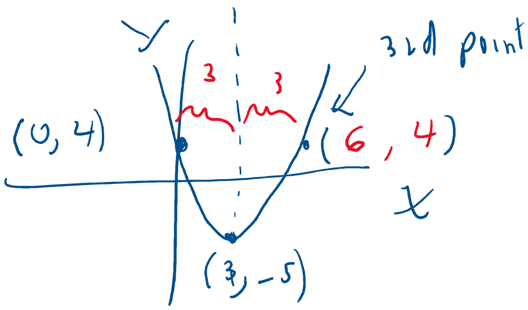
every $x \in$ domain is both a local min and a local max

absolute max $=1$ on $(-\infty, 2)$

every $x \in$ domain is both a local min and a local max

global max $= 1$ on $(-\infty, 2)$

global min $= -1$ on $(2, \infty)$



axis of symmetry
$x = 3$

2.4.1 Exercises

In Exercises 1 - 32, solve the inequality. Write your answer using interval notation.

17. $x^2 + 2x - 3 \geq 0$

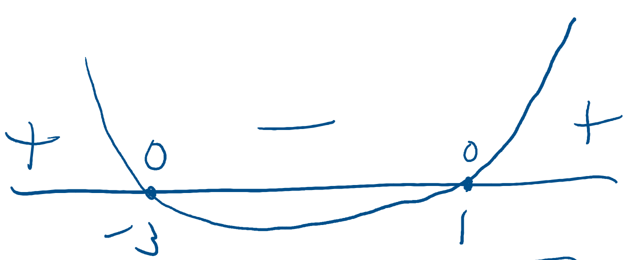
coefficient of $x^2 = 1 > 0$
$\therefore$ parabola opens up

solve $x^2 + 2x - 3 = 0$

$(x - 1)(x + 3) = 0$

$x - 1 = 0$ or $x + 3 = 0$

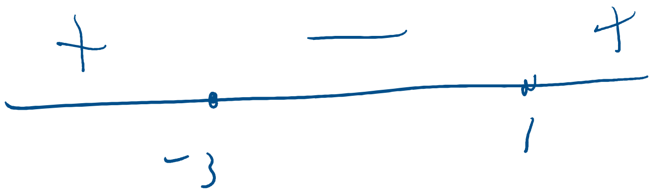
$\boxed{x = 1}$ $\boxed{x = -3}$



$\boxed{(-\infty, -3] \cup [1, \infty)}$ easiest method here

$\boxed{(-\infty, -3] \cup [1, \infty)}$

numeric test value



The zeros divide the number line into intervals $(-\infty, -3)$, $(-3, 1)$, $(1, \infty)$

Choose any convenient test value in each interval

Let $f(x) = x^2 + 2x - 3$

$f(-4) = (-4)^2 + 2(-4) - 3 = 16 - 8 - 3 = 5 > 0$

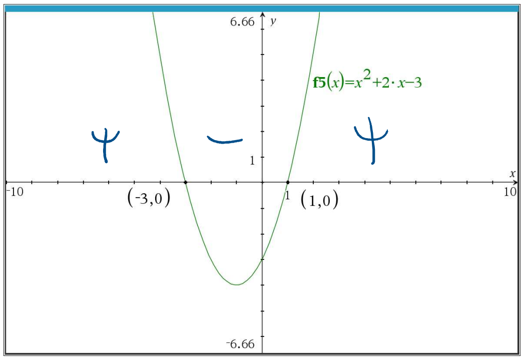
$f(0) = 0^2 + 2(0) - 3 = -3 < 0$

$f(2) = 2^2 + 2(2) - 3 = 4 + 4 - 3 = 5 > 0$

$\boxed{(-\infty, -3] \cup [1, \infty)}$

Instead of calculating test values, we can find the zeros manually, as we did above.
Then, graph the function on our calculator and check where the function is greater than or equal to zero.

calc



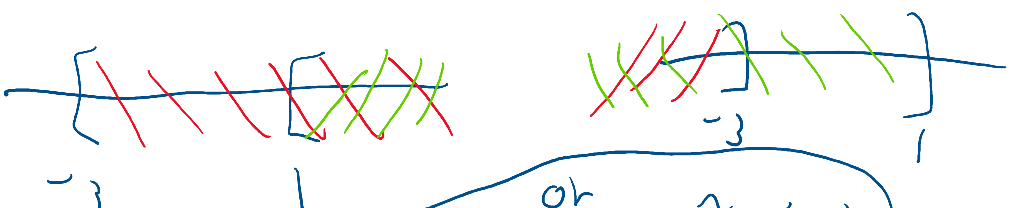
algebraic approach

$$x^2+2x-3=(x+3)(x-1)\geq 0$$

case 1 case 2

$$[x+3\geq 0 \text{ and } x-1\geq 0] \text{ or } [(x+3)\leq 0 \text{ and } (x-1)\leq 0]$$

$$[x\geq -3 \text{ and } x\geq 1) \text{ or } [x\leq -3 \text{ and } x\leq 1]$$



$x\geq 1$ or $x\leq -3$

$$\{x \mid x\geq 1 \text{ or } x\leq -3\}$$

$$=(-\infty,-3]\cup[1,\infty)$$

Solve $x^2+2x-3=0$

using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\overline{}$$
$$2a$$

$a=1$
$b=2$
$c=-3$

$$x=\frac{-2\pm\sqrt{2^2-(4)(1)(-3)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+12}}{2}$$

$$x=\frac{-2\pm\sqrt{16}}{2}$$

$$x=\frac{-2\pm\sqrt{4^2}}{2}$$

$$x=\frac{-2\pm 4}{2}$$

$$x=\frac{-2-4}{2},\ \frac{-2+4}{2}$$

$$x=\frac{-6}{2},\ \frac{2}{2}$$

$$\boxed{x=-3, 1}$$

---

solve $x^2+2x-3=0$ by completing the square

$$(x^2+2x)-3=0$$

$$\left(\frac{1}{2}\right)(2)=1$$
$$1^2=1$$

$$\left(\tfrac{2}{2}\right)\left(\tfrac{2}{2}\right) = 1$$

$$1^2 = 1$$

$$(x^2 + 2x + 1 - 1) - 3 = 0$$

$$(x^2 + 2x + 1) - 1 - 3 = 0$$

$$(x + 1)^2 - 4 = 0$$

$$(x + 1)^2 = 4$$

$$x + 1 = \pm 2$$

$$x = -1 \pm 2$$

$$x = -1 - 2, -1 + 2$$

$$\boxed{x = -3, 1}$$

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motivation for completing the square

$x^2 + 2x - 3$ want $(p + q)^2$ + constant

$$p^2 + 2qp + q^2$$

$x = p$ $x^2 = p^2$	2 is coefficient of x $2q$ = coefficient of p

$$\boxed{2 = 2q}$$

$$q = \frac{2}{2} = 1$$

$$q^2 = 1$$

$$(x^2 + 2x + 1 - 1) - 3$$

$$(x^2 + 2x + 1) - 4$$

$(x^2+2x+1)-4$

perfect

square